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RADIATIVE CORRECTIONS TO HIGH ENERGY NEUTRINO SCATTERING

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A B S T R A C T

Motivated by precise neutrino experiments, we reconsider the electromagnetic radiative corrections to the data. We investigate the usefulness and demonstrate the simplicity of the "leading log" approximation: the calculation to order $\alpha \ln(Q/\mu)$, $\alpha \ln(Q/m_q)$. Here Q is an energy scale of the over-all process, μ is the lepton mass and m_q is a hadronic mass, the effective quark mass in a parton model. We identify the questions whose answers do not depend on unknown hadron parameters like quark masses. The leading log radiative corrections to do/dy distributions and to suitably interpreted do/dx distributions are quark-mass independent. We improve upon the conventional leading log approximation and compute explicitly the largest terms that lie beyond the leading log level. In practice this means that our model-independent formulae, though approximate, are likely to be excellent estimates everywhere except at low energy or very large y . We point out that radiative corrections to measurements of deviations from the Callan-Gross relation and to measurements of the "sea" constituency of nucleons are gigantic. The QCD inspired study of deviations from scaling is of particular interest. We compute, beyond the leading log level, the radiative corrections to the QCD predictions.

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1. - INTRODUCTION, TERMINOLOGY AND OUTLOOK

The data on high-energy inclusive neutrino scattering, particularly "charged current" scattering, has become precise enough for one to fear that the electromagnetic radiative corrections may be comparable to, or larger than the error bars. Neutrinos can now be used to study small deviations from scaling, to make detailed comparisons with electroproduction, to determine precisely the parameters describing neutral currents, to study within quark-parton models various quark and gluon momentum distribution functions, etc. Needless to say, all of this high precision work makes the understanding of radiative corrections mandatory. The radiative corrections to inclusive charged current neutrino scattering have been thoroughly studied by Kiskis¹⁾ in a quark-parton model. The appearance in the results of terms of the type $\ln(Q/m_q)$, where Q is the momentum transfer and m_q a quark mass, was a source of worry to Kiskis¹⁾. For $Q=10$ GeV, say, it makes a large difference to set $m_q=350$ MeV (a typical "constituent quark" mass, $\ln(Q/m_q) = 3.35$) or $m_q=1$ MeV (a typical "current algebra" mass $\ln(Q/m_q) = 9.21$). A further understanding of this dependence of the radiative corrections on relatively unknown or ill-defined parameters is one of the motivations of our work. We will conclude that many, if not most, of the radiative corrections and physically relevant questions are in fact quark-mass and even quark-model independent.

In the past, radiative corrections have been applied to the data on elastic, quasi-elastic and deep inelastic electron scattering, and found to be rather large (up to ~ 30 per cent) in certain kinematical regions²⁾. Except for the elastic case, only bremsstrahlung and virtual effects in the electron "legs" and photon propagator corrections (see Fig. 1a) have been explicitly worked out. Three reasons to limit oneself to this set of corrections, in order of decreasing quality are^{1),2)} :

- i) the corrections form a gauge invariant set and thus make sense as part of the complete answer ;
- ii) electrons being much lighter than hadrons radiate more ; thus the contribution of processes where photons are attached to electrons should constitute the lion's share of the radiative corrections ; more precisely, the leading radiative effects from a relativistic particle of mass M are of order $(\alpha/\pi) [\ln(Q/M) + O(1)]$.; one may neglect hadronic radiative effects provided $\ln(Q/m_e) \gg \ln(Q/m_p)$ or $\ln(Q/m_\tau)$, say ; clearly this argument is not good for processes involving muons or for energies large enough for all logs to become of comparable magnitude ;

- iii) the calculation of radiative corrections involving photons attached to hadrons requires detailed knowledge of hadron dynamics.

In electroproduction, and if one accepts the above arguments, one need not compute photon bremsstrahlung from hadrons (Fig. 1b), virtual photon corrections to hadron propagators and vertex (Fig. 1c), two-photon exchange process (Fig. 1d) or interferences between the real emission of photons by hadrons and leptons (Fig. 1e).

To come back to the radiative corrections to neutrino scattering, let us at this point introduce some terminology for quick reference to Feynman diagrams (for amplitudes or "squared" amplitudes) contributing to $O(\alpha)$ corrections. A "lepton leg" correction is one where photons are only attached to leptons (examples are given in Fig. 2a,b). An "interference" is a diagram where a photon line ends both in a lepton and somewhere else, either a hadron or an intermediate vector boson, see Fig. 2c,e,f. A "hadron leg" correction is a diagram where photons are only attached to hadrons ; Fig. 2d is an example.

At first sight, the radiative corrections to neutrino scattering appear to present formidable challenges, relative to their electron scattering counterpart. First, and worst of all, the lepton leg corrections of Fig. 2a,b do not form a gauge invariant set. Electric charge flows from the leptons to the hadrons and one must include interference and other diagrams to obtain a sensible result. Second, muons are not much lighter than hadrons and the latter may radiate as much as the former. Third, the weak interaction intermediaries are also charged and a complete set of radiative corrections includes diagrams like the one in Fig. 2e. This seems to bring into play a model of the weak interactions and an extra model-dependence of the radiative corrections. Fourth, one still needs a model of the strong interactions to calculate interference and hadron leg diagrams. Last and least, the diagrams classified in Fig. 2 are not a complete set. Diagrams where photons attach only to weak intermediaries, for instance, have not been included. These left over diagrams, after renormalization, contribute only terms of order $\alpha Q^2/M_W^2$, and we forget about them in what follows.

Fortunately, the problems in the lengthy list of the previous paragraph are solved in one stroke if one limits oneself to the "leading log" radiative corrections. We proceed to define them. As we shall see, the relation between the observed cross-section σ_O and the radiatively corrected bare cross-section σ_B is of the form :

$$\sigma_0 = \sigma_B + \frac{\alpha}{2\pi} \ln\left(\frac{Q^2}{\mu^2}\right) F\{\sigma_B\} + \frac{\alpha}{2\pi} \ln\left(\frac{Q^2}{M^2}\right) H + \frac{\alpha}{2\pi} G \quad (1.1)$$

We call the "lepton log" that part of the radiative corrections that is proportional to $\ln(Q/\mu)$, where μ is the charged lepton mass. We shall prove that the lepton log "factorizes"³⁾: the calculation of its coefficient $F\{\sigma_B\}$ does not require knowledge of the dynamics of processes where photons attach to intermediate vector bosons or hadrons. We will comment more on factorization as soon as we complete the first reading of Eq. (1.1). The coefficient $F\{\sigma_B\}$ is an explicit functional of the bare cross-section and can be computed from the actual data. The remaining potentially large radiative corrections are "hadron logs", extra additive corrections proportional to $\ln(Q/M)$, where M is a typical hadronic mass. In a quark model of deep inelastic scattering the coefficient H of the hadron log is computable and M is a quark "mass" (or, more precisely, a measure of $\sqrt{p^2}$ for a quark inside a hadron). Whether the model-dependent hadron log corrections are to be made at all is, more often than not, just a matter of convention: it all depends on the interpretation of the "bare" cross-section σ_B in Eq. (1.1). We come back to this question at the end of the Introduction.

The remaining $O(\alpha)$ radiative corrections have a coefficient G in Eq. (1.1) that does not contain potentially large logarithms of the form $\ln Q/m$. The calculation of G requires a model of weak and strong interaction dynamics and is a formidable task¹⁾. We occasionally refer to the G term in Eq. (1.1) as the "constant" term.

We call the approximation of keeping only the terms of order $(\alpha \ln Q)$ in Eq. (1.1) the "leading log" approximation. This is a slight departure from conventional terminology, the leading log approximation generally refers to the sum to all orders n of perturbation theory of terms of the form $(\alpha \ln)^n$. For electrodynamic corrections at reasonable energies the two meanings numerically coincide to a high degree of precision. The leading log approximation is very similar to the electroproduction "peaking" approximation reviewed in Ref. 2). In Chapter 5 we discuss an improvement of the classical peaking approximation, consisting in the use of an optimal momentum scale (instead of Q) in Eq. (1.1). This is optimal in the sense that the potentially most dangerous constant terms (behaving as squares of logs of scaling variables) are automatically taken care of. The domain of applicability of the leading log approximation increases very substantially.

At small momentum transfers, $\ln(Q/\mu)^2, \ln(Q/m_q)^2 = O(1)$ the radiative corrections are of order α/π : somewhat smaller than the error bars of present experiments. At larger momentum transfers $\ln(Q/\mu)^2, \ln(Q/m_q)^2 \gg 1$ the radiative corrections are of order $\alpha/\pi \ln Q^2$ and become potentially large. But as the momentum transfer becomes large enough for the radiative corrections to be large $[\alpha/\pi \ln(Q^2/\mu^2) \sim \text{several per cent}]$, the leading log approximation becomes a good approximation $[\ln(Q^2/\mu^2) \gg 1]$. Thus we conclude that the leading log radiative corrections may be sufficient for the purpose of analyzing current neutrino experiments. We will devote a chapter to the analysis of the precision of the leading log approximation. The leading lepton log corrections can be implemented with no reference to a particular model of hadron dynamics. In the many cases where hadron log corrections need not be implemented, correcting the data in the leading (lepton) log approximation is sufficient, satisfactory and, as we shall see, extremely simple.

We have already mentioned a crucial property of the leading logs : factorization. The calculation of the lepton logs involves only lepton leg diagrams (in which photons attach exclusively to leptons). The previous sentence is also true with the substitution of hadrons for leptons. Factorization seems to clash with the fact that neither the ensemble of lepton leg diagrams nor the ensemble of hadron leg diagrams is a gauge invariant ensemble. The point is somewhat subtle. In a physical gauge (with only transverse photons) every interference diagram vanishes in the leading log approximation. In an arbitrary gauge the leading log contribution of the sum of interference diagrams can be related explicitly to hadron leg and lepton leg diagrams. One way or the other, all of the information to compute the leading logs can be extracted from lepton leg and hadron leg diagrams. We shall dwell on these matters in detail in Section 2.

As we remind the reader in Section 2, the leading logs arise from collinear situations ⁴⁾ where a photon is emitted close to the direction of flight of the emitting particle. A leading lepton log arises from the configuration $\vec{p} \parallel \vec{k}$ in Fig. 2a, for instance. In this situation the parent particle propagator (labelled $p+k$ in Fig. 2a) blows up and eventually gives rise to the "large" logarithm. In a physical gauge, where classical intuition and Feynman diagrams are least distant, the leading lepton logs arise from the collinear emission (or emission and reabsorption) of photons in the lepton leg diagrams of Fig. 2a (Fig. 2b). Thus the main contribution to radiation at high energy is "semi-classical" in the sense that it can be traced back to

the emission of photons by the particle in question. The typically quantum mechanical interference effects are not involved.

The lepton log radiative corrections do what one expects them to do from their "semi-classical" interpretation as radiation of collinear photons from an accelerated particle. The amount of radiation increases logarithmically with the energy scale ("acceleration"). Let x and y be the usual variables in terms of which deep inelastic processes are described :

$$x \equiv \frac{Q^2}{2P \cdot q} = \frac{-q^2}{2P \cdot q}$$

$$q \equiv P_\nu - P_\mu$$

$$y \equiv (E_\nu - E_\mu)/E_\nu \quad (1.2)$$

where P is the target four-momentum and the rest of the notation is self-explanatory. Collinear photon emission makes events migrate at fixed angle from large to smaller E_μ (small to large y). Thus the observed (uncorrected) $d\sigma/dy$ cross-section at fixed E_ν is an underestimate at small y and an overestimate at large y of the bare (radiatively corrected) cross-section. Similarly, $d\sigma/dx$ is underestimated at large x and overestimated at small x .

The measured y distributions are used in practice to extract small but particularly important combinations of structure functions : the ones that measure the "sea" constituency of the nucleon and the "longitudinal" couplings of the current (the deviations from the Callan-Gross relation). The radiative corrections to these quantities are small (a few per cent) but as large as the quantities themselves. Of all the radiative corrections we compute these are the most important in actual practice. Chapter 6 is devoted to this subject.

We now come back to the question of leading hadron logs : the radiative corrections of order $(\alpha/\pi)\ln(Q/M)$, with M a hadron or quark mass. They factorize in the sense of arising solely from hadron leg diagrams. There are three types of practical situations concerning these radiative corrections : cases where they need not be made at all, cases where they should be made, but they are in fact M independent, and cases where they must be made (the results can then be interpreted as a measurement of M).

An example where there are no hadron log corrections is the following. Let an electroproduction or neutrino scattering experiment be interpreted, after the leading lepton log radiative corrections are made, as the measurement of the total cross-sections (or structure functions) for off-shell γ 's or W 's on nucleons. Let "total" in the previous sentence mean the sum over final states with and without photons. In this framework, there are no hadron log corrections to be made. The above-mentioned interpretation of the experiment is correct up to terms of order α/π , rather than α/π times potentially large $\ln Q$ terms. It is in this spirit that the radiative corrections to electroproduction have been implemented in the past. Hadron logs only appear when one attempts to undress the hadrons from the virtual or real photons that may accompany them. The difference between bare and (electromagnetically) dressed structure functions is of $O(\alpha/\pi \ln Q/M)$, the probability for the hadrons to radiate photons. If one could observe, and desired to subtract, events with hard photons collinear to the hadron shower, hadron log radiative corrections would be necessary.

An example where hadron log corrections must be made, but turn out not to depend on a hadronic mass scale is the analysis of deviations from scaling in deep inelastic scattering. Suppose one is studying a given reaction ($\nu_\mu p \rightarrow \mu^- X$, say) at two different Q^2 scales, in an attempt to predict or to measure deviations from scaling in the inclusive structure functions, $F_1(x, Q_0^2) \neq F_1(x, Q_1^2)$. In a quark parton model, the electromagnetic hadron log corrections are a sum over quark flavours of terms proportional to $e_q^2 (\alpha/\pi) \ln(Q/m_q)$, where e_q and m_q are quark charges and effective masses. Thus, the electromagnetic corrections to the scale violating effects $F_1(x, Q_0^2) - F_1(x, Q_1^2)$ have coefficients of the form

$$\frac{\alpha}{\pi} \ln \left(\frac{Q_0}{m_q} \right) - \frac{\alpha}{\pi} \ln \left(\frac{Q_1}{m_q} \right) = \frac{\alpha}{\pi} \ln \left(\frac{Q_0}{Q_1} \right) + O(\alpha d_s) \quad (1.3)$$

The wonderful properties of logarithms make these corrections quark mass independent. The difference between the probabilities for a particle to radiate at two large energy scales is a function of the energy ratio, not the particle's mass. The mass independence (or "infra-red insensitivity") of the scaling deviations is also (the crucial) property of the QCD "radiative corrections" that underlye the predictions of logarithmic scaling deviations. The analysis of deviations from scaling is of particular importance. We shall see that the radiative corrections to this analysis may be implemented in a rather model-independent fashion even beyond the leading

log level. If QCD is right, the violations of scaling induced by the strong interactions are much bigger than the apparent violations induced by electromagnetism : $\alpha_s \gg \alpha$, with α_s the chromodynamic fine structure constant. In other words, the hadron leg radiative corrections to measurements of scaling violations are negligible.

The quark mass makes its first unavoidable appearance when one compares two or more different reactions, in an attempt study different quark distribution functions, check sum rules, etc. Quarks of different flavours interact differently with different leptonic probes, and radiate differently according to their different charges. Thus, a parton model comparison of different reactions may be re-expressed as a measurement of effective quark masses. In this respect, electrodynamic and chromodynamic radiative corrections are very different, the chromodynamic corrections are flavour-independent, while the electromagnetic ones are not.

This paper is organized as follows. In Section 2 we remind the hypothetical non-expert reader of the origin of the leading logarithms, and clarify the questions of gauge invariance and interference diagrams. In Section 3 we compute the lepton leg log radiative corrections and we analyze them numerically. In Section 4 we compute and discuss hadron leg log corrections, with emphasis on the significance of implementing them or not. In Section 5 we discuss and ascertain the accuracy of our improved leading log approximation. In Section 6 we discuss the relatively large radiative corrections to the relatively small combinations of structure functions. We compute the apparent quark-antiquark sea and the apparent Callan-Gross violations induced by radiative corrections. Section 7 is devoted to the radiative corrections to the analysis of scaling violations. This very important subject is treated beyond the leading log approximation. Section 8 deals with radiative corrections to neutral current phenomena. Conclusions and recommendations to experiment are given in Section 9.

Typically, the subject of radiative corrections, particularly approximative ones, is conceptually simple and technically cumbersome. In an attempt to be clear we have subdivided the subject matter ad nauseam and made frequent use of inelegant phrases like "this point will be further discussed in a later chapter". We apologize in advance for this sacrifice of linearity.

2. - ORIGIN AND FACTORIZATION OF THE LEADING LOGS

In this section we outline the proof and attempt to clarify the meaning of a statement made in the Introduction : the lepton log arises from lepton leg diagrams, the hadron log from hadron leg diagrams. This section is somewhat technical and perhaps to be skipped by potential readers mainly interested in results. We indulge in a rather detailed discussion because the related questions of gauge invariance and interference diagrams are the ones that have historically obscured and/or complicated the study of neutrino scattering radiative corrections.

The opening statement of this section is precisely true in a physical gauge. In a covariant gauge the correct statement is slightly more complicated : the leading logs from interference diagrams can be related via gauge invariance to information that can be extracted from hadron leg or lepton leg diagrams.

Consider an interference diagram with real photons, like the one in Fig. 2c. We proceed to outline the process of extraction of its lepton log contribution. Let $\epsilon_\mu(\lambda)$ be the photon polarization vector and let $\epsilon^\alpha T_p(\text{hadronic})$ be the amplitude for the process where the photon hooks onto the hadron blob (the right-hand half of Fig. 2c). Let the lepton and photon momenta be labelled as in Fig. 2. In explicitly calculating this interference one encounters the factor

$$\frac{(\not{p} + \not{k} + \not{\mu}) \gamma_\alpha (\not{p} + \not{\mu}) \sum_\lambda \epsilon^\alpha \epsilon^\beta T_p(\text{hadr.})}{2 p \cdot k} \quad (2.1)$$

The logarithm of the lepton mass arises from the collinear ($\vec{p} \parallel \vec{k}$) end of the phase space integration over outgoing photons, a situation where the off-shell lepton is closest to its mass shell :

$$\int \frac{d^3 k}{2k} \cdot \frac{1}{p \cdot k} \propto \int_0^1 d\cos\theta \frac{1}{E_\mu k (1 - |\vec{p}_\mu| \cos\theta / E_\mu)} \\ \propto \ln \left(1 - \frac{|\vec{p}_\mu|}{E_\mu} \right) \longrightarrow \ln (E_\mu^2 - |\vec{p}_\mu|^2) = \ln \mu^2 \quad (2.2)$$

To compute the leading logarithms it suffices to set $\mu = 0$, $p^2 = 0$ and use the massless collinear approximation $p_\mu \propto k_\mu$ in the numerator of Eq. (2.1)⁵⁾. The terms neglected in this approximation are $O(\mu)$ or $O(p.k)^\alpha$, $\alpha > 0$ and do not give rise to leading logs. In this approximation the numerator collapses into a term proportional to the photon four-momentum $(\not{p} + \not{k} + \not{\mu})\gamma_\alpha(\not{p} + \not{\mu}) \rightarrow k_\alpha$. It is now about time to choose a gauge.

In a physical gauge where the photon only enjoys transverse polarizations $k_\alpha \sum_\lambda \epsilon^\alpha_\lambda \epsilon^\beta_\lambda = 0$ there are no leading log contributions from the interference diagrams. This vanishing of collinear photon emission has a simple interpretation. The electromagnetic vertex $\bar{u}(p+k)\gamma_\alpha u(p)$ conserves lepton helicity in the relativistic limit $\mu/E \ll 1$ $[\bar{u}_L \gamma_\alpha u' = \bar{u} \gamma_\alpha u'_L]$. Thus the emission of transversally polarized collinear photons vanishes because of helicity conservation ($\frac{1}{2} \rightarrow \frac{1}{2} \pm 1$ is forbidden). The same argument does not manage to kill the logs from lepton leg diagrams like the one in Fig. 2a. There, there are two lepton propagators to fight and only one is cancelled by the "zero" of collinear photo-emission.

Let us now choose a covariant gauge : Feynman's. The polarization sum is $\sum_\lambda \epsilon^\alpha_\lambda \epsilon^\beta_\lambda = g^{\alpha\beta}$ and, in the collinear approximation, the numerator of Eq. (2.1) becomes $k^\beta T_\beta(\text{hadronic})$. One is then momentarily tempted to conclude that, as a consequence of gauge invariance ($k^\beta T_\beta(\text{hadronic}) = 0$?), the interference diagram does not give rise to leading logs. Not true, it is only a "complete" amplitude $\tilde{T}_\beta$, defined to contain photon insertions in all charged lines that satisfies the condition $k^\beta \tilde{T}_\beta = 0$. The complete $\tilde{T}$ is $T(\text{hadronic}) + T(\text{leptonic})$, where $T(\text{leptonic})$ refers to the photon insertion in the lepton leg, as in Fig. 2a. Thus $k^\beta T_\beta(\text{hadronic}) = -k^\beta T_\beta(\text{leptonic})$ and the lepton log from the interference diagram is related to the corresponding contribution of the bremsstrahlung diagram, scaled by an explicit factor encountered in computing the numerator of Eq. (2.1) in the collinear, zero mass approximation. Once again we conclude, this time in a covariant gauge, that it is not necessary to know the dynamics of photon couplings to hadrons or intermediate vector bosons to compute the leading lepton logs. The proof that hadron logs can be extracted from hadron leg diagrams follows a parallel path.

The proof of "factorization" just outlined is not complete, for it referred only to real photons, and not to virtual photon interference diagrams like the ones in Figs. 2e,f. In a physical gauge the leading logs from virtual photon interferences vanish. In a covariant gauge they are again related to

lepton leg or hadron leg diagrams, this time virtual propagator corrections like the one in Fig. 2b. The proof of these statements for virtual photons is similar to the proof for real photons that we have outlined before and we do not give it here. Perhaps the simplest way to carry it through is to study the mass "singularity" ($\ln \mu^2$) of the virtual photon diagrams in terms of the relevant absorptive parts (cuts of the diagrams in Figs. 2e,d through the charged lepton and photon lines).

The non-vanishing leading logs encountered in a covariant gauge calculation of an interference diagram can be traced back to the contribution from "unphysical" timelike and longitudinal polarizations. They cancel exactly against the corresponding contributions from bremsstrahlung and propagator correction diagrams. This is the way the physical gauge result and the covariant gauge result manage to be one and the same.

3. - THE LEPTON LEG LOG

The type of neutrino experiment we have in mind in discussing lepton leg radiative corrections is the following. The outgoing leptons are muons, not electrons. For muons the straggling radiative corrections are small ²⁾ and we do not further discuss them. Our results are also valid for outgoing electrons but, in practice, straggling corrections would have to be added to our results. The neutrino energy E_ν is either known ("narrow band beam") or measured as a sum of outgoing particles' energies : $E_\nu = E_\mu + E(\text{hadrons}) + E(\text{photons})$. If photons are present, a typical calorimetric device adds up their energies in measuring E_ν . A "directional" calorimeter may in principle be able to see hard photons associated with the direction of the outgoing lepton ⁶⁾ ; this would provide the possibility to test explicitly the lepton leg radiative corrections. The hadron-inclusive differential cross-section whose radiative corrections we discuss is $d\sigma(E_\nu)/dE_\mu d\Omega$, where E_μ is the outgoing muon lab. energy and Ω its solid angle.

Let $d\sigma_o/dE_\mu d\Omega$ be the observed cross-section (that allows for the presence of photons) and let $d\sigma_B/dE_\mu d\Omega$ be the bare cross-section. The relation between these quantities is :

$$\begin{aligned}
 \frac{d\sigma_0}{dE_\mu d\Omega} &= \frac{d\sigma_B}{dE_\mu d\Omega} + \frac{\alpha}{2\pi} \ln\left(\frac{4E_\mu^{*2}}{\mu^2}\right) \int_0^1 dz \frac{1+z^2}{1-z} \left\{ \frac{1}{z} \frac{d\sigma_B}{d\tilde{E}_\mu d\Omega} \right\} \Big|_{\tilde{E}_\mu = \frac{E_\mu}{z}}^{\Theta(z-z_{\min})} \\
 &\quad - \frac{d\sigma_B}{dE_\mu d\Omega} \Big\} + O\left(\frac{\alpha}{2\pi}\right) \\
 4E_\mu^{*2} &= 2mE_\nu(1-\gamma+xy) = [Q^2 + 2m_p E_\mu]^2 / 2m_p E_\nu \\
 z_{\min} &= \frac{E_\mu}{E_\nu} \left(1 + \frac{E_\nu}{m_p} (1 - \cos\theta_{\text{lab}})\right) \quad (3.1)
 \end{aligned}$$

where m_p is the target mass and θ_{lab} is the lab scattering angle between the outgoing muon and the neutrino beam direction. Here we have used the improved leading log variable $4E_\mu^{*2}/\mu^2$, whose merits are discussed in Chapter 5; E_μ^* is the outgoing muon centre-of-mass energy. Equation (3.1) is the complete answer to $O(\alpha)$, provided σ_B is interpreted as the total cross-section for the process weak current + target $\rightarrow$ hadrons + (eventual) photons. The quantity $d\sigma_B$ in Eq. (3.1) is the one that would be observed if the muon charge (but not the charges of hadron constituents) were to be switched off. These points are discussed in further detail in Section 4. In deriving Eq. (3.1) we have only assumed locality on the weak current, we did not invoke a model of the weak or the strong interactions, nor any of their properties, like approximate scaling.

The derivation and interpretation of Eq. (3.1) are straightforward. In a physical gauge, the first term under the integral sign in the right-hand side comes from the bremsstrahlung diagram of Fig. 2a. Muons of energy $\tilde{E}_\mu = E_\mu/z > E_\mu$ radiate photons and contribute to the uncorrected differential cross-sections for outgoing muons of energy E_μ . The leading log correction originates from collinear photons so that the angle Ω is the same in both sides of the equation. The precision of the collinear approximation will be discussed in Section 5. The cut $z \geq z_{\min}$ originates from the requirement that the final hadrons have an invariant mass equal or higher than the mass of the target hadron. The second term under the integral sign in the right-hand side of Eq. (3.1) originates from lepton propagator corrections like the one in Fig. 2b, and is thus proportional to the bare cross-section at the observed value of E_μ . The $z \rightarrow 1$ soft photon infra-red singularity cancels in the sum of virtual and real photons. Unlike in more familiar radiative corrections, there is no reference to the experimental resolution. This

is a consequence of the inclusive character of the measurement. Should the photon and/or the hadron energies be separately measured, the resolution would enter the expression for the radiative corrections.

In principle, the determination of $d\sigma_B$ from the data on $d\sigma_0$ would require an iteration procedure. For the energies involved and the precision achieved in present-day experiments, our model calculations indicate that the radiative corrections are small enough for the approximation $d\sigma_B \simeq d\sigma_0$ under the integral sign of Eq. (3.1) to be justified. This makes the experimental determination of $d\sigma_B$ from the observed $d\sigma_0$ particularly simple.

To investigate the size and general features of the radiative corrections in Eq. (3.1) we first re-express it in terms of the customary scaling variables and structure functions :

$$\frac{d\sigma_B}{dE_\mu d\Omega} = \frac{E_\mu}{2\pi m_p (E_\nu - E_\mu)} \frac{d\sigma_B}{dx dy} \quad (3.2)$$

$$\frac{\pi}{G_F^2 m_p E_\nu} \frac{d\sigma_B^{(\nu)}}{dx dy} = (1-y) F_2(x, Q^2) + y^2 x F_1(x, Q^2) + y(1-\frac{y}{2}) x F_3(x, Q^2) \quad (3.3)$$

In rewriting Eq. (3.1) we encounter the "shifted" values of x and y under a rescaling of E_μ , $E_\mu \rightarrow E_\mu/z$, at fixed angle Ω :

$$\tilde{x} \equiv x \Big|_{E_\mu \rightarrow E_\mu/z} = \frac{xy}{z+y-1} \quad (3.4)$$

$$\tilde{y} \equiv y \Big|_{E_\mu \rightarrow E_\mu/z} = \frac{z+y-1}{z} \quad (3.5)$$

The lepton log radiative correction Eq. (3.1), expressed in terms of $d\sigma/dx dy$ reads :

$$\begin{aligned} \frac{d\sigma_0}{dx dy} &= \frac{d\sigma_B}{dx dy} + \frac{\alpha}{2\pi} \ln \left(\frac{s(1-y+xy)^2}{\mu^2} \right) \int_0^1 dz \frac{1+z^2}{1-z} \left\{ \frac{y \theta(z-z_{\min})}{z(y+z-1)} \frac{d\sigma_B}{dx dy} \Big|_{\substack{x=\tilde{x} \\ y=\tilde{y}}} \right. \\ &\quad \left. - \frac{d\sigma_B}{dx dy} \right\} + O\left(\frac{\alpha}{2\pi}\right) \end{aligned} \quad (3.6)$$

$$S = 2 m_p E_\nu$$

For illustration purposes and to get an idea of the magnitude of the corrections, we parametrize $d\sigma_B/dxdy$ à la quark parton model in terms of a valence quark momentum distribution $V(x)$ and a sea quark distribution $S(x)$:

$$\left(\frac{G_F^2 m_p E_\nu}{\pi}\right)^{-1} \frac{d\sigma_B^\nu}{dxdy} = V(x) + S(x) [1 + (1-y)^2]$$

$$\left(\frac{G_F^2 m_p E_\nu}{\pi}\right)^{-1} \frac{d\sigma_B^{\bar{\nu}}}{dxdy} = V(x)(1-y)^2 + S(x)[1 + (1-y)^2] \quad (3.7)$$

The above expressions assume exact scaling and an exact Callan-Gross relation ($2xF_1 = F_2$). In practice, when Eq. (3.1) is used to correct the data, it is not necessary to make these assumptions. We will subsequently return to the question of scaling violations and Callan-Gross violations.

In Eqs. (3.7), we have chosen to use the rough parametrization ⁷⁾ :

$$V(x) = \sqrt{x} \cdot (1-x)^3$$

$$S(x) = .076 \cdot (1-x)^{6.5} \quad (3.8)$$

The normalized radiative correction

$$\Delta(x, y, E_\nu) = \frac{\frac{d\sigma_0(E_\nu)}{dxdy} - \frac{d\sigma_B(E_\nu)}{dxdy}}{\frac{d\sigma_B(E_\nu)}{dxdy}} \quad (3.9)$$

is rather insensitive to the choice of parametrization, with one exception : the y dependence of $\Delta(x, y, E_\nu)$ for antineutrinos at large y is sensitive to the choice of the sea $S(x)$. There, the choice $S(x)=0$ would result in misleadingly large radiative corrections.

The quantity $\Delta(x, y, E_\nu)$ is shown in Figs. 3a and 3b for neutrino and antineutrino scattering, respectively, at a "typical" energy $E_\nu = 100$ GeV. The radiative corrections at this energy are seen to be at the level of a few per cent and to be generally larger for x and y near the kinematical limits. As we discuss in detail in Chapter 5, the expected precision of our improved leading log calculation is $\sim 10\%$ of the result everywhere except at very high y .

In Figs. 4a and 4b we have plotted the quantity

$$\Delta(y, E_\nu) = \frac{\int dx \left[\frac{d\sigma_o(E_\nu)}{dx dy} - \frac{d\sigma_B(E_\nu)}{dx dy} \right]}{\int dx \frac{d\sigma_B(E_\nu)}{dx dy}} = \frac{\frac{d\sigma_o(E_\nu)}{dy} - \frac{d\sigma_B(E_\nu)}{dy}}{\frac{d\sigma_B(E_\nu)}{dy}} \quad (3.10)$$

for neutrino and antineutrino scattering, respectively. The energy dependence of $\Delta(y, E_\nu)$ is seen to be approximately linear in $\ln(E_\nu/\mu)$. As anticipated the observed cross-section $d\sigma_o/dy \approx (1 + \Delta(y, E_\nu)) d\sigma_B/dy$ is an over-estimate (under-estimate) of the "true" radiatively corrected cross-section $d\sigma_B/dy$ at large y (small y). In practice, not too much attention should be paid to Fig. 4. In computing $\Delta(y, E_\nu)$ we have made an x integration in the whole range $0 < x < 1$. Different experiments may require different x cuts. To give an idea of the possible effect of these cuts we show in Fig. 4 the quantity Δ of Eq. (3.10) at $E_\nu = 200$ GeV with the x integrals extending only over the range $0.05 < x < 0.9$. In the analysis of actual data, the lepton leg radiative corrections should be implemented at the level of the double differential cross-section by use of Eq. (3.1). The radiatively corrected double differential cross-section may then be partially integrated and/or re-expressed in terms of scaling variables in the customary fashion.

In Figs. 5a and 5b we have plotted the quantity

$$\Delta(x, E_\nu) = \frac{\int dy \left[\frac{d\sigma_o(E_\nu)}{dx dy} - \frac{d\sigma_B(E_\nu)}{dx dy} \right]}{\int dy \frac{d\sigma_B(E_\nu)}{dx dy}} = \frac{\frac{d\sigma_o(E_\nu)}{dx} - \frac{d\sigma_B(E_\nu)}{dx}}{\frac{d\sigma_B(E_\nu)}{dx}} \quad (3.11)$$

for neutrino and antineutrino scattering, respectively. The energy dependence of $\Delta(x, E_\nu)$ is seen to be approximately linear in $\ln(E_\nu/\mu)$. As anticipated the observed cross-section $d\sigma_0/dx \approx (1 + \Delta(x, E_\nu))d\sigma_B/dx$ is an over-estimate (under-estimate) of the "true" radiatively corrected cross-section $d\sigma_B/dx$ at small x (large x). The interpretation of Fig. 5 is subject to the same words of caution as the ones given in Fig. 4 at the end of the previous paragraph. Also shown in Fig. 5 is the quantity Δ of Eq. (3.11) at $E_\nu = 200$ GeV with the y integral extending over the range $0.05 < y < 0.9$.

4. - THE HADRON LEG LOG

As we discussed in the Introduction, the relation between an observed cross-section σ_0 and a radiatively corrected bare cross-section σ_B is of the form

$$\sigma_0 - \sigma_B = \frac{\alpha}{2\pi} \left\{ \ln\left(\frac{Q^2}{\mu^2}\right) F\{\sigma_B\} + \ln\left(\frac{Q^2}{M^2}\right) H + G \right\} \quad (1.1)$$

The first term in the right-hand side is the lepton log discussed in the previous chapter. We now turn to the analysis of the second term in the right-hand side of Eq. (1.1) : the hadron log. We have given in Chapter 2 an outline of the proof that the hadron logs factorize : they arise from diagrams where photons attach exclusively to hadrons.

To compute or even to discuss the hadron leg radiative corrections requires a model of hadrons. An aim of this chapter is to find ways to have the radiative corrections - at least at the leading log level - depend as little as possible on the details of the hadronic model. In an attempt to be clear we first discuss radiative corrections in the context of a quark parton model with no strong interaction induced scaling violations. In Chapter 7, we will discuss scaling violations in the context of QCD and the effect of radiative corrections on their analysis.

In a quark parton model the cross-sections for $\nu(\bar{\nu})$ scattering in the scaling regime are :

$$\frac{\pi}{G_F^2 m_p E_\nu} \frac{d\sigma^\nu}{dx dy} = \sum_i x \{ q^i(x) + \bar{q}^i(x) (1-y)^2 \}$$

$$\frac{\pi}{G_F^2 m_p E_{\bar{\nu}}} \frac{d\sigma^{\bar{\nu}}}{dx dy} = \sum_i x \{ \bar{q}^i(x) + q^i(x) (1-y)^2 \} \quad (4.1)$$

where the sum runs over quark flavours i , and $xq(x)$ [$x\bar{q}(x)$] denote the type- i quark (antiquark) momentum distributions. The relation between an observed quark distribution $q_{OB}^i(x)$ and a "true" radiatively corrected bare distribution $q_{BB}^i(x)$ is, in the leading log approximation :

$$q_{OB}^i(x) = q_{BB}^i(x) + e_i^2 \frac{\alpha}{2\pi} \ln\left(\frac{Q^2}{m_i^2}\right) \int_0^1 dz \frac{1+z^2}{1-z} \left\{ \frac{1}{z} q_{BB}^i\left(\frac{x}{z}\right) \theta(z-x) - q_{BB}^i(x) \right\} \quad (4.2)$$

where e_i and m_i are the type- i quark electric charge and mass. Here $q_{OB}^i(x)$ stands for the "observed" quark distribution for the process current+nucleon $\rightarrow$ anything. We already have, in extracting $q_{OB}^i(x)$ from the data, implemented the lepton log radiative corrections. We write a double "bare" subindex in the right-hand side of Eq. (4.2) to emphasize that we are now going one step further and radiatively correcting q_{OB}^i to extract the "true" quark distribution one would observe if the quark charges could be switched off. We notice en passant an important difference between hadron leg and lepton leg corrections. The leading log hadron leg corrections do not distort the y distributions, while the corresponding lepton leg corrections do ; compare Eq. (3.6) with Eqs. (4.1) and (4.2).

Presented with Eqs. (4.1) and (4.2) one may foresee absolute disaster. First, doing the hadron leg radiative corrections to Eq. (4.1) seems to require detailed assumptions on the relative contributions of quarks of different charges to the structure functions. Second, and much more fearsome, is the apparent necessity to invoke explicit values for quark masses. We gave in the Introduction an explicit example in which different possible assumptions about light quark masses affected the values of radiative corrections by a factor close to three. As we have repeatedly announced there is a way out of these limitations and we proceed to discuss it in detail.

In a physical gauge, the first term under the integral sign in Eq. (4.2) originates from the incoming quark bremsstrahlung corrections depicted in Fig. 6a. The quark is seen by the lepton probe at $Q^2/2mv = x$, a value to which it has descended from a higher one of x/z by the emission of a photon. Bremsstrahlung by the outgoing quark does not affect the definition of (inclusive) structure functions or quark distributions. The last term under the integral sign of Eq. (4.2) originates from the virtual corrections depicted in Fig. 6b and cancels the bremsstrahlung infra-red singularity. What the radiative correction of Eq. (4.2) accomplishes is the undressing of the quark distribution : the extraction of the bare quark distribution $q_{BB}^i(x)$ from the observed one $q_{OB}^i(x)$, which includes the possibility that photons be coupled to quarks. The expensive price one pays to see the electromagnetically undressed quarks is the introduction of quark charges and masses, that govern the probability of photon emission. But we are in a position to avoid paying this price.

Suppose we decide to define inclusive structure functions as the total "cross-sections" for the scattering of weak or electromagnetic currents on hadrons. What we mean by this apparent tautology is the extension of the meaning of "inclusive" from "hadron inclusive" to "hadron and photon inclusive". That is, we study the process $\text{current} + \text{nucleon} \rightarrow \text{anything}$ and include all events in "anything", irrespective of whether photons do or do not accompany the final hadronic state. Somewhat tacitly, this is the way that the radiative corrections to electron scattering have been implemented in the past. At the level of radiative corrections to quark momentum distributions an "electromagnetically inclusive" definition of the final states means the acceptance of $q_{OB}^i(x)$ and not $q_{BB}^i(x)$ as the quantity one wants to measure. It is only at this model-dependent level that not switching off the quark electric charges may imply a loss of information.

With our electromagnetically inclusive definition of structure functions, their radiative corrections at fixed Q^2 need not be implemented at all. One need not go through the analysis of structure functions in terms of sums of contributions of different quark types. Quark masses or even quarks need not be talked about. Hadron leg radiative corrections may be completely ignored. All one should do to properly define and measure the weak current hadronic cross-sections is the lepton leg radiative corrections. It should be clear that all of the above statements are more general than the quark parton model used in this discussion. As far as we can guess, they are correct in any renormalizable theory of the strong interactions. They are

also true, for instance, for the "elastic" contribution, for which the final state consists of a nucleon, a muon and an eventual photon. The statements in this paragraph make full use of factorization and are only true at the leading log approximation. This is the only serious limitation of a very simple way to avoid the model-dependence of hadronic radiative corrections.

To avoid an explicit quark mass dependence and other evils in the calculation of radiative corrections, we have temporarily accepted not to subtract from the observed inclusive structure functions the theoretically calculated contribution of events with photons emitted at the hadronic vertex. We have not used Eqs. (4.1) and (4.2) to attempt to extract the electromagnetically bare $q_{BB}^i(x)$. There are two reasons why, at the present state of development of theory and experiment, the loss of information involved in not extracting $q_{BB}^i(x)$ from the data is not so great. First, suppose a large number of reactions are studied with high precision to disentangle the contributions $q_{OB}^i(x)$ of quarks of different types to the structure functions. The electromagnetically corrected $q_{BB}^i(x)$ would still depend on the assumed values of the quark masses. Whether one can gather enough data to turn the argument around and measure quark masses is an interesting question that we do not pursue any further in this paper. The second reason not to go too far in electromagnetically correcting quark distributions is our relative ignorance of strong interaction dynamics. To discuss this point we arrogantly assume that QCD is right, but admit that its long-distance or confining properties are not yet understood. In this scenario the x dependence of quark distribution functions, a "long-distance" property, is not predictable. What is calculable is their Q^2 dependence. For a "non-singlet" combination, of which

$$N^i(x, Q^2) \equiv q^i(x, Q^2) - \bar{q}^i(x, Q^2)$$

is an example, the leading order QCD prediction is ⁸⁾

$$N^i(x, Q^2) \simeq N^i(x, m^2) + \frac{4}{3} \frac{\alpha_s(Q^2)}{2\pi} \ln\left(\frac{Q^2}{m^2}\right) \int_0^1 dz \frac{1+z^2}{1-z} \left\{ \frac{1}{z} N^i\left(\frac{x}{z}, m^2\right) \theta(z-x) - N^i(x, m^2) \right\} \quad (4.3)$$

where α_s is the strong analogue of α and $N^i(x, m^2)$ is not calculable. Notice the similarity of Eqs. (4.2) and (4.3), the latter can be interpreted as the former, with reference to Fig. 6 and the substitution of photons for gluons. The only major difference between the two equations is the flavour-independence of the strong interactions : in QCD the strong quark charge is universal. To exploit Eq. (4.3) one must at present limit oneself to the comparison between two relatively close momentum scales Q_1 and Q_2

$$N^i(x, Q_1^2) - N^i(x, Q_2^2) \approx \frac{4}{3} \frac{\alpha_s(Q_1^2)}{2\pi} \ln \frac{Q_1^2}{Q_2^2} \int_0^1 dz \frac{1+z^2}{1-z} \left\{ \frac{1}{z} N^i\left(\frac{x}{z}, Q_1^2\right) \Theta(z-x) - N^i(x, Q_1^2) \right\} + O(\alpha_s^2) \quad (4.4)$$

The quark mass dependence (also referred to as long-distance or infra-red sensitivity) is gone from Eq. (4.4). We shall see that the hadron leg radiative corrections to the comparison of structure functions at two different momentum scales are smaller than the theoretical uncertainty involved in Eq. (4.4) or its presently known improved versions. Employ in QED the same trick used in QCD to go from Eq. (4.3) to Eq. (4.4), that is subtract Eq. (4.2) at $Q=Q_2$ from the same expression at $Q=Q_1$ to obtain

$$q_{08}^i(x, Q_1^2) - q_{08}^i(x, Q_2^2) = e_i^2 \frac{\alpha}{2\pi} \ln \frac{Q_1^2}{Q_2^2} \int_0^1 dz \frac{1+z^2}{1-z} \left\{ \frac{1}{z} q_{08}^i\left(\frac{x}{z}, Q_1^2\right) \Theta(z-x) - q_{08}^i(x, Q_1^2) \right\} + O(\alpha^2) \quad (4.5)$$

The Q^2 dependence induced by hadron leg electromagnetic radiative corrections is of order $3/4 e_i^2 \alpha/\alpha_s$ relative to the one induced by first order strong QCD. This is no doubt a smaller correction than the unknown and necessarily neglected higher order and higher twist QCD effects. Yet the electromagnetic corrections to the study of scaling violations can be explicitly implemented even beyond the leading log level, the $O(\alpha/2\pi)$ "constant" terms drop from difference equations like Eq. (4.5). Since the analysis of scaling deviations is of paramount importance and may improve both theoretically and experimentally, one may take the attitude that one should make the radiative corrections to scaling deviations, even if they are small, to be confronted with pure strong interaction effects. This attitude we take in Chapter 7.

The conclusion of the discussion in this chapter is the following. In our present state of partial ignorance about the strong interactions, the most crucial information one can extract from inclusive structure functions

concerns their evolution with Q^2 . The effect of hadron leg electromagnetic corrections on this evolution is calculable (quark mass independent) but smaller than the leeway allowed by our present ignorance of higher order strong interaction effects. For completeness, and hoping for improvement of both theory and experiment, we compute these corrections in Chapter 7. A less seasonable conclusion is that an electromagnetically inclusive definition of inclusive structure functions (a definition in which the usual "anything" includes photons as well as hadrons) obviates the necessity of hadron leg electromagnetic corrections at the leading log level. It is interesting and useful to compare electron and neutrino scattering data. This comparison between (lepton-leg radiatively corrected) electron data and neutrino data only makes sense if the radiative corrections have been treated on similar grounds. For the neutrino case this means ignoring hadron leg corrections, legal at the leading log level.

5. - PRECISION OF THE IMPROVED LEADING LOG RADIATIVE CORRECTIONS

In this chapter we discuss the optimal way to implement a leading log calculation of radiative corrections and we ascertain the precision of this particular approximation. We discuss lepton logs and hadron logs in turn. As in Chapter 2 the statements are precisely correct in a physical gauge ; when the dust settles, the answer is gauge invariant : we do not exhaust the reader with the gymnastics of being gauge-invariant throughout.

In practice, the precision of a leading log approximation may seriously deteriorate close to the edges of the (x,y) kinematical square, that is when x or y tend to 0 or 1, with the other variable fixed. We refer in what follows to these limits as the "edges". We have repeatedly seen that a leading lepton log radiative correction yields answers of the type :

$$\sigma_0 - \sigma_B = \frac{\alpha}{2\pi} \ln \frac{Q^2}{\mu^2} F(x,y,\sigma_B) + K(x,y) \quad (5.1)$$

As in Chapter 2, the $\ln Q^2/\mu^2$ term arises from the $\cos\theta$ angular integration in the phase space of the unobserved photon. The μ^2 mass term originates from the strictly collinear lepton-photon configuration. The quantity Q^2 is a "typical" momentum scale for the process, it originates

from the "other end" of the $\cos\theta$ angular integration, it must be there for purely dimensional reasons and is customarily left somewhat unspecified. At this stage, for no particular reason, we have chosen Q^2 to be the observed neutrino-to-muon momentum transfer. The traditional excuse for this lack of precision is that the meaning of the mass scale " Q^2 " must be found beyond the leading mass singularity level. We shall see that we can further specify this mysterious mass scale in an extremely useful manner. As usual, the quantity $F(x,y,\sigma_B)$ in Eq. (5.1) can be explicitly written down, see Eqs. (3.1) or (3.6). The function $F(x,y,\sigma_B)$ diverges at the edges as $\ln x$, $\ln(1-x)$, $\ln y$ but has no singularity as $y \rightarrow 1$ at fixed x . The constant term $K(x,y)$, defined as having no mass ($\mu \rightarrow 0$) singularities is potentially divergent at the edges as $\ln^2 x$, $\ln^2(1-x)$, $\ln^2 y$, $\ln^2(1-y)$, but not worse. The reason for this limited singular behaviour is that the radiative correction is a photon momentum triple integral of the form $d^3k/\text{propagators}$. The double logs originate from doubly singular configurations $\{\int_a dz/z\}^2$ where the photon is both soft and collinear relative to a single fermion line. Typically, singular results ($a \rightarrow 0$) can be reached at the kinematical limits : the "edges". In the leading log approximation the "third" photon momentum integral : the azimuthal $d\phi$ integration is not done exactly, but just approximated by a factor 2π . The exact $d\phi$ integration would not change the singularity structure of the answer, at most it would disguise double logs as Spence functions with the same singular behaviour at the edges. Given the no-worse-than-doubly-logarithmic behaviour of the unspecified constant term $K(x,y)$ in Eq. (5.1), dropping it in practice can be expected to be a good approximation to the extent that

$$\ln \frac{Q^2}{\mu^2} \{ \ln x, \ln(1-x), \ln y \} \gg \ln^2 x, \ln^2(1-x), \ln^2 y, \ln^2(1-y) \quad (5.2)$$

Of these edge conditions, the most restrictive is the $y \rightarrow 1$ one

$$\ln \frac{Q^2}{\mu^2} \approx \ln \frac{2m E_\nu x}{\mu^2} \gg \ln^2(1-y)$$

where we have continued to interpret Q^2 as momentum transfer. To give an example, at typical values $E_\nu = 100$ GeV, $x \sim 1/3$ one may hope for a precision better than ϵ in the radiative correction itself, provided that

$$\ln \frac{2 m E_\nu x}{\mu^2} \simeq 8.64 \gtrsim \frac{1}{\epsilon} \ln^2(1-y)$$

For $\epsilon = 20\%$ we are restricted to $y < 0.73$. For the same typical values and estimated precision we are to have $y > 0.18$. The $x \rightarrow 0, 1$ edges should be avoided to a similar extent. These conditions are very sad and restrictive, one may think that to obviate them it is necessary to do a lengthy calculation beyond the leading log level and give up all hope of model independence. Not so !

Fortunately, the simplicity, factorizability and model independence of the leading log calculation can all be extended to include the potentially most dangerous (doubly logarithmic) constant terms. We jump to the answer and then proceed to justify it. Change Eq. (5.1) to read

$$\sigma_0 - \sigma_B = \frac{\alpha}{2\pi} \ln \frac{s(1-y+xy)^2}{\mu^2} F(x, y, \sigma_B) + K'(x, y) \quad (5.3)$$

$$S = 2 m_p E_\nu$$

with $F(x, y, \sigma_B)$ the same as before and an explicit "improved" mass scale $s(1-y+xy)^2$ in the mass singularity logarithm. The advantage of the "improved" leading log expression Eq. (5.3) is that the newly defined constant term $K'(x, y)$ can only diverge at the edges at most as a single log : $\ln x$, $\ln(1-x)$, $\ln y$ or $\ln(1-y)$. This is true independently of whether $K'(x, y)$ originates from lepton leg diagrams or interference diagrams. The first term on the right-hand side of Eq. (5.3) now contains all possible lepton leg double logs. The improved mass singularity variable $s(1-y+xy)^2/\mu^2$ is simply found by correctly taking into account the kinematical limits in the photon phase space $d \cos \theta$ angular integration, rather than extending it from $\theta = 0$ to "some angle" corresponding to the momentum scale Q^2 . Explicitly,

$$\frac{s(1-y+xy)^2}{\mu^2} \simeq \frac{(p+k)_{\max}^2 - \mu^2}{(p+k)_{\min}^2 - \mu^2} = \frac{4 E_\nu^*{}^2}{\mu^2} \quad (5.4)$$

where the notation for muon and photon four-momenta (p and k) is as in Fig. 1a. We have alternatively written Eq. (5.4) in terms of E_μ^* , the muon energy in the neutrino-nucleon centre-of-mass system. The reason for the reduced (single log) edge singularity behaviour of $K'(x,y)$ in Eq. (5.3) relative to the (double log) behaviour of $K(x,y)$ in Eq. (5.1) is the following. Once again all double logs can be traced back to blowing propagators of single fermion lines in doubly singular collinear and soft photon limits. Double logs, arising from squares of amplitudes (propagators of single fermion lines) are factorizable, like the mass singularity logs. The characteristics of double logs are purely kinematical and can be found by doing the kinematics correctly. It has not escaped our attention that the chromodynamic calculations are the ones where our improved leading log technology may eventually be most useful.

Once we use the improved peaking, or leading log expression Eq. (5.3), the conditions for it to be a good approximation are not very restrictive :

$$\ln \frac{S(1-y+xy)^2}{\mu^2} \left\{ \ln x, \ln(1-x), \ln y \right\} \gg \ln x, \ln(1-x), \ln y, \ln(1-y) \quad (5.5)$$

At the "edges" these conditions read

$$\ln \frac{S}{\mu^2} \gg 1 \quad (5.6)$$

$$\ln \frac{S}{\mu^2} \gg \ln(1-y) \quad (5.7)$$

There are no longer any stringent conditions as $x \rightarrow 0,1$ at fixed y or $y \rightarrow 0$ at fixed x (other than very weak conditions stemming from having neglected masses, of the type $x \gg \mu/\sqrt{s}$, etc.). At a typical energy $E_\nu = 100$ GeV, Eq. (5.6) reads $9.74 \gg 1$ so that we may expect the improved leading log approximation to be good at the 10% level of the radiative correction itself, everywhere except at high y . As y approaches 1 the approximation deteriorates. Equation (5.7) indicates that we may expect a precision better than $\epsilon = 20\%$ provided

$$\ln \frac{s}{\mu^2} \approx 9.74 \approx \frac{1}{\epsilon} \ln(1-y) = 5 \ln(1-y)$$

or $y < 0.86$. If we demand $\epsilon = 30\%$ the limit moves to $y < 0.95$. These are not serious limitations on existing neutrino experiments, that have trouble of their own in measuring high y (relatively soft muon) events. Incidentally, the $y=1$ limit also destroys the reliability of the peaking approximation in electroproduction radiative corrections²⁾. There the problem is much more serious than in neutrino scattering : as $y \rightarrow 1$ the exchanged photon gets closer to its mass shell and the integrand to be approximated gets an extra non-collinear "peak"²⁾.

One may think that the improved leading log approximation gets into trouble when we proceed to discuss hadron logs. The reason is that the improved hadron log mass singularity variable $(Q^2/m_q^2 z(1-z))$ is not the same as the improved lepton log variable. Explicit "good approximation" conditions as Eq. (5.5) are not the same for hadron and lepton logs. Fortunately, there is no problem here, once more there are no hadron leading log corrections (mass singularities or kinematic double logs) if one decides as we discussed in Chapter 4, to define the hadron weak structure functions as electromagnetically inclusive quantities. In the study of scaling violations to be performed in Chapter 7, where we explicitly correct for hadron-associated photons, the difference between improved or unimproved hadron and lepton leading log variables drops from the final answer.

We conclude that the improved leading log radiative corrections are factorizable, model-independent, simple to implement and sufficiently precise for the analysis of current experiments.

6. - APPARENT "SEA" AND APPARENT "CALLAN-GROSS VIOLATIONS" INDUCED BY RADIATIVE EFFECTS

Radiative corrections are small but their relative effect is enormous on small but particularly interesting combinations of structure functions : the combinations that measure the "sea" of quark-antiquark pairs and the deviations from the Callan-Gross relation.

Let the bare differential cross-section for neutrino and anti-neutrino scattering, on an isoscalar target, Eq. (3.3) be rewritten in the form :

$$\frac{\pi}{G_F^2 m_p E_\nu} \frac{d\sigma_\nu^B}{dx dy} = \frac{1}{2} \left\{ \sigma_L(x, Q^2) + \sigma_R(x, Q^2)(1-y)^2 + 2\sigma_S(x, Q^2)(1-y) \right\} \quad (6.1)$$

$$\frac{\pi}{G_F^2 m_p E_\nu} \frac{d\sigma_\bar{\nu}^B}{dx dy} = \frac{1}{2} \left\{ \sigma_R(x, Q^2) + \sigma_L(x, Q^2)(1-y)^2 + 2\sigma_S(x, Q^2)(1-y) \right\} \quad (6.2)$$

where we have used the approximation of exact charge symmetry. The condition $\sigma_S = 0$ ($2xF_1 = F_2$) is generally referred to as the Callan-Gross relation. It holds exactly in the limit where the nucleon constituents are free spinors (the $\alpha_s \rightarrow 0$ limit of QCD) and have negligible "Fermi" motion ($\langle p_T \rangle / Q \rightarrow 0$). In the standard theory with only left-handed charged currents coupled to quarks, $\sigma_R = 0$ holds in the "naïve" limit where the nucleon constituency is limited to "valence" quarks, with no quark-antiquark "sea".

The quantities σ_R , σ_L and σ_S are experimentally extracted exploiting the y dependence in Eqs. (6.1) and (6.2). We define the x integrated quantities

$$R(x_0, x_1) \equiv \frac{\int_{x_0}^{x_1} \sigma_R(x) dx}{\int_{x_0}^{x_1} \{ \sigma_L(x) + \sigma_R(x) + 2\sigma_S(x) \} dx} \quad (6.3)$$

$$r(x_0, x_1) \equiv \frac{2 \int_{x_0}^{x_1} \sigma_S(x) dx}{\int_{x_0}^{x_1} \{ \sigma_L(x) + \sigma_R(x) + 2\sigma_S(x) \} dx} \quad (6.4)$$

Present experimental estimates in the scaling region are compatible with r vanishing at the level of a few per cent ("exact Callan-Gross") and $R(0.05, 0.9) = \text{few per cent}$ (small sea contamination). For the scaling parton model fit of Eqs. (3.7), for instance, $r = 0$ and $R(0.05, 0.9) = 6.4\%$. We shall see that radiative corrections induce an apparent $r \neq 0$ and $R \neq 0$ which are also at the few per cent level.

The relation between observed and bare cross-section is, as usual,

$$\frac{d\sigma_o}{dx dy} = \frac{d\sigma_B}{dx dy} + \frac{\alpha}{2\pi} \ln \frac{s(1-y+xy)^2}{\mu^2} F(x, y) \quad (6.5)$$

where the expression for $F(x, y)$ is given in Eq. (3.6). In this chapter we concentrate on the y dependence of the differential cross-sections. Only the lepton leg corrections affect this y dependence at the leading log level. Thus, we have only written in Eq. (6.5) the lepton leg corrections.

The form of the radiative correction in Eq. (6.5) is not a second degree polynomial in $(1-y)$ like the ones appearing in the bare cross-sections Eqs. (6.1) and (6.2). Yet, to investigate in a very rough manner the apparent corrections to σ_L , σ_R and σ_S induced by radiative corrections, we may imitate experimental procedure and fit the explicit form of the radiative correction with such a second degree polynomial :

$$\frac{\alpha}{2\pi} \ln \frac{s(1-y+xy)^2}{\mu^2} F^{\nu}(x, y) \approx \frac{1}{2} \left\{ \sigma_L^{\text{RAD}}(\nu) + \sigma_R^{\text{RAD}}(\nu) (1-y)^2 + 2 \sigma_S^{\text{RAD}}(\nu) (1-y) \right\} \quad (6.6)$$

$$\frac{\alpha}{2\pi} \ln \frac{s(1-y+xy)^2}{\mu^2} F^{\bar{\nu}}(x, y) \approx \frac{1}{2} \left\{ \sigma_R^{\text{RAD}}(\bar{\nu}) + \sigma_L^{\text{RAD}}(\bar{\nu}) (1-y)^2 + 2 \sigma_S^{\text{RAD}}(\bar{\nu}) (1-y) \right\} \quad (6.7)$$

This procedure must be carried independently for neutrinos and antineutrinos since the radiative corrections are not charge-symmetric, $\sigma_R^{\text{RAD}}(\nu) \neq \sigma_R^{\text{RAD}}(\bar{\nu})$, etc. To present results of an analysis along these lines we define, in analogy to Eqs. (6.3) and (6.4), the apparent x averaged ratios R and r induced by radiative corrections

$$R^{\text{RAD}}(x_0, x_1, S) \approx \frac{\int_{x_0}^{x_1} \sigma_R^{\text{RAD}}(x, S) dx}{\int_{x_0}^{x_1} \{ \sigma_L(x) + \sigma_R(x) + 2 \sigma_S(x) \} dx} \quad (6.8)$$

$$r^{\text{RAD}}(x_0, x_1, S) \approx \frac{2 \int_{x_0}^{x_1} \sigma_S^{\text{RAD}}(x, S) dx}{\int_{x_0}^{x_1} \{ \sigma_L(x) + \sigma_R(x) + 2 \sigma_S(x) \} dx} \quad (6.9)$$

Notice that we are normalizing to the bare cross-section. To estimate R^{RAD} and r^{RAD} , we use as input the scaling quark parton model fits of Eqs. (3.7). We make a minimum χ^2 fit of a polynomial of the form Eq. (6.7) to the y dependence of the radiative correction in the interval $0.1 < y < 0.9$ in bins of width $\Delta y = 0.1$. This way we estimate the radiatively induced corrections to σ_S , σ_R and σ_L .

Suppose that, following once more experimental tradition, we fit the radiative corrections with a second degree $(1-y)$ polynomial where, by assumption, the σ_S term is absent. In this case we get, at a typical energy $E_\nu = 100$ GeV

$$R_\nu^{\text{RAD}}(0.05, 0.9, E_\nu = 100) \simeq -5.2\% \quad (6.10)$$

$$R_{\bar{\nu}}^{\text{RAD}}(0.05, 0.9, E_{\bar{\nu}} = 100) \simeq 0.7\% \quad (6.11)$$

and $r_\nu^{\text{RAD}} = r_{\bar{\nu}}^{\text{RAD}}$ by construction. The fit of the y dependence is reasonable for neutrinos and quite bad for antineutrinos. The apparent radiatively induced fractional "sea" of Eq. (6.11) and particularly Eq. (6.10) is quite large in comparison with the corresponding bare cross-section input values $R_\nu(0.05, 0.9) = R_{\bar{\nu}}(0.05, 0.9) = 6.4\%$. The "large" structure function σ_L is also affected by radiative corrections at the same absolute level, i.e., a much smaller percentage level.

A more fair procedure, leading to rather good y polynomial fits of the radiative corrections, is to allow for $\sigma_S^{\text{RAD}} \neq 0$. In this case the results for the apparent fractional sea are rather different

$$R_\nu^{\text{RAD}}(0.05, 0.9, E_\nu = 100) \simeq -6.8\% \quad (6.12)$$

$$R_{\bar{\nu}}^{\text{RAD}}(0.05, 0.9, E_{\bar{\nu}} = 100) \simeq 1\% \quad (6.13)$$

The apparent violations of the Callan-Gross relation are at the level

$$r_\nu^{\text{RAD}}(0.05, 0.9, E_\nu = 100) = 1.7\% \quad (6.14)$$

$$Y_{\bar{\nu}}^{\text{RAD}}(.05, .9, E_{\bar{\nu}}=100) = 8.0\% \quad (6.15)$$

These results, compared with the bare input value ($r_{\nu} = r_{\bar{\nu}} = 0$) are rather large.

We have seen that the radiative corrections mock up a sea contribution ($R \neq 0$) and a violation of the Callan-Gross relation ($r \neq 0$) which are not charge symmetric and may be as large as several per cent. The bare values of R and r themselves are expected to be at the few per cent level at presently available energies making the potential effect of radiative corrections quite formidable. Our methods and results in this section, quite clearly, are only indicative. We conclude that radiative corrections are to be carefully implemented on the data before any serious attempts are made to determine the sea constituency or study the Callan-Gross relation. Published uncorrected and even radiatively corrected results along these lines should be taken cum grano salis.

7. - EFFECTS OF RADIATIVE CORRECTIONS ON THE STUDY OF SCALING VIOLATIONS

At the time of this writing experiment indicates that the charged weak currents are "conventional" : they are compatible with being entirely left-handed and having the postulated Glashow-Iliopoulos-Maiani structure ⁹⁾. The study of the structure of the weak current is de-emphasized in favour of its use as a tool to analyze the strong interactions and the structure of the nucleon. QCD, our present candidate for a theory of the strong interactions, makes definite predictions for the Q^2 evolution of the moments of structure functions. The simplest and easiest to test predictions concern "non-singlet" structure functions of which $x F_3$ (for neutrinos or anti-neutrinos on any given target) is an example ¹⁰⁾. Very beautiful data analyses along these lines have recently been published ^{11), 7)}.

For the sake of clarity we briefly recall the QCD predictions. Define the moments of the bare structure function

$$\int_0^1 (x F_3^{\text{BB}}(x, Q^2)) x^n dx \equiv \eta_n^{\text{BB}}(Q^2) \quad (7.1)$$

where x , in this chapter, is to be understood as Nachtmann's ξ scaling variable ¹²⁾. The QCD prediction, to the leading log level [all effects of order $(\alpha_s \ln Q^2)^n$ included] is :

$$M_n^{BB}(Q^2) = M_n^{BB}(Q_0^2) \left[\frac{\ln Q^2/\Lambda^2}{\ln Q_0^2/\Lambda^2} \right]^{-d_n} \left[1 + O\left(\frac{\alpha_s}{\pi}\right) + O\left(n \frac{\Lambda^2}{Q^2}\right) \right] \quad (7.2)$$

$$d_n = G \left(4 \sum_{j=1}^{n+2} \frac{1}{j} - 3 - \frac{2}{n+2} + \frac{2}{n+3} \right) \quad (7.3)$$

where Λ defines the strength of the strong interactions. For n flavours of colour triplets

$$G = \frac{4}{33-2n} \quad (7.4)$$

$$\alpha_s(Q^2) = 12\pi/(33-2n) \ln(Q^2/\Lambda^2) \quad (7.5)$$

The measured value of Λ is of the order of half a GeV.

In this chapter we concentrate on the effect of radiative corrections to Eq. (7.2). These corrections, as we shall see, can be explicitly computed even beyond the leading log level. They are particularly model-independent for isoscalar targets. What follows is a lengthy derivation. The trusting reader interested in results may jump to Eq. (7.16).

We follow experimental procedure and extract xF_3 from a difference of neutrino and antineutrino cross-sections on an isoscalar target. In the approximation of exact charge symmetry, the relation between the observed cross-section difference and the bare structure function xF_3^B is :

$$\begin{aligned} \frac{\pi}{2G_F^2 m_p E} \left(\frac{d\sigma_\nu^0 - d\sigma_{\bar{\nu}}^0}{dx dy} \right) &= F_3^{BB}(x, Q^2) y (1-y/2) + \\ &+ \frac{\alpha}{2\pi} \ln \frac{Q^2(1-y+xy)^2}{\mu^2 xy} \int_0^1 dz \frac{1+z^2}{1-z} \left\{ \frac{y \Theta(z-z_{\min})}{z(y+z-1)} \tilde{x} F_3^{BB}(\tilde{x}, \frac{Q^2}{z}) \tilde{y} (1-\tilde{y}/2) - \right. \end{aligned}$$

$$- x F_3^{BB}(x, Q^2) y(1-y/2) \Big\} + \frac{d}{2\pi} y(1-y/2) \sum_i e_i^2 \int_0^1 dz \ln \frac{Q^2}{m_i^2 z(1-z)} \cdot$$

$$\cdot \frac{1+z^2}{1-z} \left\{ x F_{3,i}^{BB}\left(\frac{x}{z}, Q^2\right) \theta(z-x) - x F_{3,i}^{BB}(x, Q^2) \right\} + O\left(\frac{d}{2\pi}\right) \quad (7.6)$$

where $\tilde{x}$, $\tilde{y}$ and $z_{\min}$ are defined as in Eqs. (3.4), (3.5), (3.6). The symbols e_i and m_i stand for quark charges and masses as in Chapter 4. We have made the Q^2 dependence explicit because we are eventually interested in comparing two different momentum transfer scales. The function $x F_{3,i}^{BB}$ represents the contribution to the bare structure functions of quarks and antiquarks of flavour i :

$$x F_3^{BB} = \sum_i x F_{3,i}^{BB} = \sum_i (x q_i^{BB}(x) - x \bar{q}_i^{BB}(x)) \quad (7.7)$$

Here, unlike in the rest of this paper, we are setting up to compute radiative corrections involving photon couplings to hadrons. The quark mass dependence and the unknown $O(\alpha/2\pi)$ corrections will drop from the radiative corrections to the chromodynamic prediction, Eq. (7.2).

As usual, the moments of an equation like Eq. (7.6) are simpler than the equation itself. Define the following y averaged x^n moments :

$$m_n^0(Q^2) = \frac{3}{2} \int_0^1 dy \int_0^1 x^n dx \left[\frac{\pi}{G^2 m_p E} \frac{d\sigma_v^0 - d\sigma_{\bar{v}}^0}{dx dy} \right] \quad (7.8)$$

$$m_n^{BB}(Q^2) = \int_0^1 dx (x F_3^{BB}(x, Q^2)) x^n \quad (7.9)$$

It is the first quantity that is experimentally interpreted as a measurement of the second. When radiative corrections are taken into account the relation between bare and observed moments is :

$$M_n^0(Q^2) = M_n^{BB}(Q^2) \left[1 + A_n \frac{\alpha}{2\pi} \ln \frac{Q^2}{\mu^2} \right] \left[1 + b_n \frac{\alpha}{2\pi} \sum_i e_i^2 \ln \left(\frac{Q^2}{m_i^2} \right) f_i^n \right].$$

$$\cdot \left[1 + \frac{\alpha}{2\pi} K_n + \frac{\alpha}{2\pi} O \left\{ m_i^2/Q^2, \mu^2/Q^2 \right\} \right]$$

(7.10)

We explain the meaning of the many new symbols in Eq. (7.10) in turn.

The coefficients A_n are moments of the lepton-log correction :

$$A_n \equiv \frac{3}{2} \int_0^1 dy^2 (1-y/2) \int_0^1 dz \frac{1+z^2}{1-z} \left\{ \left(\frac{zy}{1-z+yz} \right)^n \left(\frac{\ln Q^2/\Lambda^2 z}{\ln Q^2/\Lambda^2} \right)^{-d_n} - 1 \right\} \equiv$$

$$\equiv a_n + \alpha_n$$

(7.11)

$$a_n \equiv \frac{3}{2} \int_0^1 dy^2 (1-y/2) \int_0^1 dz \frac{1+z^2}{1-z} \left\{ \left(\frac{zy}{1-z+yz} \right)^n - 1 \right\} \quad (7.12)$$

$$\alpha_n \equiv \frac{3}{2} \int_0^1 dy^2 (1-y/2) \int_0^1 dz \frac{1+z^2}{1-z} \left(\frac{zy}{1-z+yz} \right)^n \left[\left(\frac{\ln Q^2/\Lambda^2 z}{\ln Q^2/\Lambda^2} \right)^{-d_n} - 1 \right] \quad (7.13)$$

The a_n are pure numbers, the α_n are formally $O(\alpha_s)$ corrections to them that, though calculable, will turn out to be negligible in practice. The inclusion of the α_n term affects the object of interest, the chromodynamic prediction Eq. (7.2), as a correction of order $\alpha\alpha_s$. This is smaller than the α_s^2 corrections (and, presumably, the α_s^3 corrections) that we have not included in Eq. (7.2). Hereinafter we drop the α_n term. The coefficients b_n are moments of the hadron log correction and they are related to QED anomalous dimensions. Not surprisingly the b_n are, up to a group factor, identical to the QCD non-singlet anomalous dimensions of Eq. (7.3) :

$$b_n \equiv -\frac{1}{2} \left[4 \sum_{j=1}^{n+2} \frac{1}{j} - 3 - \frac{2}{n+2} + \frac{2}{n+3} \right] \quad (7.14)$$

The quantity f_i^n is the fractional contribution to the n^{th} moment of quarks of flavour i :

$$f_i^n = \frac{\int_0^1 (x F_{3,i}^{BB}) x^n dx}{\int_0^1 (x F_3^{BB}) x^n dx} \quad (7.15)$$

The "constant" terms (as opposed to the leading log terms) appear in Eq. (7.10) as a veritable constant : K_n . Hidden in the K_n lies the difference between the Q^2 dependence of interest here and the dependence on the "improved" leading log variable. For the lepton log piece, for instance

$$\ln \frac{s(1-y+xy)^2}{\mu^2} = \ln \frac{Q^2}{\mu^2} + \ln \frac{(1-y+xy)^2}{xy}$$

The second term in the right-hand side of the above expression ends up in the moment relation Eq. (7.10) as part of the K_n term. Ditto for hadron logs. All that remains in Eq. (7.10) beyond the "constant" K_n level are terms that vanish as inverse powers of Q^2 .

Equation (7.10) presents the usual characteristics : the lepton log radiative corrections can be explicitly performed but the hadron log radiative corrections require knowledge of quark masses (m_i) and detailed dynamics (f_i^n). But imagine writing Eq. (7.10) at two momentum scales Q^2 and Q_0^2 and taking the ratio. The result is :

$$\frac{m_n^0(Q^2)}{m_n^0(Q_0^2)} = \frac{m_n^{BB}(Q^2)}{m_n^{BB}(Q_0^2)} \left[1 + C_n \frac{\alpha}{2\pi} \ln \frac{Q^2}{Q_0^2} \right] \cdot \left[1 + O(\alpha_s) \right] \left[1 + \frac{\alpha}{2\pi} O \left\{ \frac{m_i^2}{Q^2} - \frac{m_i^2}{Q_0^2} ; \frac{M^2}{Q^2} - \frac{M^2}{Q_0^2} \right\} \right] \quad (7.16)$$

$$C_n = a_n + \frac{5}{18} b_n \quad (7.17)$$

Lo !, the quark mass dependent logs, the detailed dynamics (f_i^n) and the constant terms (K_n) are all gone in one stroke. Equations (7.16) and (7.17) are therefore the exact expression of the radiative corrections to the moments, up to the order α_s level and up to "twist-four" corrections vanishing as inverse powers of Q . The model dependence on how differently flavoured quarks add up to $x F_3^{BB}$, Eq. (7.7), has dropped for an isoscalar target, for which

$$\sum_i e_i^2 f_i = \frac{1}{2} (e_u^2 + e_d^2) = \frac{5}{18} \quad (7.18)$$

The last equality holds for the conventional fractional quark charges.

The numerical values of a_n , b_n and c_n for a few values of n are given in the Table.

n	0	1	2	3	4	5	6	7
a_n	0	-1.875	-2.815	-3.448	-3.927	-4.312	-4.635	-4.913
b_n	-1.333	-2.083	-2.617	-3.033	-3.376	-3.668	-3.922	-4.147
c_n	-0.370	-2.454	-3.542	-4.290	-4.865	-5.331	-5.724	-6.065

We can read from the Table and from Eqs. (7.16), (7.17) that the radiative correction to the ratio of the first (fifth) moments at $Q_0^2 = 3 \text{ GeV}^2$ and $Q^2 = 100 \text{ GeV}^2$ is 1% (2.3%). The leading QCD prediction, Eq. (7.2), for these bare ratios is 39% (58%). While it is satisfactory to be able to make precise and model-independent radiative corrections to the chromodynamic moment predictions, we must conclude that radiative corrections are likely to be smaller than higher order and higher twist chromodynamic effects. These statements apply to the analysis of scaling violations in neutrino scattering. It is easy to extend them to the analysis of electron scattering experiments, where hadron leg radiative corrections have never been implemented.

8. - RADIATIVE CORRECTIONS TO "NEUTRAL CURRENT" DATA

Consider the neutral current processes

$$\nu + \mathcal{N} \rightarrow \nu + \Gamma$$

$$\bar{\nu} + \mathcal{N} \rightarrow \bar{\nu} + \Gamma$$

No radiative corrections are to be implemented on the corresponding data if the final state Γ is defined as being electromagnetically inclusive, as we explained in Chapter 4 for charged current scattering. Scaling violations in neutral currents, if ever observable, can be radiatively corrected much in the same way as charged current scaling violations (Chapter 7).

9. - CONCLUSIONS AND RECOMMENDATIONS TO EXPERIMENT

A task of inclusive charged current neutrino experiments is to measure the squares of matrix elements of weak currents J_μ between an initial nucleon N and an inclusive final state Γ :

$$W_{\alpha\beta}^B = \sum_{\Gamma} \frac{1}{\delta^{(4)}(P_r - q - P)} \langle N(P) | J_\alpha^\dagger | \Gamma(P_r) \rangle \langle \Gamma(P_r) | J_\beta | N(P) \rangle. \quad (9.1)$$

At the level at which radiative corrections are ignored, the cross-section is proportional to W^B :

$$\sigma_B \sim l^{\alpha\beta} W_{\alpha\beta} \quad (9.2)$$

$$l_{\alpha\beta} = \sum_{\text{spins}} \langle \nu | J_\alpha | \mu \rangle \langle \mu | J_\beta^\dagger | \nu \rangle \quad (9.3)$$

When radiative corrections are taken into account, the factorized expression Eq. (9.2) no longer holds.

Radiative corrections increase approximately logarithmically with energy and are potentially large, in particular in the determination of the small but important combinations of structure functions σ_R and σ_L in the

explicit form of $W_{\alpha\beta}^B$. The photons involved in radiative corrections inextricably link charged leptons and hadrons and invalidate the factorized form Eq. (9.2). To implement radiative corrections one needs to know how the electromagnetic current couples to hadrons. In principle this entails a strong model dependence of the corrections, involving in particular a dependence on quark masses. A dilemma is faced :

- i) radiative corrections may be relatively large and should be implemented, but
- ii) their implementation and actual numerical incidence on the data appears to be very model-dependent.

We have shown a way out of this dilemma : the radiative corrections are not model and quark mass dependent provided :

- i) one restricts oneself to the potentially large leading log radiative corrections of order $\alpha \ln s/\mu^2$ or order $\alpha \ln^2$ (dimensionless variables) ;
- ii) the final state Γ in Eq. (9.1) is interpreted as being electromagnetically inclusive : the sum extends over hadrons and photons.

Under these conditions a form of factorization is recovered, the observed cross-section σ_0 is still a linear functional of $W_{\alpha\beta}^B$ (or the bare cross-section σ_B) :

$$\sigma_0 \sim \sigma_B + \frac{\alpha}{2\pi} \ln \left(\frac{2m_p E_\nu}{\mu^2} \right) \mathcal{F}(\sigma_B) \quad (9.4)$$

This expression has the same form as the "lepton leg" radiative corrections traditionally implemented on electroproduction data.

We have improved upon the classical leading log or peaking approximation of radiative corrections to include the potentially most dangerous terms that may occur beyond the leading mass singularity level. In practice this means that we may expect our radiative correction formulae to be very good approximations for all x and for all but very large values of y [for which $|\ln(1-y)|$ becomes comparable to $\ln 2m E_\nu/\mu^2$].

We conclude with two recommendations :

- i) the leading lepton log radiative corrections of Eq. (3.1) should be separately implemented on neutrino and antineutrino data before any attempt is made to analyze them in terms of separate structure functions ; this may have a large impact on measurements of the "sea" and of "Callan-Gross deviations" ;
- ii) the radiative corrections to the evolution of moments of structure functions, Eqs. (7.16), (7.17) should perhaps be implemented, even if small, for the sake of elegance and completeness.

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FIGURE CAPTIONS

Figure 1 Diagrams appearing in the calculation of radiative corrections to charged lepton scattering.

- a) Lepton leg and photon propagator corrections ;
- b) photon bremsstrahlung from hadrons ;
- c) virtual photon corrections to hadron legs ;
- d) two-photon exchange ;
- d) a real photon lepton-hadron interference.

Figure 2 Diagrams appearing in the calculation of radiative corrections to neutrino scattering.

- a),b) lepton leg corrections ;
- c),e),f) interference diagrams ;
- d) hadron leg correction.

Figure 3 The doubly differential normalized radiative correction defined in Eq. (3.9) :

- a) for neutrino scattering ;
- b) for antineutrinos.

Figure 4 Examples of the y dependence of the x integrated normalized radiative correction defined in Eq. (3.10). Full lines are results integrated in the whole range $0 < x < 1$. The dashed line indicates the sensitivity to x cuts and is integrated in the range $0.05 < x < 0.9$:

- a) for neutrinos ;
- b) for antineutrinos.

Figure 5 Examples of the x dependence of the y integrated normalized radiative correction of Eq. (3.11). Full lines are results integrated in the whole range $0 < y < 1$. The dashed line indicates the sensitivity to y cuts and is integrated in the range $0.05 < y < 0.9$:

- a) for neutrinos ;
- b) for antineutrinos.

Figure 6 Diagrams contributing to leading hadron leg logs in a physical gauge :

- a) hadron leg incoming quark real photon bremsstrahlung ;
- b) hadron leg incoming quark virtual photon correction.

Fig. 1

Fig. 2

a)

b)

Fig. 3

Fig. 4

Fig. 5

Fig. 6