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FLAVOUR MIXING AND QUARK MASSES IN SEQUENTIAL FLAVOURDYNAMICS

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A B S T R A C T

We discuss the mixing of quark flavours in sequential flavour dynamics in case of an arbitrary number of quark generations. For n generations the dominant weak interaction mixing is described by $(n-1)$ mixing angles, which are computed in terms of the quark masses.

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The leptons and quark flavours observed thus far seem to come in three generations:

$$\begin{aligned}
 \text{I.} \quad & \begin{pmatrix} \nu_e & | & u \\ e^- & | & d \end{pmatrix} \\
 \text{II.} \quad & \begin{pmatrix} \nu_\mu & | & c \\ \mu^- & | & s \end{pmatrix} \\
 \text{III.} \quad & \begin{pmatrix} \nu_\tau & | & t \\ \tau^- & | & b \end{pmatrix}
 \end{aligned} \tag{1}$$

(we have assumed that a t-quark of charge $2/3$ exists with a mass larger than ~ 8 GeV). The lefthanded members of these generations form doublets of the weak isospin, apart from small weak mixing angles.

The mass ratios of the leptons and quarks belonging to different generations are observed to be large:

$$\begin{aligned}
 \frac{m_s}{m_d} &\approx 20 & \frac{m_\mu}{m_e} &\approx 210 \\
 \frac{m_b}{m_s} &\approx 29 & \frac{m_\tau}{m_\mu} &\approx 17 \\
 \frac{m_c}{m_u} &\approx 280 & \frac{m_t}{m_c} &\approx 7.
 \end{aligned} \tag{2}$$

Furthermore one has:

$$\begin{aligned}
 \frac{m_s}{m_d} &\sim \frac{m_b}{m_s} \\
 \frac{m_\mu}{m_e} &\sim \frac{m_\tau}{m_\mu}
 \end{aligned} \tag{3}$$

which implies that the logarithms of the lepton and quark masses when plotted as a function of the generation show almost straight lines ^{1, 2)}.

It has been suggested that the weak mixing angles are functions of the quark masses ^{2, 3)}. A particularly simple situation arises if the lepton and quark masses are generated solely by the weak interaction mixing. In the six-quark case this situation is realised if the quark mass matrix, written in terms of the weak interaction eigenstates takes the form ^{2, 3)}:

$$\overline{(u_o, c_o, t_o)}_L \begin{pmatrix} 0 & A & 0 \\ A & 0 & B \\ 0 & B & C \end{pmatrix} \begin{pmatrix} u_o \\ c_o \\ t_o \end{pmatrix}_R + \text{h.c.} \quad (4)$$

when $|A| \ll |B| \ll |C|$ (a similar ansatz is supposed to be valid for the (d, s, b)-system). The phases of the matrix elements A, B, and C can be absorbed by suitable redefinition of the quark fields, and we shall assume that the numbers A, B, and C are real and positive. If the quark mass matrix has the form given in eq. (4) the weak mixing angles are given in terms of the quark masses ^{2, 3)}, and one finds results which are in good agreement with the experimental constraints.

In this note we should like to discuss the quark masses and flavor mixing for an arbitrary, but finite number of generations. The quarks are denoted as follows:

$$\begin{pmatrix} U_1 & U_2 & \dots & U_n \\ D_1 & D_2 & \dots & D_n \end{pmatrix}.$$

where $\begin{pmatrix} U_1 \\ D_1 \end{pmatrix} = \begin{pmatrix} u \\ d \end{pmatrix}$, $\begin{pmatrix} U_2 \\ D_2 \end{pmatrix} = \begin{pmatrix} c \\ s \end{pmatrix}$ etc.

Sometimes we shall use the short notation $Q_i = \begin{pmatrix} U_i \\ D_i \end{pmatrix}$. We shall suppose that formulas like eq. (3) are valid in general:

$$\frac{m_2}{m_1} \sim \frac{m_3}{m_2} \sim \dots \sim \frac{m_{i+1}}{m_i} \dots \sim \frac{m_{n-1}}{m_n} \quad (5)$$

(m_i : mass of quark U_i or D_i respectively).

The weak interaction eigenstates are denoted by U_1^0, U_2^0 , etc. In terms of the latter the quark mass matrix is supposed to have the following form:

$$\begin{pmatrix} \overline{U_1^0}, \overline{U_2^0}, \overline{U_3^0}, \dots, \overline{U_n^0} \end{pmatrix}_L \begin{pmatrix} 0 & a_1 & 0 & 0 & \dots & 0 \\ a_1 & 0 & a_2 & 0 & \dots & 0 \\ 0 & a_2 & 0 & \dots & \dots & 0 \\ 0 & 0 & \dots & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots & 0 & a_{n-1} \\ 0 & 0 & 0 & \dots & a_{n-1} & a_n \end{pmatrix} \begin{pmatrix} U_1^0 \\ U_2^0 \\ U_3^0 \\ \dots \\ U_n^0 \end{pmatrix}_R + h.c. \quad (6)$$

where $a_1 \ll a_2 \ll a_3 \dots \ll a_n$. The coefficients a_i are taken to be real, since their phases can be absorbed in redefining the quark fields.

Generalizing the method used in ref. (2, 3) a mass matrix of the type (6) can be obtained using the flavor group $SU(2) \times SU(2) \times U(1)$. Here the quark masses are obtained by their coupling to Higgs scalars transforming like $(1/2, 1/2)$ under $SU(2)_L \times SU(2)_R$. Let us introduce n such scalars S_k ($k = 1, \dots, n$) and suppose that the interactions of the quarks with the scalar fields obeys the following $[U(1)]^n$ -symmetry:

$$\begin{aligned} Q_{1L}^0 &\rightarrow e^{i\alpha_1} Q_{1L}^0 & Q_{1R}^0 &\rightarrow e^{-i\alpha_1} Q_{1R}^0 \\ Q_{2L}^0 &\rightarrow e^{i\alpha_2} Q_{2L}^0 & Q_{2R}^0 &\rightarrow e^{-i\alpha_2} Q_{2R}^0 \\ &\vdots & & \vdots \\ Q_{n-1,L}^0 &\rightarrow e^{i\alpha_{n-1}} Q_{n-1,L}^0 & Q_{n-1,R}^0 &\rightarrow e^{-i\alpha_{n-1}} Q_{n-1,R}^0 \end{aligned} \quad (7)$$

$$\begin{aligned}
Q_{n,L(R)}^0 &\longrightarrow e^{i\alpha_n} Q_{n,L(R)} \\
S_k &\longrightarrow e^{i(\alpha_{k+1} + \alpha_k)} S_k \quad (k=1, 2, \dots, n-1) \\
S_n &\longrightarrow S_n
\end{aligned}$$

The Yukawa interaction which generates the quark masses and obeys the $[U(1)]^N$ -symmetry as well as the L-R symmetry is as follows:

$$\begin{aligned}
\mathcal{L}^{int} = & \sum_{k=1}^{n-1} f_k \bar{Q}_{k,L} S_k Q_{k+1,R} + f_k^* \bar{Q}_{k+1,L} S_k Q_{k,R} \\
& + f_n \bar{Q}_{n,L} S_n Q_{n,R} + h.c.
\end{aligned} \tag{8}$$

where the f_k are the Yukawa coupling constants. The coefficients α_n in eq. (6) are given by $|f_k| < 0 < |f_{k+1}|$.

It is easy to diagonalize the symmetric matrix (6), provided we neglect all terms which are of higher order in (a_i/a_{i+1}) . In this case one finds

$$m_i \cdot m_{i+1} = a_i^2 \quad (i = 1, \dots, n-1),$$

where m_i denotes the mass of U_i . (In case of the D-flavors we replace $m \rightarrow \tilde{m}$, $a \rightarrow \tilde{a}$). The masses m_i can be expressed in terms of the matrix elements a_i as follows:

$$m_i = \frac{[c-1-i]}{[c-1-i]} \prod_{r=1}^{[c-1-i]} a_{i+2r}^2 - a_n^2 \tag{9}$$

where $[c]$ denotes the integer part of c .

We have remarked earlier that the logarithms of the quark masses when plotted as functions of the generation index are nearly straight lines. Thus it may be useful to study the particular case where the logarithms of the quark masses are exactly straight lines as functions of the generation index i , e.g.

$$m_i = \varepsilon^{2(n-i)} m_0 \quad (\varepsilon < 1). \quad (10)$$

It is easy to see that in this case one has

$$\frac{a_i}{a_n} \approx \varepsilon^{2(n-i)+1}, \quad (11)$$

and the quark mass matrix (6) takes the special form:

$$\begin{pmatrix} 0 & \varepsilon^{2n-1} & 0 & 0 & \cdot & \cdot & 0 \\ \varepsilon^{2n-1} & 0 & \varepsilon^{2n-3} & 0 & \cdot & \cdot & 0 \\ 0 & \varepsilon^{2n-3} & 0 & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & 0 & \varepsilon^3 & 0 \\ \cdot & \cdot & \cdot & \cdot & \varepsilon^3 & 0 & \varepsilon \\ 0 & 0 & \cdot & \cdot & 0 & \varepsilon & 1 \end{pmatrix} \quad (12)$$

(we have normalized all matrix elements such that $a_n = 1$).

The structure of the matrix (12) reminds us of a perturbation expansion in terms of the small parameter ε , which may be related to $\alpha_w = g^2/4\pi$ (g : gauge coupling constant). Thus one may speculate that the quark masses are generated by radiative corrections. Of course, at the present time this is just a conjecture; no definite model has yet been constructed, which would give a mass matrix similar to the one given in (12).

Finally we calculate the rotation matrix needed to diagonalize the matrix (6). We suppose that all ratios (m_i/m_{i+1}) are small and of the same order, and take into account only the lowest non-vanishing terms of order (m_i/m_{i+1}) . Thus one obtains the following orthogonal matrix, which diagonalizes the U-quarks

$$O[U] \equiv \begin{pmatrix} 1 & (-1)^n \alpha_{1,2} & 0 & \cdot & \cdot & \cdot & 0 \\ -(-1)^n \alpha_{1,2} & 1 & (-1)^{n-1} \alpha_{2,3} & \cdot & \cdot & \cdot & 0 \\ 0 & -(-1)^{n-1} \alpha_{2,3} & 1 & \cdot & \cdot & \cdot & 0 \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & 1 & -\alpha_{n-2,n-1} & 0 \\ \cdot & \cdot & \cdot & \cdot & \alpha_{n-2,n-1} & 1 & \alpha_{n-1,n} \\ 0 & 0 & \cdot & \cdot & 0 & -\alpha_{n-1,n} & 1 \end{pmatrix} \quad (13)$$

where $\alpha_{i,i+1} \equiv \sqrt{\frac{m_i}{m_{i+1}}}$ and U denotes the U -quark mass eigenstates. Analogously one diagonalizes the D -quarks using an orthogonal matrix $O[D]$, which is obtained from (13) by the replacement $\alpha_{i,i+1} \rightarrow \beta_{i,i+1}$.

In order to arrive at the matrix (6) we had to absorb the phases of the coefficients a_i by redefining the quark fields. Taking into account these phase transformations the weak coupling matrix takes the form

$$\bar{U} O^{-1}[U] P O[D] D \quad (14)$$

where P is a diagonal $n \times n$ matrix, whose diagonal elements consist of the phase factors mentioned above. As a consequence the dominant weak currents are:

$$\bar{U}_{i,L} (-\theta_{i-1,i} D_{i-1,L} + D_{i,L} + \theta_{i,i+1} D_{i+1,L}) \quad (15)$$

where the angles $\theta_{i,i+1}$ are given by

$$\begin{aligned} \theta_{i,i+1} &= | \alpha_{i,i+1} - e^{i\delta_i} \beta_{i,i+1} | \\ &\approx \left| \sqrt{\frac{m_i}{m_{i+1}}} - e^{i\delta_i} \sqrt{\frac{\tilde{m}_i}{\tilde{m}_{i+1}}} \right| \end{aligned} \quad (16)$$

($\tilde{m}_i$: mass of the i -th D -quark; δ_i : phases given by the matrix P in eq. (14)). Since the phase factors cannot be expressed in terms of the quark masses, the angles θ can vary between $|\alpha - \beta|$ and $\alpha + \beta$.

We can describe the weak interaction mixing as follows. The dominant weak current acts between the U-flavor U_i and the D-flavor D_i . There exist small Cabibbo-like transitions between the flavor U_i and the flavors D_{i-1} and D_{i+1} , described by the angles $\theta_{i-1,i}$ and $\theta_{i,i+1}$ respectively, as shown below:

All other weak transitions (e.g. $U_i \rightarrow D_{i-2}$, etc.) are very small and have been neglected in our analysis.

In the cases $n = 2, 3$ our scheme reduces to the one discussed in ref. (2). For $n = 2$ the angle $\theta_1 = \theta_{1,2}$ (Cabibbo angle) is given by

$$\theta_{1,2} = \left| \sqrt{\frac{m_d}{m_s}} - e^{i\delta_1} \sqrt{\frac{m_u}{m_c}} \right|. \quad (17)$$

For $n = 3$ one is dealing in our approximation with two mixing angles $\theta_1 = \theta_{1,2}$ (see eq. (17)) and

$$\theta_2 = \theta_{2,3} = \left| \sqrt{\frac{m_s}{m_b}} - e^{i\delta_2} \sqrt{\frac{m_c}{m_t}} \right| \quad (18)$$

In an eight-quark theory given by the doublets

$$\begin{pmatrix} u & | & c & | & t & | & g \\ d & | & s & | & b & | & h \end{pmatrix}$$

we have three mixing angles, besides θ_1 and θ_2

$$\theta_3 = \theta_{3,4} = \left| \sqrt{\frac{m_b}{m_h}} - e^{i\delta_3} \sqrt{\frac{m_t}{m_g}} \right|. \quad (19)$$

Finally we should like to emphasize the following point. We have based our considerations on the assumption that the order of magnitudes of the ratios

of the masses of two consecutive flavors are comparable, i.e. typically of order $m_b/m_s \sim 20$. This implies that the masses of the seventh and eighth flavors are rather large ($\gtrsim 50$ GeV). Only one new flavor, the t-quark, is expected to have a mass such that it can be detected in the PETRA and PEP experiments to be carried out in the near future.

References

- 1) J.D. Bjorken, SLAC-PUB-2195 (September 1978).
- 2) H. Fritzsch, CERN preprint (March 1979).
- 3) See e.g.:
 - H. Fritzsch, Phys.Lett. 73B, 317 (1978);
 - T. Hagiwara et al., Phys.Lett. 76B, 602 (1978);
 - H. Georgi and D.V. Nanopoulos, Harvard preprint 78/A039 (1978).

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