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STRONG RADIATIVE CORRECTIONS TO ANNIHILATIONS OF QUARKONIA IN QCD

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ABSTRACT

The significance of the lowest order QCD predictions for the annihilations of heavy quark-antiquark bound states is analyzed. The calculation of the leading strong radiative corrections to the hadronic versus electromagnetic annihilation rate ratio R of pseudo-scalar quarkonium is presented. In terms of the coupling constant α_s , as defined in the minimal subtraction scheme, we find

$$R = R^{(0)} (1 + 22.14 \alpha_s / \pi) .$$

The physical significance of this result is discussed by comparing it with the calculation of the non-leading effects in α_s on the scaling violations in deep inelastic scattering. A bad convergence of the relative perturbative expansion is found, demanding for its safe application a value of the relevant momentum definitely higher than that of charmonium physics.

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1. INTRODUCTION

In some limited domain of applications perturbative quantum chromodynamics (QCD) as a theory of strong interactions is currently subjected to quantitative tests. This kind of activity -- even though we are still faced with major physical problems whose solution is only supposedly contained in QCD -- clearly acquires an increasing importance: the effort to transform important but rather qualitative evidence for the theory into quantitative tests is a highly desirable aim, which it becomes more and more worth while to pursue.

It is within this prospective that we address our attention in this paper to the following application of perturbative QCD; the use of asymptotic freedom to explain qualitatively the Zweig rule, in particular to get semi-quantitative predictions for the widths of heavy quark-antiquark bound states^{1,2)}.

In general terms the underlying physical picture can be summarized as follows. A heavy quark-antiquark bound state is characterized by two distances: a) its mean radius r_c of the order of 1 fermi, a distance scale at which the effective coupling strength for the binding has become large; b) the Compton wavelength of the constituents or their inverse mass $1/m$. Now, to the extent that $1/m$ is much smaller than r_c , it makes sense to think that the binding effects can be factored out from the annihilation processes, which are essentially local relatively to r_c . The computation of the decay matrix elements is then done in analogy with the QCD perturbation computation of $\sigma_{\text{tot}}(e^+e^- \rightarrow \text{hadrons})$, when no attempt is made at giving a realistic description of the final states. In leading order one then describes, for example, the $1^{--} Q\bar{Q}$ -bound states as the quark-antiquark annihilation at rest into three on-mass-shell gluons.

Of course a necessary condition for the applicability of perturbation theory is that no large dynamical factor enters in higher orders to make the expansion break down. In view of this, one would like to perform a general analysis of the problem with the purpose of controlling, for example, the effects of threshold singularities or of eventual infrared singularities connected with the zero binding limit, by a suitable separation of binding and annihilation effects. This is not what we attempt to do in this work. Here we take in fact the more modest and more pragmatical attitude of explicitly computing the corrections to some of the relevant annihilation processes, with the twofold purpose of making a first step in the above-mentioned direction, but also -- and more importantly for us -- of getting the actual size of these corrections.

At present the comparison²⁾ between the leading order predictions and the experimental information is controversial. In fact, within the ψ family, the apparent success of calculated decay rate ratios for the $3P$ states³⁾ between themselves and for ψ' versus ψ is to be contrasted: a) with the failure in the

prediction for the hadronic versus electromagnetic annihilation rates of η_c (the theoretical ratio is at least an order of magnitude larger than the experimental one); and b) with the need of a value for the strong coupling constant α_s which is about twice as small as the one deduced from analyses of deep inelastic scattering, in order to get the analogous rate ratio for ψ (here too, if using a "normal" value of α_s , the theory overestimates the hadronic decay by essentially an order of magnitude). In turn for the γ , the existing evidence⁴⁾ for three jets in the final states^{*)} (suggestive of the three-gluon decay mechanism) contrasts with the fact that the hadronic width may turn out to be larger than what one would obtain by scaling the ψ width down to the γ mass according to the standard asymptotic freedom formulae^{**)}.

With all this in mind we report in this work the results of the calculation of the leading strong radiative corrections to the hadronic versus electromagnetic annihilation rate ratio of pseudoscalar quarkonium. Such a calculation should be seen as a first step in exploring the significance of the theoretical model we have been talking about, beyond its lowest order results. In general it should be kept in mind that at present there is almost no experience in these kinds of calculations: the only computed effect up to non-leading order in α_s is that of the anomalous dimensions of the local operators of lowest twist⁶⁾ relevant, for example, in the prediction of scaling violations in deep inelastic scattering. In particular the rate of convergence of these expansions -- which in turn determines the minimum momentum where perturbation theory is safely applicable -- is essentially unknown.

The paper is organized as follows. In Section 2 the problem of the separation of binding and annihilation effects is dealt with to the order in question. The solution which emerges naturally from perturbation theory up to non-leading order in α_s is not, however, straightforwardly extendable to any order; we have nothing to say for this last problem. In Section 3 the results of the calculation are described in some detail, showing the cancellation of all infrared and mass singularities. Finally, in Section 4, we discuss the physical interpretation of the results obtained by connecting them with the mentioned prediction of the scaling violations in deep inelastic scattering. We are faced with a bad convergence of the perturbative expansion; its reliable application at a quantitative level demands a value of the relevant momentum definitely greater than the 3 GeV of charmonium physics.

*) One should better say negative evidence for two jets or normal phase-space distributions.

***) The presently preferred value of about 50 keV⁵⁾ is more than two times larger than the value scaled from the ψ width.

2. STATEMENT OF THE PROBLEM

As briefly explained in the Introduction, the lowest order result for the hadronic versus electromagnetic annihilation rates of pseudoscalar quarkonium is obtained by taking the ratio of the annihilation cross-sections of an on-shell quark-antiquark pair at rest -- after the extraction of the flux factor proportional to the relative velocity v -- in a colour singlet state and spin $S = 0$ into two gluons or two photons, respectively, in the Born approximation (see the diagrams of Fig. 1)

$$v \sigma^{(0)}(Q\bar{Q} \rightarrow 2 \text{ gluons}) = \frac{8\pi}{g^2 m^2} \alpha_s^2 \quad (1)$$

$$v \sigma^{(0)}(Q\bar{Q} \rightarrow 2 \text{ photons}) = \frac{4\pi}{m^2} e_Q^4 \alpha^2$$

where e_Q is the electric charge of the constituent quark in units of electron charge.

The order α_s corrections to the photonic annihilation cross-section are readily obtained from the known electromagnetic corrections to this process⁷⁾ -- relevant in positronium physics -- via the trivial replacement $\alpha \rightarrow (\frac{4}{3})\alpha_s$. The result is

$$\sigma^{(1)}(Q\bar{Q} \rightarrow 2 \gamma) = \sigma^{(0)}(Q\bar{Q} \rightarrow 2 \gamma) \times \left[1 + \frac{4}{3} \frac{\alpha_s}{\pi} \left(\frac{\pi^2}{4v} - 5 + \frac{\pi^2}{4} \right) \right] \quad (2)$$

where we show explicitly the divergence for zero relative velocity v of the quarks, contained in the diagram with the gluon exchanged between the incoming fermion legs (Fig. 2a). Such a divergence is there to remind us of the binding that actually occurs between the quark-antiquark pair; in a bound state calculation it has to be absorbed in the wave function.

Let us turn now to the "hadronic" annihilation cross-section. As a result of our calculation, for a $J^{PC} = 0^{-+}$ $Q\bar{Q}$ -state we get up to non-leading order in α_s

$$\begin{aligned} \sigma^{(1)}(Q\bar{Q} \rightarrow \text{"had."}) &\equiv \\ &\equiv \sigma^{(1)}(Q\bar{Q} \rightarrow 2 \text{ gl}) + \sigma^{(1)}(Q\bar{Q} \rightarrow 3 \text{ gl}) + \\ &+ \sum_l \sigma^{(1)}(Q\bar{Q} \rightarrow 1 \text{ gl} + q_l \bar{q}_l) = \sigma^{(0)}(Q\bar{Q} \rightarrow 2 \text{ gl}) \left[1 + \frac{\alpha_s}{\pi} \left(\frac{\pi^2}{3v} + \frac{1}{2} \right) \right] \end{aligned} \quad (3)$$

where again we have the same $v \rightarrow 0$ divergence as before and f is a finite number which obviously depends on the definition of the renormalized coupling constant $\alpha_s \rightarrow q_s$ stands for any quark lighter than Q . We then have

$$\frac{\sigma^{(1)}(Q\bar{Q} \rightarrow \text{"had"})}{\sigma^{(1)}(Q\bar{Q} \rightarrow 2\gamma)} = \frac{\sigma^{(0)}(Q\bar{Q} \rightarrow \text{"had"})}{\sigma^{(0)}(Q\bar{Q} \rightarrow 2\gamma)} \left[1 + \frac{\alpha_s}{\pi} \left(\frac{1}{2} - \frac{\pi^2}{3} + \frac{20}{3} \right) \right] \quad (4)$$

with a correction factor which is finite as $v \rightarrow 0$. We take this same factor as the correction to the ratio of the decay widths too.

The analogous statement in pure QED, concerning for example the ratio between the annihilation widths of orto- and para-positronium, could be an exact statement, in the sense that the neglected terms are of higher order in α . This is so because in QED the binding too is essentially a perturbative phenomenon and therefore the dependence on α of the individual widths is in principle calculable to any accuracy; in particular the calculability of the mean velocity of the constituents $v \sim \alpha$ allows the controlling of relativistic binding effects and say that they do not enter up to the order in question. The same is obviously not true in QCD: the intuition that we have developed on the dynamics of the heavy quark-antiquark bound states tells us that their widths will be controlled by two small dimensionless numbers α_s and $1/mr_c$, not connected in a simple way. What we are trying to do here is an expansion in α_s including non-leading effects, but neglecting higher orders in $1/mr_c$, which we do not know at the moment how to control. Needless to say it is crucial, for this program to be successful, that we calculate the corrections to a ratio of decay rates, since it is only there that the $1/v$ divergences displayed in Eqs. (2) and (3) for the individual annihilation cross-sections drop out; in other words we have nothing to say for the strong radiative corrections to the individual decay widths and in turn for the non-relativistic wave function at the origin, to which the widths are proportional. This obviously requires a deeper understanding of the binding problem.

3. DESCRIPTION OF THE CALCULATION

The diagrams whose imaginary parts need be computed to get $\sigma^{(1)}(Q\bar{Q} \rightarrow \text{"had"})$ as defined by Eq. (3) are shown in Fig. 2. In spite of the fact that they are already quite a number, many other contributing diagrams are not shown since, for the particular kinematical situation we are interested in, their contributions are taken into account simply by giving an appropriate multiplicity factor to the individual graphs of Fig. 2 ^{*)}. Since we study the decays of a $Q\bar{Q}$ -bound state, the

^{*)} In particular the crossing of the gluon legs in the two-gluon cut graphs amounts to a factor of 2.

quark lines in Fig. 2 are Q-lines everywhere except in the loop of Fig. 2j which is meant to represent any kind of quark line (i.e. a sum of diagrams differing by the mass of the quark loop). As we stated already in the previous section, the imaginary parts of the diagrams shown receive contributions from two-gluon cuts, three-gluon cuts and a "light" quark-antiquark pair plus one-gluon cuts, corresponding respectively to $\sigma^{(1)}(Q\bar{Q} \rightarrow 2 \text{ gl})$, $\sigma^{(1)}(Q\bar{Q} \rightarrow 3 \text{ gl})$ and $\sigma^{(1)}(Q\bar{Q} \rightarrow q_e \bar{q}_e + 1 \text{ gl})$ of Eq. (3).

Without entering a technical description of the calculation, we merely state the different sources of ambiguities which arise in the intermediate steps as well as the way we resolve them. This serves also to define some of the intermediate results that we will give; the final physical result will be free of any ambiguity.

- a) Gauge invariance⁸⁾: the contributions of the individual diagram are calculated in the Feynman gauge, which means that we have to add a ghost contribution both to $\sigma^{(1)}(Q\bar{Q} \rightarrow 2 \text{ gl})$ and $\sigma^{(1)}(Q\bar{Q} \rightarrow 3 \text{ gl})$ through the diagram of Fig. 2i".
- b) Ultraviolet regularization: the diagrams with vertex or propagator insertions are regularized in the ultraviolet by going to n dimensions and introducing a scale μ through the standard replacement of the bare coupling constant $g_0 \rightarrow g_0 \mu^{(n-4)/2}$. In the following we will give the results for the regularized, unrenormalized diagrams, which will then show a pole $1/(n-4)$ and a finite part. To be precise the finite part will not contain a piece coming from the interference of the pole term in the loop insertion with a term vanishing like $(n-4)$ in the skeleton diagram; it is in this way that a consistent renormalization procedure -- the so-called "minimal renormalization scheme" -- is obtained by simply dropping the pole terms in the over-all result^{*)}.
- c) Infrared regularization: severe infrared divergences show up in the intermediate steps of the calculation, especially in the separation of the different cuts. Such divergences are regulated by giving to the gluons a small mass λ , very much in the same way as in ordinary QED calculations. Some finite contributions arising from integrands with a factor λ^2 in the numerator appear in the intermediate steps, but they cancel at the very end. As a matter of fact it may be interesting to know that for each of the pure QCD

*) To be precise the two-gluon cut diagrams $G(n)$ with a divergent loop insertion are calculated in n dimensions and written as $G(n) = G_0(n)f(n)$, where $G_0(n)$ is the skeleton diagram in n dimensions too. The factor $f(n)$ for $n \approx 4$ shows up a pole, $f(n) \approx [A/(n-4)] + B$. The unrenormalized contribution is obtained by writing $G_{\text{unr}}(n) \approx G_0(4)[A/(n-4)] + B$.

diagrams shown in Fig. 2 the full imaginary parts are finite in the infrared in spite of the fact that the individual cuts are not (see below for the details). Of course the cancellation of the infrared divergences for the total cross-section substantiates the physical significance of the calculation.

We finally come to the presentation of the results. The over-all result for the unrenormalized quantity obtained by summing up the different partial contributions (see below), is

$$\frac{\sigma^{(1)}(Q\bar{Q} \rightarrow \text{"had"})}{\sigma^{(0)}(Q\bar{Q} \rightarrow \text{"had"})} =$$

$$= 1 + \frac{g_0^2}{4\pi^2} \left[\left(\frac{\pi^2}{3\nu} + \frac{\pi^2}{3} - \frac{20}{3} \right) + \left(\frac{25}{6} P - \frac{2}{3} \ln 2 - \frac{13}{8} \pi^2 + \frac{61}{2} \right) \right] \quad (5)$$

where $P = [2/(4 - n)] - \gamma + \ln 4\pi + 2 \ln (u/2m)$, and $\gamma = 0.577$ is the Euler constant. Since we have in mind the η_c , hereafter we take three quark flavours lighter than Q , and their zero mass limit is considered. In view of possible applications to heavier quark bound states (for example, the B_c of the T family), the result is easily modified by means of the more detailed results given in Table 1.

To compute the strong radiative corrections to the photonic decay process, the relevant diagrams are those of Fig. 2a-d with the two gluons in the final states replaced by two photons; as anticipated in the Introduction, they reproduce the old QED result⁷⁾, apart from a trivial replacement of coupling constant, so that

$$\frac{\sigma^{(1)}(Q\bar{Q} \rightarrow 2\gamma)}{\sigma^{(0)}(Q\bar{Q} \rightarrow 2\gamma)} = 1 + \frac{g_0^2}{4\pi^2} \left(\frac{\pi^2}{3\nu} + \frac{\pi^2}{3} - \frac{20}{3} \right) \quad (6)$$

From Eqs. (5) and (6), our main result is obtained

$$\frac{\sigma^{(1)}(Q\bar{Q} \rightarrow \text{"had"})}{\sigma^{(1)}(Q\bar{Q} \rightarrow 2\gamma)} =$$

$$= \frac{\sigma^{(0)}(Q\bar{Q} \rightarrow \text{"had"})}{\sigma^{(1)}(Q\bar{Q} \rightarrow 2\gamma)} \left[1 + \frac{g_0^2}{4\pi^2} \left(\frac{25}{6} P - \frac{2}{3} \ln 2 - \frac{13}{8} \pi^2 + \frac{61}{2} \right) \right] \quad (7)$$

We give in Table 1 the separate contributions of the single-cut graphs to the above equations. From such contributions the different cross-sections $\sigma^{(1)}(Q\bar{Q} \rightarrow 2 \text{ gluons})$, $\sigma^{(1)}(Q\bar{Q} \rightarrow 3 \text{ gluons})$ and $\sigma^{(1)}(Q\bar{Q} \rightarrow q\bar{q} + 1 \text{ gluon})$ are easily worked out: the cancellation of infrared and mass singularities which takes place in summing up the three contributions is worth noticing.

4. DISCUSSION OF THE RESULTS AND CONCLUSIONS

To convert the result of Eq. (7) into a physically sensible statement, the renormalization has to be performed, which requires the replacement of the bare coupling g_0 with its expression $g_0(g, n)$ in terms of the renormalized coupling g , so that the pole $1/(n-4)$ be removed.

Defining, for example, g according to the minimal subtraction scheme, i.e.

$$g \simeq g_0 \left(1 + \frac{g_0^2}{16\pi^2} \frac{25}{3} \frac{1}{n-4} \right) \quad (8)$$

the following renormalized result is obtained*) ($\alpha_s \equiv g^2/4\pi$)

$$\frac{\Gamma(Q\bar{Q} \rightarrow \text{"had"})}{\Gamma(Q\bar{Q} \rightarrow 2\gamma)} \equiv R = \frac{2}{g e_q^4} \left(\frac{\alpha_s}{\alpha} \right)^2 \times \quad (9)$$

$$\left[1 + \frac{\alpha_s}{\pi} \left(\frac{25}{6} \left(\ln 4\pi - \gamma + 2 \ln \frac{\mu}{2m} \right) - \frac{2}{3} \ln 2 - \frac{13}{8} \pi^2 + \frac{61}{2} \right) \right]$$

Numerically, taking μ equal to the natural scale of the problem $\mu = 2m$, we get

$$R \equiv R_0 \left(1 + \frac{\alpha_s}{\pi} r_1 \right) = \frac{2}{g e_q^4} \left(\frac{\alpha_s(2m)}{\alpha} \right)^2 \left(1 + 22.14 \frac{\alpha_s(2m)}{\pi} \right) \quad (10)$$

The unusually large size of the coefficient of the correction term is apparent. Such number, however, is not very meaningful in itself, since it obviously depends on the definition of the renormalized coupling α_s , which in turn is not mandatory. This is especially so because, with "normal" values of $\alpha_s(2m)$, the correction in Eq. (10) is overwhelmingly large. What is meaningful in absolute terms is the comparison of the above prediction for R with an independent prediction developed to the same non-leading order in α_s .

Let us consider then to this purpose the already mentioned Q^2 dependence of the moments of the deep inelastic structure functions. Making reference to the original literature for the details⁹⁾, we recall the following basic equation holding for the moments $M^n(Q^2)$ of the vector-axial vector interference structure function $F_3(x, Q^2)$ measured in deep inelastic $\nu N, \bar{\nu} N$ scattering

$$L^n(Q) = Q \frac{d}{dQ} \ln M^n(Q^2) \simeq - L_0^n \frac{\alpha_s}{\pi} \left[1 + \frac{\alpha_s}{\pi} \left(l_1^n - \frac{1}{2} \left(11 - \frac{2}{3} \beta \right) \ln \frac{Q}{\mu} \right) \right] \quad (11)$$

*) Hereafter, as explained in the Introduction, we replace the ratio of the cross-sections with the ratio of the widths.

Here again -- as the non-leading correction r_1 to the lowest order ratio R -- the coefficients ℓ_1^n depend on the definition of α_s ; what does not depend on such definition is the combination $(r_1 - 2\ell_1^n)$, since it appears in the expansion of the physical quantity R versus another physical quantity $L^n(Q)/L_0^n \approx \alpha_s(Q)/\pi$, obtained by eliminating the coupling constant between Eqs. (9) and (11)

$$R \approx \frac{2\pi^2}{9\ell_q^4 d^2} \left(\frac{L^n(2m)}{L_0^n} \right)^2 \left[1 - \frac{L^n(2m)}{L_0^n} (r_1 - 2\ell_1^n) \right] \quad (12)$$

The coefficients $(r_1 - 2\ell_1^n)$ are given for the first few moments in Table 2. Notice that -- if the scaling violations are indeed well described by Eq. (11)* -- the right-hand side of Eq. (12) is actually independent from n , up to corrections of relative order $[L^n(2m)/L_0^n]^2$; the same applies to R .

For the sake of the present discussion, it is enough to know for the first few moments ($n = 3, 4, 5$), that¹⁰⁾ **)

$$-L^n(Q \approx 2.5 \text{ GeV})/L_0^n \approx 0.15 \div 0.20$$

It is apparent that, for a charmed quark mass of about $m = \frac{1}{2}(2.5-3) \text{ GeV}$, we get a *positive* correction to the lowest order result of *overwhelmingly large size*, actually larger than the leading order. This puts in serious trouble the significance of the expansion***). Demanding that the correction in Eq. (12) be less than 50% requires m to be such that $-L^n(2m)/L_0^n \approx \alpha_s(2m)/\pi \lesssim 0.07$: the T mass is at the border of the acceptable region.

These are the definite conclusions of our calculation. One can ask now if there is any more general lesson that we can learn from it. We can envisage the following sources -- not independent -- for the large size of the correction that we have obtained:

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- *) This seems to emerge from the comparison with the experimental data¹⁰⁾.
 - **) A value of about 0.2 is obtained from the bubble chamber experiments (see Bosetti et al.), whereas a somewhat lower value is got by extrapolating to $Q \approx 2.5 \text{ GeV}$ the results of the CDHS collaboration (see de Groot et al.).
 - ***) In Ref. 9 the emphasis was put on the difference between the coefficient of the correction (22.14) in terms of the coupling constant defined as in the minimal subtraction scheme and the average size of $(r_1 - 2\ell_1^n) \approx 6-7$. This allowed us to say that also from the charmonium calculation the unnaturalness of such a coupling constant was emerging. It has been pointed out there how such a choice could lead to misleading conclusions. It is true, on the other hand, that 6-7 is not in the present context a small number, and this has an absolute meaning, as explained at length.

- a) The large number of diagrams due to the number of possible different interactions. What operates here is also the constancy of the sign for all the pure QCD diagrams (see Table 1), which may very well be an accident.
- b) The number of degrees of freedom. Notice that the correction r_1 is proportional to the Casimir invariant $C_A = N$ for $SU(N)$. (The part of the correction proportional to $C_F = (N^2 - 1)/2N$ drops out when taking the ratio of hadronic versus electromagnetic decay.)
- c) The fact that we have computed a quantity which starts from α_s^2 , which partly brings about the large number of diagrams and partly causes a higher instability of the expansion, because of the stronger dependence of the corrections from the ultraviolet cut-off.

In view of all this, we feel allowed to make some *speculative* considerations. We would expect the expansion of the hadronic versus the leptonic width of ψ to be even more unstable than the one we have done for the pseudoscalar, since it starts from α_s^3 . On the contrary, one can perhaps attach physical significance to the other predictions of the model within the ψ family, which start from zeroth or first order in α_s ; this can explain the mentioned successes of this theoretical picture, whereas in general -- at the charmonium level -- the suppression of the hadronic decay modes shows up only as a qualitative feature of asymptotic freedom.

As already stated, the T family should be at the border of the allowed region for the applicability of perturbation theory. Precisely in view of the applications to the T family it is interesting to ask the following question, which is raised by the importance of the radiative corrections that we find: At which mass can one safely say that quarkonium decay is dominated by the minimum number of gluon jets as suggested by lowest order perturbation theory? We hope to be able to answer this question for pseudoscalar quarkonium in a future publication.

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Table 1

Partial contributions to $\sigma^{(1)}(Q\bar{Q} \rightarrow \text{"had"})$ from individual diagrams of Fig. 2
(q = quark, gl = gluon, gh = ghost)

Diagram*)	Cut particles	Contribution to imaginary part**)	Numerical value ***)
a)	2 gl	$C_F \left[\frac{1}{4v} \pi^2 + 2 \ln \frac{\lambda}{2m} + 2 \ln 2 - 2 \right]$	-0.61
b)	2 gl	$C_F \left[-\frac{1}{2} P + 3 \ln 2 - 1 \right]$	1.08
c)	2 gl	$C_F \left[-\frac{1}{2} P - 2 \ln \frac{\lambda}{2m} - 3 \ln 2 - 2 \right]$	-4.08
d)	2 gl	$\left(C_F - \frac{1}{2} C_A \right) \left[P - 2 \ln 2 + \frac{1}{4} \pi^2 \right]$	1.08
e)	2 gl	$C_A \left[\frac{3}{2} P - \mathcal{L} - \ln 2 + 3 - \frac{1}{18} \pi^2 \right]$	0.98
	3 gl	$C_A \left[+ \mathcal{L} - \frac{1}{2} - \frac{1}{36} \pi^2 \right]$	
f)	2 gl	$C_A \left[+ \mathcal{L} - 2 \ln 2 - \frac{5}{72} \pi^2 \right]$	0.46
	3 gl	$C_A \left[- \mathcal{L} - \frac{7}{2} + \frac{11}{18} \pi^2 \right]$	
g)	2 gl	$C_A \left[- 2 \mathcal{L} - 2 \ln 2 + \frac{7}{18} \pi^2 \right]$	1.81
	3 gl	$C_A \left[+ 2 \mathcal{L} + 7 - \frac{19}{18} \pi^2 \right]$	
h)	2 gl	0	0
i + i' + i'')	2 gl	$C_A \left[\frac{5}{6} P - \frac{5}{3} \ln \frac{\lambda}{2m} - \frac{5}{18} + \frac{\pi}{2\sqrt{3}} \right]$	2.56
	3 gl + 1 gl, gh, $\overline{\text{gh}}$	$C_A \left[+ \frac{5}{3} \ln \frac{\lambda}{2m} + \frac{17}{6} - \frac{\pi}{2\sqrt{3}} \right]$	
j****)	2 gl	$C_F \left[-\frac{f}{4} P + \frac{1}{2} \sum_{i=1}^{f-1} \ln \frac{m_i}{2m} - \frac{1}{2} \ln 2 \right]$	-2.35
	1 gl, $q_i \bar{q}_i$	$C_F \left[-\frac{1}{2} \sum_{i=1}^{f-1} \ln \frac{m_i}{2m} - \frac{2}{3} (f-1) \right]$	
k + k' + k'' + k''')	3 gl	$C_A \left[+ \frac{5}{2} - \frac{5}{24} \pi^2 \right]$	0.44

Here $P = \frac{2}{4-n} + \ln 4\pi - \gamma + 2 \ln \frac{\mu}{2m}$, $\mathcal{L} = \ln^2 \frac{\lambda}{2m} + \ln \frac{\lambda}{2m} + \frac{\pi}{2\sqrt{3}}$

*) The proper multiplicity factor is included.

**) Normalized to $8\pi/3 (g_0^2/4\pi)^2 \cdot (g_0^2/4\pi^2)$; $C_F = (N^2 - 1)/2N$ and $C_A = N$ for SU(N).

***) Numerical value of the finite part within the parenthesis.

****) f stands for the total number of quarks, including Q.

Table 2

Numerical values of the coefficients $(r_1 - 2\ell_1^n)$
as defined in the text

n	2	3	4	5	6
$r_1 - 2\ell_1^n$ *)	11.08	8.22	6.80	5.86	5.14

*) In the ℓ_1^n 's we have taken the number of relevant
flavours equal to 4. Excluding charm amounts to
a reduction of $(r_1 - 2\ell_1^n)$ by about 10%.

Figure captions

Fig. 1 : Unitarity diagrams for $\sigma^{(0)}(Q\bar{Q} \rightarrow 2 \text{ gluons})$.

Fig. 2 : Representative diagrams whose imaginary parts contribute to $\sigma^{(1)}(Q\bar{Q} \rightarrow \text{"hadrons"})$.

Fig. 1

Fig. 2