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JETS IN NEUTRINO DEEP INELASTIC SCATTERING

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A B S T R A C T

Jet cross-sections are calculated to the first order in QCD for neutrino production. We give thrust and sphericity distributions and calculate angular correlations between jet and leptonic planes.

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1. INTRODUCTION

The QCD approach to strong interactions gives a theoretical meaning to the parton model and the concept of jets. But besides the two-quark jet picture, the QCD perturbation theory implies the existence of three or more jet events due to gluon bremsstrahlung. However, quarks and gluons are so far unobserved and prefer to convert into usual hadrons through non-perturbative mechanisms; this makes quantitative perturbative QCD results difficult to compare directly with data. A great amount of work has been devoted to the study of QCD jets and mainly to e^+e^- annihilation into hadrons. Indeed this is the cleanest place to study QCD effects: the non-perturbative mechanisms only occur in the final state (and an inclusive treatment gets rid of them), the lab. frame is also the rest frame of the final state where jets are well separated and the only energy scale in the problem is the accurately measured "virtual photon mass" Q . When dealing with deep inelastic processes, things get more complicated for at least three reasons: First, non-perturbative effects play a role in both the initial and final states; second, the hadronic energy of the final state W is an additional independent variable of the problem; and lastly, the jet rest frame can be reached by a boost along $\vec{q}$ and this calls for accurate measurements of Q and W which are not achieved experimentally.

In this paper we present the jet predictions of QCD, in the first order of perturbation theory, in neutrino production. As is well known, in addition to the usual two-jet behaviour (see Fig. 1), a three-jet structure is expected in this order from the diagrams of Fig. 2. In Section 2 we very briefly define variables and recall kinematic details. In Section 3, we discuss differential cross-sections in terms of variables which were introduced to get rid of infrared sensitivity from QCD calculated observables, namely: the thrust or maximum directed momentum [1]

$$T = 2 \max \frac{\sum \tilde{p}_L^i}{\sum |\vec{p}_i|} \quad (1.1)$$

where $\sum$ denotes the sum over all particles in the final state and $\tilde{\sum}$ the sum over all particles in one hemisphere. The momenta p_L^i are parallel to a jet axis, normal to a plane defining the hemispheres and chosen to maximize T ; the sphericity [2]

$$S = \left(\frac{4}{\pi}\right)^2 \min \left(\frac{\sum |\vec{p}_T^i|}{\sum |\vec{p}_i|} \right)^2 \quad (1.2)$$

where the p_T 's are transverse to a jet axis chosen to minimize S .

Following the work of De Rújula, Ellis, Floratos and Gaillard [3] in e^+e^- annihilation, we calculate $d\sigma/dT$, $\langle S \rangle$ and $\langle 1 - T \rangle$ as given by the first order in QCD and give some evaluation of the non-perturbative effects, which appear to be rather large.

In Section 4, we present results on angular asymmetries. Georgi and Politzer [4] have proposed to study within QCD the angular correlations between the lepton plane and an outgoing hadron. So far, this has been done only in the context of semi-inclusive processes [4-6]. We study these correlations in inclusive processes, in terms of jet cross-sections.

Finally, in Section 5, we briefly give our conclusions.

2. KINEMATICS

In leptonproduction, the parton cross-section in the first order of perturbation theory can be expressed in terms of the usual kinematical variables:

$$Q^2 = -q^2, \quad y = \frac{p_1 \cdot q}{p_1 \cdot k_1}, \quad z = \frac{p_1 \cdot p_3}{p_1 \cdot q}, \quad x = \frac{Q^2}{2p_1 \cdot q}, \quad \phi$$

where p_1 , p_3 , k_1 , and q are defined in Fig. 2 and ϕ is the angle between $\vec{k}_{1T}$ (transverse with respect to $\vec{q}$) and $\vec{p}_3$ (see Fig. 3).

The differential parton cross-section is to first order in α_s [5]

$$\frac{d\sigma^i}{dx dy dz} = \alpha_s(Q^2) \frac{G_F^2 Q^2}{4\pi^3 y} (L_i + M_i \cos \phi + N_i \cos 2\phi) \quad (2.1)$$

where the index i refers to the nature of incoming parton (quark, antiquark, gluon; see Fig. 2) and the coefficients L_i , M_i , N_i are, in the case of neutrino scattering:

$$L_q^{(\nu)} = \frac{4}{3} \left\{ \frac{1+x^2z^2}{(1-x)(1-z)} + 4xz(1-y) + (1-x)(1-z)(1-y)^2 \right\} \quad (2.2a)$$

$$L_{\bar{q}}^{(\nu)} = \frac{4}{3} \left\{ (1-x)(1-z) + 4xz(1-y) + \frac{1+x^2z^2}{(1-x)(1-z)} (1-y)^2 \right\} \quad (2.2b)$$

$$L_g^{(\nu)} = \frac{1}{2} \left\{ [x^2 + (1-x)^2] \left[\frac{z}{1-z} + \frac{1-z}{z} (1-y)^2 \right] + 8x(1-x)(1-y) \right\} \quad (2.2c)$$

$$M_q^{(\nu)} = -\frac{4}{3} \sqrt{1-y} [xz + (1-x)(1-z)(1-y)] 4\sqrt{\frac{xz}{(1-x)(1-z)}} \quad (2.3a)$$

$$M_{\bar{q}}^{(\nu)} = -\frac{4}{3} \sqrt{1-y} [xz(1-y) + (1-x)(1-z)] 4\sqrt{\frac{xz}{(1-x)(1-z)}} \quad (2.3b)$$

$$M_g^{(\nu)} = -\frac{1}{2} \sqrt{1-y} (1-2x) [(1-z)(1-y) - z] 4\sqrt{\frac{x(1-x)}{z(1-z)}} \quad (2.3c)$$

$$N_q^{(\nu)} = N_{\bar{q}}^{(\nu)} = \frac{4}{3} (1-y) \frac{xz}{2} \quad (2.4a)$$

$$N_g^{(\nu)} = \frac{1}{2} (1-y) 4x(1-x) \quad (2.4b)$$

in the case of antineutrino scattering

$$L_q^{(\bar{\nu})} = L_{\bar{q}}^{(\bar{\nu})}, \quad M_q^{(\bar{\nu})} = M_{\bar{q}}^{(\bar{\nu})}, \quad N_q^{(\bar{\nu})} = N_{\bar{q}}^{(\bar{\nu})}, \quad N_g^{(\bar{\nu})} = N_g^{(\nu)} \quad (2.5)$$

$$L_g^{(\bar{\nu})} = \frac{1}{2} \left\{ [x^2 + (1-x)^2] \left[\frac{1-z}{z} + \frac{z}{1-z} (1-y)^2 \right] + 8x(1-x)(1-y) \right\} \quad (2.6a)$$

$$M_g^{(\bar{\nu})} = -\frac{1}{2} \sqrt{1-y} (1-2x) [(1-z) - z(1-y)] 4 \sqrt{\frac{x(1-x)}{z(1-z)}} \quad (2.6b)$$

The cross-section for the lepton scattering on a nucleon is then simply obtained by integration over the incident parton variable x [4]

$$\frac{d\sigma}{dx_n dy dz d\varphi} = \sum_i \int d\xi \int dx \delta(x_n - \xi x) f_i(\xi) \frac{d\sigma^i}{dx dy dz d\varphi} \quad (2.7)$$

where $f_i(\xi)$ is the probability distribution of type i parton inside the nucleon with a fraction of momentum ξ , and $x_H = Q^2/2Pq$ (P is the nucleon four-momentum). We give in the appendix the functions f_i that we have used. Since we are interested in jet cross-sections, we do not have to introduce decay functions for outgoing partons, as one should do when dealing with semi-inclusive processes. We finally recall the zeroth order cross-section

$$\frac{d\sigma_{(0)}^{(\nu)}}{dx_n dy} = \frac{G_F^2 Q^2}{\pi y} \left[\binom{1}{(1-y)^2} f_q(x_n) + \binom{(1-y)^2}{1} f_{\bar{q}}(x_n) \right] \quad (2.8)$$

3. THRUST AND SPHEROCITY DISTRIBUTIONS*)

The three jets are more easily seen in the final hadronic state rest frame defined by $\vec{p} + \vec{q} = \vec{0}$. We can then use another set of variables

$$W^2 = (p+q)^2 \quad \text{and} \quad x_i = \frac{2E_i}{W}$$

namely the total available energy in the hadronic final state and the fraction of energy of each jet (E_i is the energy of one of the outgoing partons, that is, in a jet language, the energy of one of the jets). We have the usual constraint

$$x_1 + x_2 + x_3 = 2 \quad (3.1)$$

*) While completing this work we received a preprint by G. Ranft and J. Ranft on the same subject, where different results are obtained [7].

Starting from (2.1) to (2.7) we integrate over ϕ and changing the variables according to

$$z = 1 - \frac{1-x_3}{x_1}, \quad x_H = 1 - \frac{1-\xi}{x_1} \quad (3.2)$$

we get

$$\frac{d\sigma}{dx_1 dx_3 d\xi dy} = \alpha_s(Q^2) \frac{Q^2}{2\pi^2 y} \frac{1}{x_1^3} \sum_i \frac{1-\xi}{\xi} f_i(\xi) L_i\left(x=1-\frac{1-\xi}{x_1}, y, z=1-\frac{1-x_3}{x_1}\right) \quad (3.3)$$

Then using

$$\xi = 1 - \frac{W^2}{W^2 + Q^2} x_1, \quad y = \frac{W^2 + Q^2}{2E M_{\text{proton}}} \quad (3.4)$$

we finally obtain
$$\frac{d\sigma}{dx_1 dx_3 dQ^2 dW^2}$$

It is straightforward to change into T and S variables using [3]

$$T = \max(x_1, x_2, x_3)$$

$$S = \frac{64}{\pi^2} (1-x_1)(1-x_2)(1-x_3)/T^2 \quad (3.5)$$

which gives the three-jet cross-section in terms of thrust and sphericity:

$$\frac{d\sigma}{dQ^2 dW^2 dS dT}$$

However, this cross-section is too differential to be useful for comparison with experiments, so we instead consider

$$\frac{d\sigma}{dQ^2 dW^2 dT}$$

and normalize it by dividing with the factor

$$\frac{d\sigma^{(0)}}{dQ^2 dW^2}$$

given by (2.8). Due to the complexity of the coefficients, it is not possible to obtain closed analytic forms for this cross-section, as it is in e^+e^- annihilation.

We show in Fig. 4 the Q^2 dependence of

$$\left[\frac{d\sigma_{(0)}}{dQ^2 dW^2} \right]^{-1} \frac{d\sigma}{dQ^2 dW^2 dT}$$

at fixed W ($W = 10$ GeV) which turns out to be small. This is as one could expect from a naive point of view: Q^2 is the "natural" variable when dealing with the

parton cross-sections (for example, the diagrams of Fig. 2) and this is why we took the strong coupling constant to depend on Q^2 ; but W^2 is the natural variable when dealing with the three jets, that is, with the total hadronic final state. In a way, x dependence should conspire to damp the Q^2 dependence of α_s . We are unable to explain in a simple way how this is realized practically. We have also noted that at high Q^2 , this damping is suppressed and the cross-section varies as α_s .

We then integrate over Q^2 . Our results are shown in Figs. 5 and 6. The perturbative QCD results cannot be directly compared with experimental data [8], because of infrared and non-perturbative effects. In the following, we will closely follow the procedure adopted by De Rújula et al. [3].

As we have seen before, thrust is an infrared safe quantity. But integrating over T the

$$\frac{d\sigma}{dW^2 dT}$$

that we have computed (that is, by putting all masses to zero) gives an infinite answer. This infinity is only the reflection of the usual infrared singularity. As is well known, the correct procedure is as follows. To stay away from the infrared singularity at $T = 1$, one integrates $d\sigma/dW^2 dT$ for $2/3 \leq T \leq 1 - \Delta T$ where $\Delta T \neq 0$. The answer is finite. Then giving a mass to the gluon, one computes $d\sigma/dW^2 dT$ for $1 - \Delta T \leq T \leq 1$ including the vertex and propagator gluon corrections to the two-jet term and integrates it over this range. Just as in the Sterman and Weinberg formula [9], these corrections cancel the mass singularities so that one can put the mass of the gluon again to zero. Summing both integrals, one gets a finite answer as $\Delta T \rightarrow 0$. Unfortunately, when ΔT is too small, say $\Delta T \leq \Delta T_{\text{pert.}}$, the first integral

$$\frac{1}{\sigma_{(0)}} \int_{\frac{2}{3}}^{1-\Delta T} \frac{d\sigma}{dW^2 dT} dT$$

is greater than one, and perturbation theory fails. So one can only trust $d\sigma/dT dW^2$ for $T < 1 - \Delta T_{\text{pert.}}$. We have checked that in our case

$$\frac{1}{\sigma_{(0)}} \int_{\frac{2}{3}}^{1-\Delta T} \frac{d\sigma}{dW^2 dT} dT < \frac{1}{2} \quad \text{for } \Delta T > \Delta T_{\text{pert.}} \sim 0.03$$

In addition, one has to face non-perturbative effects which convert quark and gluon jets into jets of hadrons with a non-zero transverse momentum. This gives smearing both of the two-jet and three-jet distributions.

First, the observed thrust distribution for two-jet events is no longer localized at $T = 1$ but is a smooth distribution with finite width $(\Delta T)_{np}$ which can be evaluated from the p_T spectrum of outgoing hadrons. Typically [3]

$$(\Delta T)_{np} \simeq \frac{1}{2} n(W) \frac{\langle p_T \rangle}{W} \quad (3.6)$$

where $n(W)$ is the average total multiplicity and $\langle p_T \rangle$ the mean transverse momentum. For applications, we took $\langle p_T \rangle$ to be 300 MeV and [10]

$$n(W) = \frac{3}{2} \left(\frac{2}{3} + 1.28 \ln W^2 \right)$$

As noted earlier, the relevant variable in the final hadron state is W , not Q . In order to mimic in a simple way the non-perturbative smearing of two-jet events distributions, we choose a simplified parametrization

$$\frac{1}{\sigma_{(0)}} \left(\frac{d\sigma}{dT} \right)_{NP} = \frac{2}{(\Delta T)_{np}^2} e^{-\frac{(1-T)^2}{(\Delta T)_{np}^2}} (1-T) \quad (3.7)$$

The $(1 - T)$ factor accounts for the fact that massive particles cannot be produced collinearly. The experimental behaviour in e^+e^- suggests taking a Gaussian.

And finally one should note that as $(\Delta T)_{np} \rightarrow 0$,

$$\frac{1}{\sigma_{(0)}} \left(\frac{d\sigma}{dT} \right)_{NP} \rightarrow \delta'(1-T)$$

that is a function localized at $T = 1$.

Secondly, to smear the three-jet predictions, one has to average the QCD results over bins of width $(\Delta T)_{np}$.

Therefore, in Figs. 5 and 6, we give three curves. One of them is the two-jet non-perturbative result [as given by (3.7)]. The second one is the genuine QCD prediction for three-jet events and the third one is the smeared version. Figure 5 is relative to presently available W 's. One can readily see that the QCD distribution stays always much lower than the non-perturbative two jet. We do not think that these crude results (as regards the non-perturbative procedure) would be drastically changed had we taken a realistic Monte-Carlo, because of the large size of $(\Delta T)_{np}$ over which all results must be smeared.

It can be interesting to compare our results to the situation in e^+e^- annihilation that seems much more encouraging. Two facts conspire to give this difference of behaviour: the QCD result itself is smaller in neutrino reaction than in e^+e^- by roughly a factor 2; the multiplicity is higher in neutrino-production [so is $(\Delta T)_{np}$].

Figure 6 gives $d\sigma/dT$ for higher W 's. At $W = 28$ GeV, the QCD tail clearly emerges from the background of two-jet events.

We now turn to estimates for $\langle S \rangle$ and $\langle 1 - T \rangle$. Unlike $1/\sigma d\sigma/dT$, its S and $(1 - T)$ moments are finite, so $\langle S \rangle$ and $\langle 1 - T \rangle$ can be calculated reliably. They have to be compared with the corresponding values given by two-jet non-perturbative distributions as given by (3.7):

$$\begin{aligned} \langle 1 - T \rangle_{NP} &= \frac{\sqrt{n}}{2} \Delta T_{NP} \\ \langle S \rangle_{NP} &\simeq \left(\frac{4}{n}\right)^2 \frac{\langle p_T \rangle^2}{W^2} \langle n(W) \rangle^2 \end{aligned} \quad (3.8)$$

Both are shown in Figs. 7 and 8. QCD values are much lower than the two-jet background and the situation only slightly improves at high W . However, one should note that this background is very sensitive to the values of $\langle p_T \rangle$ and $n(W)$ that one chooses. As a matter of fact, for high energies ($W > 15$ GeV), we have just used for $n(W)$ the parametrization given in Ref. 10, which could turn out to be wrong in that region.

4. ANGULAR CORRELATIONS

Dealing with inclusive processes, there are two ways to look at the angular asymmetries that arise from Eqs. (2.1) to (2.7). The first one is to study the distribution of the angle between a given jet, say a quark (gluon) jet, and the lepton plane: this is roughly the angle ϕ that we have already defined in Section 2 (see also Fig. 3). The other one is based on the fact that, to first order in α_s , the three outgoing partons lie in the same plane. This situation is spoilt by higher order perturbation theory and non perturbative effects. But if one believes in the cautious applicability of perturbative QCD, one should expect the outgoing hadrons to be roughly in the same plane. One can therefore study the angle Φ from the lepton plane to the hadronic plane just defined (see Fig. 3).

One should note that, ϕ and Φ being azimuthal angles, their distribution are identical in all the frames that are deduced from each other by a boost along $\vec{q}$: in particular the lab. frame and the final hadron state rest frame.

4.1 Angular correlations between a given jet and the lepton plane

In this subsection, we disregard the jet corresponding to the fragmentation of the nucleon and we pay attention only to the other two jets.

We start with a quark jet. The corresponding angle ϕ is just the angle ϕ already defined in Section 2. Its distribution is slightly modified by the fact that the diagram of Fig. 2c gives rise to two quark jets so that one has to sum over the two contributions. It is proportional to

$$\sum_{i=q,\bar{q}} \left[L_i(z) + M_i(z) \cos \varphi_q + N_i(z) \cos 2\varphi_q \right] + \left[L_g(z) + L_g(1-z) \right] + \left[M_g(z) - M_g(1-z) \right] \cos \varphi_q + \left[N_g(z) + N_g(1-z) \right] \cos 2\varphi_q \quad (4.1)$$

A good measurement of these correlations is given by $\langle \cos \phi \rangle_q$ and $\langle \cos 2\phi \rangle_q$. One gets from (2.1), (2.7) and (4.1)

$$\langle \cos \phi \rangle_q = \frac{\alpha_s(Q^2)}{d\sigma_{(0)}} \frac{G_F^2 Q^2}{4\pi^2 y} \sum_{i=q,\bar{q}} \int_{x_H}^1 \frac{dx}{x} f_i\left(\frac{x_H}{x}\right) \int_0^1 M_i(x, y, z) dz \quad (4.2)$$

where $d\sigma_{(0)}/dx_H dy$ is given by (2.8). Similarly

$$\langle \cos 2\phi \rangle_q = \frac{\alpha_s(Q^2)}{d\sigma_{(0)}} \frac{G_F^2 Q^2}{4\pi^2 y} \int_{x_H}^1 \frac{dx}{x} \left[\sum_{i=q,\bar{q}} f_i\left(\frac{x_H}{x}\right) \int_0^1 N_i(x, y, z) dz + f_g\left(\frac{x_H}{x}\right) \int_0^1 2N_g(x, y, z) dz \right] \quad (4.3)$$

The complete analytical expressions for $\langle \cos \phi \rangle_q$ and $\langle \cos 2\phi \rangle_q$ are rather complicated and we do not write them here.

As noted earlier by Cahn [11] the incorporation of a primordial transverse momentum into the naive parton model, generates a ϕ distribution that is similar to the one predicted by QCD. The main difference comes from the Q^2 dependence. In QCD calculation, $\langle \cos \phi \rangle_q$ and $\langle \cos 2\phi \rangle_q$ depend on Q^2 via the running coupling $\alpha_s(Q^2)$ ($\Lambda = 500$ MeV). On the other hand, in the improved parton model [11]

$$\langle \cos \phi \rangle \propto - \frac{P_T}{Q} \quad \langle \cos 2\phi \rangle \propto \frac{P_T^2}{Q^2} \quad (4.4)$$

For $Q^2 \sim 5 \text{ GeV}^2$ the parton model contribution to $\langle \cos \phi \rangle_q$ is larger than that of QCD. As Q^2 gets larger, the $1/Q$ and $\alpha(Q^2)$ behaviour are not sufficiently different to make the QCD effect dominant even at $Q^2 \sim 100 \text{ GeV}^2$.

For $\langle \cos 2\phi \rangle_q$ the $1/Q^2$ dependence of the parton model then favours the emergence of QCD effects in the range of available Q^2 , but the global effect is only of a few percent.

In Figures 9 and 10a we give the values of $\langle \cos \phi \rangle_q$ and $\langle \cos 2\phi \rangle_q$ for both models integrated over x_H as a function y . For numerical purposes the transverse momentum p_T was taken to be 400 MeV.

We then come to the gluon jet. Since the outgoing jets (partons) lie in a plane and since $\vec{p}_2$ and $\vec{p}_3$ are on opposite sides of $\vec{q}$ (see Fig. 3), the angle ϕ is now equal to $\phi + \pi$. The diagrams of Fig. 2c do not contribute to the distribution which is then proportional to

$$\sum_{i=q,\bar{q}} \left[L_i(z) - M_i(z) \cos \varphi_g + N_i(z) \cos 2\varphi_g \right] \quad (4.5)$$

One gets

$$\langle \cos \varphi \rangle_g = - \frac{\alpha_s(Q^2)}{\frac{d\sigma_{\cos}}{dx_H dy}} \frac{G_F^2 Q^2}{4\pi^2 y} \sum_{i=q,\bar{q}} \int_{x_H}^1 \frac{dx}{x} f_i\left(\frac{x_H}{x}\right) \int_0^1 N_i(x, y, z) dz = - \langle \cos \varphi \rangle_q \quad (4.6)$$

$$\langle \cos 2\varphi \rangle_g = \frac{\alpha_s(Q^2)}{\frac{d\sigma_{\cos}}{dx_H dy}} \frac{G_F^2 Q^2}{4\pi^2 y} \sum_{i=q,\bar{q}} \int_{x_H}^1 \frac{dx}{x} f_i\left(\frac{x_H}{x}\right) \int_0^1 N_i(x, y, z) dz \quad (4.7)$$

The corresponding values, integrated over x_H are shown in Figs. 9 and 10b. As one can see, the behaviours of quark and gluon jet asymmetries are rather different. The scattered quark is produced with a transverse momentum opposite to k_{1T} whereas the gluon tends to be on the same side. Moreover, the presence of the diagram of Fig. 2c makes $\langle \cos 2\phi \rangle_q$ and $\langle \cos 2\phi \rangle_g$ depend very differently on y .

Of course we do not claim that such jet asymmetries will cleanly test QCD. There is no experimental evidence for a difference of behaviour between quark and gluon jets, although one thinks [12] that multiplicities in gluon jets are higher than in quark jets. But as soon as one can distinguish them, and one should be helped by the fact that they are lying in two different half-planes separated by $\vec{q}$ (see Fig. 3), these asymmetries will be worth studying.

4.2 Angular correlation between the lepton plane and the hadron plane

It is straightforward to get the distribution of the angle Φ from the one of ϕ . If one denotes by

$$\frac{dn}{d\phi}(\phi) \quad (\text{resp. } \frac{dN}{d\Phi}(\Phi))$$

the number of events with ϕ (resp Φ) between ϕ and $\phi + d\phi$ (resp. Φ and $\Phi + d\Phi$), we get from (2.1)

$$\frac{dn}{d\phi} \propto L + M \cos \phi + N \cos 2\phi \quad (4.8)$$

Since

$$\frac{dn}{d\Phi}(\Phi) = \frac{dn}{d\phi}(\Phi) + \frac{dn}{d\phi}(\Phi + \pi)$$

we obtain

$$\frac{dn}{d\Phi} \propto 2 (L + N \cos 2\Phi) \quad (4.9)$$

Putting all the factors back, we finally get, restricting Φ to the range $]-\frac{\pi}{2}, +\frac{\pi}{2}]$

$$\langle \cos \Phi \rangle = \frac{\alpha_s(Q^2)}{\frac{d\sigma_{\cos}}{dx_n dy}} \frac{G_F^2 Q^2}{\pi^3 y} \sum_{i=g,q} \int_{x_n}^1 \frac{dx}{x} f_i\left(\frac{x_n}{x}\right) \int_0^1 \left[L_i(x, y, z) + \frac{1}{3} N_i(x, y, z) \right] dz \quad (4.10)$$

$$\langle \cos 2\Phi \rangle = \frac{\alpha_s(Q^2)}{\frac{d\sigma_{\cos}}{dx_n dy}} \frac{G_F^2 Q^2}{4\pi^2 y} \sum_{i=g,q} \int_{x_n}^1 \frac{dx}{x} f_i\left(\frac{x_n}{x}\right) \int_0^1 N_i(x, y, z) dz \quad (4.11)$$

Unfortunately, as one can check from (2.2), because of the presence of the L_i 's in (4.10), $\langle \cos \Phi \rangle$ suffers infrared divergences. The reason is simple: when the gluon becomes collinear with one of the two quarks (z or $x \rightarrow 1$), the hadron plane is no longer defined and neither is $\langle \cos \Phi \rangle$. $\langle \cos 2\Phi \rangle$ stays finite because it still has the symmetry of the problem (as $\langle \cos \Phi \rangle_{g,q}$). To get rid of this infrared divergence, we must use some cut-offs. Instead of using this procedure on both x and z variables, we chose to come back to the variables of the last section, namely S and T and to put a cut-off on T , preventing it to go to 1. The nice feature of this choice is that only one cut-off is needed: if $T < 1$, the three jets are theoretically well separated and the hadron plane is well defined.

We present our results in Figs. 11-13. As we have just noted, the cut-off procedure is only needed in the case of $\langle \cos \Phi \rangle$. For example, Fig. 12 shows $\langle \cos \Phi \rangle$ integrated over S , T ($2/3 \leq T \leq 1 - \Delta T$) and Q^2 as a function of y . As ΔT increases, the asymmetries decrease (we tend to favour the spherical events) but even for a cut at $T = 0.87$ the effect is sizeable (several percents).

What are the non-perturbative effects? If we first disregard the primordial p_T , the infrared divergence arising from Eq. (4.9) gets translated into the experimental world in the following sense. Since there are two jets, the outgoing quark (we do not consider the recoil jet) gives a jet structure that should be roughly symmetrical under rotations around the jet axis and there would be no possibility of finding a position for the hadron plane.

If we now include a primordial p_T , the thrust in the two-jet events is shifted from 1 to

$$T = 1 - \frac{2p_T^2}{W^2} \quad (4.12)$$

The change is then negligible: for $p_T = 600$ MeV, $W = 6$ GeV, $T = 0.98$ (this remark is also valid for the cross-sections that we have computed in Section 3). Of course, the distribution in T gets smoothed around this value of T , in the same way as in Section 3. But if one tries to determine a plane using this single

jet, that is, restricting oneself to the hemisphere opposite to the recoil jet, one should face the same problems as before (translating once again the infrared singularity).

5. CONCLUSIONS

As seen in the preceeding sections, deep inelastic processes may not be a very good place to test QCD properly at least for the available energies. Many effects which had been advocated as clear QCD effects are fused in the non-perturbative background. The structure function of the nucleon are rather accurately known at present and the problem we are facing is how quarks and gluons hadronize. For instance, our results are strongly dependent on $(\Delta T)_{np}$ and $(\Delta S)_{np}$. We think, however, that $\langle \cos \Phi \rangle$, the asymmetry between the lepton and hadron plane is a rather clear cut case on the theoretical side. Whether it is easily measurable is of course an experimental issue. The hadron plane can be defined as the plane which minimizes the momentum flowing out of it (this is related to the acoplanarity defined in [6]). This definition holds both in the lab. and the jet rest frame. In the lab. frame the target fragments are expected to be spherically distributed and then not to influence the determination of the plane. The cuts in T select events with significant p_T where QCD structure should emerge. We therefore hope that measuring Φ asymmetries will provide a test for perturbative QCD.

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Note that in Fig. 5c of this paper, the correct scale is
 $\frac{1}{\sigma} \frac{d\sigma}{dT} \times 10$ and not $\frac{1}{\sigma} \frac{d\sigma}{dT} \times 10^2$.
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APPENDIX

The parton distribution functions which we fold in (2.7) were extracted from CDHS collaboration results

$$\frac{d}{dQ^2} F_2(x) = A \frac{(1-x)^3}{\sqrt{x}} + B \frac{(1-x)^7}{x}$$

$$\frac{d}{dQ^2} F_3(x) = B \frac{(1-x)^7}{x}$$

The normalization constants A and B were determined using the measured valence quark momentum

$$\int_0^1 x F_3(x) dx \simeq \frac{3}{4} \int_0^1 F_2(x) dx$$

and the momentum sum rule:

$$\int_0^1 F_2(x) dx \simeq \frac{1}{2}$$

No Q^2 dependence was taken into account in our calculation. The gluon distribution function was taken to be

$$\frac{d}{dQ^2} F_2(x) = \frac{5}{2} \frac{(1-x)^4}{x}$$

We checked that modifying the exponents of $(1-x)$ in these functions by one unit changed our results by at most 10-15 per cent.

Figure captions

- Fig. 1 : The zeroth order contribution to two-jet production.
- Fig. 2 : a,b,c) Contribution of the first order QCD perturbation theory to three-jet events:
a) Incident quark. b) Incident antiquark. c) Incident gluon.
- Fig. 3 : Kinematical configuration of lepton hadron scattering in the hadronic final state rest frame, showing the angles ϕ and Φ ($-\pi/2 \leq \phi \leq \pi/2$).
- Fig. 4 : Illustration of the Q^2 dependence of $(1/\sigma)(d\sigma/dT)$ for fixed W . The dashed curve is the Q^2 integrated version of this T distribution.
- Fig. 5 : a,b,c) $(1/\sigma)(d\sigma/dT)$ for various values of W , integrated over $Q^2 \geq 1 \text{ GeV}^2$. Data from [8] correspond to $6 < W < 14 \text{ GeV}$, NP stands for the non-perturbative two-jet contribution as calculated from (3.7). The dashed curves correspond to the QCD contribution smeared over T bins of width $(\Delta T)_{NP}$ indicated in the bottom of the figure.
- Fig. 6 : a,b,c) Analogous to Fig. 5 for large values of W at which the QCD tail begins to show up.
- Fig. 7 : $\langle 1 - T \rangle$ as a function of W . The left-hand side of the figure corresponds to our estimates for E_{lab} in the range 50-150 GeV, while in the right-hand side $E_{lab} = 10,000 \text{ GeV}$. Data points are from [8]. QCD dashed lines are estimates for fixed Q^2 , the full line is integrated over Q^2 , the NP dashed lines are our guess for the non-perturbative contribution.
- Fig. 8 : $\langle S \rangle$ as a function of W , notations are as in Fig. 9.
- Fig. 9 : Average values of $\langle \cos \phi \rangle$ for the quark jet calculated in QCD and the improved parton model.
- Fig. 10 : a,b) Analogous to Fig. 9 for $\langle \cos 2\phi \rangle$.
- Fig. 11 : The angular asymmetry between the hadron and lepton planes as a function of y for different cuts in thrust.
- Fig. 12 : Same as above, $\langle \cos \phi \rangle$ as a function of W .
- Fig. 13 : Expectations for hadron distribution around the leptonic plane in QCD and in the improved parton model as measured by $\langle \cos 2\Phi \rangle$.

Fig. 1

(a)

(b)

(c)

Fig. 2

Fig. 3

Fig. 4

Fig. 5

Fig. 6

Fig. 7

Fig. 8

Fig. 9

Fig. 10

Fig. 11

Fig. 12

Fig. 13