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HADRONIC LEPTON-PAIR PRODUCTION BEYOND THE LEADING ORDER
IN PERTURBATIVE QCD

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A B S T R A C T

Hadronic lepton-pair production in perturbative QCD is calculated to order $\bar{g}^2$ to be usable in combination with the existing calculation of the anomalous dimension of order $\bar{g}^4$. The structure function of the subprocess $q + \bar{q} \rightarrow \gamma + \text{anything}$ in an arbitrary covariant gauge for gluons is found to have gauge-dependent contributions which are exactly cancelled by contributions from operator matrix elements to give the gauge-independent "coefficient function" for lepton-pair production in the minimal subtraction scheme. This provides a consistency check of the perturbative QCD recipe. A comparison is made with other existing calculations.

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1. INTRODUCTION

The scaling violations of deep inelastic lepton-hadron scattering have been analyzed¹⁾ to make precise quantitative tests of quantum chromodynamics (QCD)²⁾. The recent progress in perturbative QCD³⁾⁻⁷⁾ has opened an intriguing possibility for precise tests of QCD in a vast variety of hard processes even when the operator-product expansion⁸⁾ does not apply. Among them the hadronic lepton-pair production experiment⁹⁾ such as $pp \rightarrow \mu\mu X$ is providing perhaps the most accurate and the most extensive data at high energies. The purpose of our paper is to present a concrete formula for structure functions of hadronic lepton-pair production to the order of $\bar{g}^2$ in the same renormalization scheme as that of the existing calculation¹⁰⁾ of the anomalous dimension of order $\bar{g}^4$ (minimal subtraction scheme). In the course of the calculation we obtain a consistency check of the perturbative QCD in terms of the gluon gauge dependence.

The operator-product expansion^{2),8)} gives the structure function W^{DI} (νW_2 etc.) of deep inelastic lepton-hadron scattering as a convolution of the coefficient function c and the inverse Mellin transform f of hadron matrix elements of twist-2 operator renormalized at Q^2

$$W^{\text{DI}}(x, Q^2) = \int_x^1 \frac{dx}{x} c(x, \bar{g}^2) f\left(\frac{x}{x}, Q^2\right) \quad (1.1)$$

The Q^2 dependence of the function f (we can regard f as the Q^2 dependent parton distribution function¹¹⁾) is controlled by the anomalous dimension. It is now well known that both the coefficient function c of order $\bar{g}^2$ and the anomalous dimension of order $\bar{g}^4$ depend on the renormalization prescription and the gluon gauge in general^{10),12)}. Only when we combine the coefficient function and the anomalous dimension calculated in the same prescription and gauge, can we obtain a consistent result for the structure function of the hadron which is independent of the renormalization prescription and the gluon gauge. So far the anomalous dimension of order $\bar{g}^4$ has been obtained only in the minimal subtraction scheme¹⁰⁾.

Perturbative QCD³⁾⁻⁷⁾ provides similar formulas as (1.1) for various hard processes such as hadronic lepton-pair production. Since the same anomalous dimension as in deep inelastic lepton-hadron scattering controls the Q^2 dependence of the parton distribution function f , these "coefficient functions" should depend on the renormalization prescription

and the gluon gauge in general. We calculate the coefficient function for lepton-pair production in the minimal subtraction scheme which can consistently be used with the existing calculation of the anomalous dimension of order $\bar{g}^4$. Previous calculations¹³⁾⁻¹⁵⁾ of hadronic lepton-pair production did not use the minimal subtraction scheme and therefore should not be used with the existing calculation¹⁰⁾ of the anomalous dimension of order $\bar{g}^4$.

Since the coefficient function is independent of the target state, it can be obtained by calculating structure functions of the (slightly off-shell) parton (quark or gluon) target provided operator matrix elements between the parton state are subtracted¹²⁾. For the (slightly off-shell) parton target, both structure functions and operator matrix elements depend on the gauge of gluons. In the minimal subtraction scheme, however, the anomalous dimension of the twist-2 operator is known to be gauge independent¹⁶⁾. Therefore, to make Eq. (1.1) consistent, the coefficient function should also be gauge independent. The coefficient function of deep inelastic scattering has been calculated in an arbitrary covariant gauge¹²⁾ and found to be independent of the gauge parameter α . In order to see the consistency of the perturbative QCD recipe, we calculate the lepton-pair production structure function of parton target and beam in an arbitrary covariant gauge for gluons. We explicitly demonstrate that the structure function has gauge-dependent contributions which exactly cancel gauge-dependent contributions from operator matrix elements between parton states. Thus we are able to show that the coefficient function for lepton-pair production is in fact independent of the gauge parameter α .

In the next section the perturbative QCD recipe is described for hadronic lepton-pair production and coefficient functions of sub-processes $q + \bar{q} \rightarrow \gamma + \text{anything}$ and $q + \text{gluon} \rightarrow q + \gamma$ are presented in an arbitrary covariant gauge of gluons. In Section 3 lepton-pair mass spectrum is calculated and is compared with the previous works. The structure function of the sub-process $q + \bar{q} \rightarrow \gamma + \text{anything}$ is described in Appendix A and its gauge term in an arbitrary covariant gauge is calculated in Appendix B. Appendix C is devoted to studying other structure functions which can be obtained by measuring lepton angular distributions.

2. PERTURBATIVE QCD FOR LEPTON-PAIR PRODUCTION

2.1 Recipe of perturbative QCD

The lepton-pair production cross section from the collision of hadrons P_1 and P_2 is given by a structure function^{*)} W

$$q_0 \frac{d\sigma}{dq^2 d^3\vec{q}} = \frac{4\alpha^2}{3\lambda(q^2)^2} W(x_1, x_2, x_{12}, q^2),$$

$$W(x_1, x_2, x_{12}, q^2) = -g_{\mu\nu} W^{\mu\nu}, \quad (2.1)$$

$$W^{\mu\nu} = \frac{\pi q^2}{2} \int \frac{d^4y}{(2\pi)^4} e^{iqy} \langle P_1, P_2 | J^\mu(y) J^\nu(0) | P_1, P_2 \rangle$$

(2.2)

where q is the four momentum of the lepton-pair as illustrated in Fig. 1, α is the fine structure constant and

$$\lambda = (P_1 + P_2)^2 \simeq 2(P_1 \cdot P_2),$$

$$x_i = \frac{q^2}{2(P_i \cdot q)}, \quad (2.3)$$

$$x_{12} = \frac{2(P_1 \cdot q)(P_2 \cdot q)}{q^2(P_1 \cdot P_2)}.$$

In the centre-of-mass frame of P_1 and P_2 , we have

*) Our state is normalized as $\langle P | P' \rangle = (2\pi)^3 2P_0 \delta^{(3)}(P - P')$.
Our metric is $g^{\mu\nu} = [1, -1, -1, -1]$.

$$q_T^2 / q^2 = X_{12} - 1$$

$$q^2 / P = (X_1 - X_2) X_{12} \quad (2.4)$$

$$\tau \equiv q^2 / \Delta = X_1 X_2 X_{12}$$

where $P_1^\mu = (P, 0, 0, P)$, and $q^\mu = (q^0, \vec{q}_T, q^Z)$. Since we have shown recently that the correction due to the non-vanishing hadron mass is small for q_T - as well as q^2 - distributions of lepton-pairs¹⁷⁾, we neglect hadron and quark masses in this paper. Hereafter we will denote structure functions with hadron target and beam by capital letters, and those with partons (quark or gluon) by small letters $w^{\mu\nu}$, etc.

Ellis et al.⁵⁾ have recently shown by using the axial gauge that mass singularities of inclusive cross sections with parton beam, target or produced particles factor to all orders in perturbation theory of QCD. We can express the factorization in the case of lepton-pair production taking the generalized Bjorken limit

$$q^2 \rightarrow \infty, \quad x_1, x_2, x_{12} \text{ fixed} \quad (2.5)$$

as

$$w_{ij}(x_1, x_2, x_{12}, q^2) = \int_{x_1}^1 \frac{dz_1}{z_1} A_i\left(\frac{x_1}{z_1}, \frac{q^2}{p_1^2}\right) \int_{x_2}^1 \frac{dz_2}{z_2} A_j\left(\frac{x_2}{z_2}, \frac{q^2}{p_2^2}\right) \cdot C_{ij}(z_1, z_2, x_{12}) \quad (2.6)$$

where all the mass singularities of parton $i(P_1)$ and $j(P_2)$ are contained in process-independent quantities A_1 and A_2 respectively. The C_{ij} depends on q^2 only through the running coupling constant $\bar{g}^2(q^2)$ of the renormalization group equation²⁾ and is hereafter called the coefficient function for lepton-pair production. Equation (2.6) is illustrated

in Fig. 2. The finite part of A_i can be specified more precisely by applying perturbative QCD to deep inelastic lepton-hadron scattering where the operator-product expansion also works. By comparing (2.6) with the result of the operator-product expansion, we see that the moment of A_i

$$A_i^{\ell} \left(\frac{q^2}{p_i^2} \right) = \int_0^1 dx x^{\ell-1} A_i \left(x, \frac{q^2}{p_i^2} \right) \quad (2.7)$$

can be identified as the matrix element of the twist-2 operators $O_i^{\mu_1 \dots \mu_{\ell}}$ renormalized at q^2

$$\langle p_i | O_i^{\mu_1 \dots \mu_{\ell}} | p_i \rangle = p_i^{\mu_1 \dots \mu_{\ell}} A_i^{\ell} \left(\frac{q^2}{p_i^2} \right). \quad (2.8)$$

Following the perturbative QCD recipe^{3),4)} we assume that the mass singularity of the parton p_i in the moment $A_i^{\ell} (q^2/p_i^2)$ can be canceled by multiplying the factor $\bar{f}_i^{\ell}$ which carries all the information of the target or beam hadron. Thus we obtain a mass-singularity-free moment $f_i^{\ell} (q^2)$ of the q^2 dependent i -th parton distribution function $f_i(x, q^2)$

$$f_i^{\ell} (q^2) = \bar{f}_i^{\ell} A_i^{\ell} (q^2 / p_i^2),$$

$$f_i^{\ell} (q^2) = \int_0^1 dx x^{\ell-1} f_i(x, q^2). \quad (2.9)$$

In perturbative QCD lepton-pair production structure function W of the hadron collision is assumed to be given by the convolution of the coefficient function C_{ij} in Eq. (2.6) (of the collision of parton i and j) and the q^2 dependent parton distribution function f_i and f_j (i, j = quark, antiquark or gluon)

$$W(X_1, X_2, X_{12}, q^2) = \frac{1}{3} \sum_{i,j} \int_{X_1}^1 \frac{dx_1}{x_1} f_i\left(\frac{X_1}{x_1}, q^2\right) \cdot \int_{X_2}^1 \frac{dx_2}{x_2} f_j\left(\frac{X_2}{x_2}, q^2\right) C_{ij}(x_1, x_2, X_{12}). \quad (2.10)$$

We have taken the trace of color indices to give $1/3$ in Eq. (2.10). By applying the similar recipe of perturbative QCD to deep inelastic lepton-hadron scattering, we can identify the moment of the q^2 dependent parton distribution f_i as the hadronic matrix element of the twist-2 operators $O_i^{\mu_1 \dots \mu_\ell}$ renormalized at q^2

$$\langle P | O_i^{\mu_1 \dots \mu_\ell} | P \rangle = P^{\mu_1} \dots P^{\mu_\ell} f_i^\ell(q^2). \quad (2.11)$$

whose q^2 dependence is controlled by the anomalous dimension of the operator. It is now well known that the anomalous dimension of order $\bar{g}^4$ depends on the renormalization prescription and the gluon gauge in general^{10),12)}. These dependences should be cancelled by those of the coefficient function C_{ij} to give the observable hadronic structure function W . Therefore, to be consistent we have to calculate the coefficient function C_{ij} in the minimal subtraction scheme which is used in the only existing calculation¹⁰⁾ of the anomalous dimension of order $\bar{g}^4$.

2.2 The sub-process quark + antiquark $\rightarrow$ photon + anything

To obtain the coefficient function C_{ij} we should calculate the structure function w of parton target and beam and subtract the contributions of the operator matrix elements A_i by using Eq. (2.6). To order $\bar{g}^2$ the structure function of parton target and beam has a kinematical delta-function due to the fact that a massless particle recoils against the virtual photon

$$w_{ij}(x_1, x_2, x_{12}) = \delta\left(\frac{1}{x_{12}} - x_1 - x_2 + x_1 x_2\right) \bar{w}_{ij}(x_1, x_2) \quad (2.12)$$

Let us first calculate the structure function of the sub-process quark + antiquark $\rightarrow$ photon + anything. The Born term¹⁸⁾ is $q + \bar{q} \rightarrow \gamma$ as shown in Fig. 3 and for unit charge quark^{*)}

$$\bar{w}_{q\bar{q}}^{(0)}(x_1, x_2) = \delta(1-x_1) \delta(1-x_2) \quad (2.13)$$

To order $\bar{g}^2$ the structure function has contributions from diagrams in Figs. 4 and 5. We take quarks and gluons massless, but make incident quark and antiquark slightly off-shell to avoid mass singularities

$$\omega_i = -p_i^2 / q^2 > 0 \quad (2.14)$$

The calculation is performed in the minimal subtraction scheme using the dimensional regularization and in an arbitrary covariant gauge for gluons: the gauge parameter α appears in the gluon propagator as

$$D^{\lambda\sigma}(k) = \frac{1}{k^2} \left(-g^{\lambda\sigma} + \frac{1-\alpha}{k^2} k^\lambda k^\sigma \right) \quad (2.15)$$

After the extraction of mass singularities which is described in the Appendices A and B, we drop terms of order ω_i . We obtain the structure function of the subprocess quark + antiquark $\rightarrow$ photon + anything to order $\bar{g}^2$

*) In the following we treat quark as if it had unit charge.

$$\begin{aligned} \bar{w}^{(1)}(x_1, x_2) = & \frac{\bar{g}^2 C_2(R)}{16\pi^2} \left[\left\{ - \left(\frac{4\pi^2}{3} + 2 \right) \delta(1-x_1) \delta(1-x_2) \right. \right. \\ & + \ln \frac{1}{\omega_2} \delta(1-x_1) \left(3 \delta(1-x_2) + 2 \frac{1+x_2^2}{(1-x_2)_+} \right) + (1 \leftrightarrow 2) \\ & - \delta(1-x_1) \left(2 + 2 \frac{1+x_2^2}{(1-x_2)_+} \ln x_2 \right) - (1 \leftrightarrow 2) \\ & + 2 \frac{x_1^2 + x_2^2}{(1-x_1)_+ (1-x_2)_+} \left. \right\} \\ & + (1-\alpha) \left\{ \delta(1-x_1) \frac{1-2x_2}{(1-x_2)_+} + (1 \leftrightarrow 2) \right\} \left. \right], \end{aligned} \quad (2.16)$$

where

$$\frac{1}{(1-x)_+} = \lim_{\varepsilon \rightarrow +0} \left(\frac{1}{1-x+\varepsilon} - \ln \frac{1}{\varepsilon} \cdot \delta(1-x) \right), \quad (2.17)$$

and

$$C_2(R) = \frac{4}{3}. \quad (2.18)$$

It is interesting to note that the structure function of the sub-process $q + \bar{q} \rightarrow \alpha + \text{anything}$ is gauge dependent.

The twist-2 operator $O_{\psi}^{\mu_1 \dots \mu_\ell}$ for the quark field ψ is given by using the covariant derivative D^μ

$$O_{\psi}^{\mu_1 \dots \mu_\ell} = \frac{i^{\ell-1}}{2\ell!} \left[\bar{\psi} \gamma^{\mu_1} D^{\mu_2} \dots D^{\mu_\ell} \psi + \text{permutations} \right]. \quad (2.19)$$

Diagrams in Fig. 6 contribute to order $\bar{g}^2$ to the matrix element between quark states ¹²⁾ and gives the reduced matrix element in Eq. (2.8) as ^{*})

^{*}) We define $\int_0^1 dx f(x) \ln(1-x)/(1-x)_+$
as $\int_0^1 dx (f(x) - f(1)) \ln(1-x)/(1-x).$

$$\begin{aligned}
 A(x, \frac{q^2}{p^2}) &= A^{(0)} + A^{(1)} \\
 A^{(0)} &= \delta(1-x) \\
 A^{(1)} &= \frac{\bar{g}^2 C_2(R)}{16\pi^2} \left[\left\{ \ln \frac{1}{\omega} \left(3\delta(1-x) + 2 \frac{1+x^2}{(1-x)_+} \right) \right. \right. \\
 &\quad + \left(7 - \frac{2\pi^2}{3} \right) \delta(1-x) - 4 + 2x \\
 &\quad - 2 \frac{1+x^2}{1-x} \ln x - 2 \frac{1+x^2}{(1-x)_+} \ln(1-x) \Big\} \\
 &\quad \left. + (1-\alpha) \frac{1-2x}{(1-x)_+} \right]
 \end{aligned} \tag{2.20}$$

Here we set the effective renormalization point of the minimal subtraction scheme at q^2

$$4\pi\mu^2 e^{-\gamma_E} = q^2 \tag{2.21}$$

We can obtain the coefficient function from Eq. (2.6) by neglecting terms of order $\bar{g}^4$ and higher

$$C(x_1, x_2, x_{12}) = \delta\left(\frac{1}{x_{12}} - x_1 - x_2 + x_1 x_2\right) \bar{C}(x_1, x_2) \tag{2.22}$$

$$\bar{C}^{(0)}(x_1, x_2) = \delta(1-x_1) \delta(1-x_2)$$

$$\begin{aligned}
 \bar{C}^{(1)}(x_1, x_2) &= \bar{W}^{(1)}(x_1, x_2) - A^{(1)}(x_1) \delta(1-x_2) \\
 &\quad - A^{(1)}(x_2) \delta(1-x_1).
 \end{aligned} \tag{2.23}$$

We see that gauge dependent terms in Eq. (2.16) and (2.20) exactly cancel each other and we obtain the gauge independent coefficient function in the minimal subtraction scheme

$$\begin{aligned}
 \bar{C}_{q\bar{q}}^{(1)}(x_1, x_2) &= \bar{C}_{\bar{q}q}^{(1)}(x_1, x_2) \\
 &= \frac{\bar{g}^2 C_2(R)}{16\pi^2} \left\{ -16 \delta(1-x_1) \delta(1-x_2) \right. \\
 &\quad + 2 \delta(1-x_1) \left(1-x_2 + \frac{1+x_2^2}{(1-x_2)_+} \ln(1-x_2) \right) + (1 \leftrightarrow 2) \\
 &\quad \left. + 2 \frac{x_1^2 + x_2^2}{(1-x_1)_+ (1-x_2)_+} \right\} \quad (2.24)
 \end{aligned}$$

The minimal subtraction scheme gives the gauge-independent anomalous dimension¹⁶⁾ which controls the q^2 dependence of the parton distribution function in the perturbative QCD formula (2.10). Therefore the gauge independence of the coefficient function for lepton-pair production in the minimal subtraction scheme provides a consistency check of our recipe of perturbative QCD.

The quark and antiquark annihilation contribution to the structure function can be isolated by taking the difference of charge states of target and beam hadrons such as

$$(W(pp) - W(np)) - (W(pn) - W(nn)) \quad (2.25)$$

and it can be called the non-singlet structure function.

2.3 The sub-process quark + gluon $\rightarrow$ quark + photon

To order $\bar{g}^2$ we obtain contributions to the lepton-pair production coefficient function also from the sub-process $q + \text{gluon} \rightarrow q + \gamma$ shown in Fig. 7. The structure function of the collision of a quark (or anti-quark) p_1 and a gluon p_2 is found to be

$$\begin{aligned} \overline{w}(x_1, x_2) = & \frac{\overline{g}^2}{16\pi^2} \frac{1}{2} \left[\ln \frac{1}{\omega_2} \delta(1-x_1) (2-4x_2+4x_2^2) \right. \\ & - \delta(1-x_1) \{ (2-4x_2+4x_2^2) \ln x_2 + 2 \} \\ & \left. + 2 \frac{x_1 \{ x_2^2 + (x_1+x_2-2x_1x_2)^2 \}}{(1-x_1)_+ (x_1+x_2-x_1x_2)} \right]. \end{aligned} \quad (2.26)$$

To obtain the coefficient function we have to subtract the contributions from the twist-2 quark operator (2.19) sandwiched between gluon states^{*)}

$$\begin{aligned} A_G^\psi(x) = & \frac{\overline{g}^2}{16\pi^2} \frac{1}{2} \left\{ \ln \frac{1}{\omega_2} (2-4x+4x^2) - 2 - 4x + 4x^2 \right. \\ & \left. - (2-4x+4x^2) (\ln x + \ln(1-x)) \right\}. \end{aligned} \quad (2.27)$$

where we again put the effective renormalization point at q^2 [Eq. (2.21)]. One should note that (2.27) is obtained by taking contributions only from the anticlockwise quark loops with single flavor as in Fig. 8 for the quark-gluon collision because quark and antiquark of each flavor are distinguished here. Therefore, the coefficient function of the collision of quark p_1 and gluon p_2 is given by

*) We have calculated Eqs. (2.26) and (2.27) by contracting the gluon legs λ and σ . More precisely, we have applied a projection operator $P_{\lambda\sigma} = -g_{\lambda\sigma} / (2 - \epsilon)$ where $\epsilon = 4 - D$ is the difference of space-time dimension from 4. The factor $(2 - \epsilon)^{-1}$ was not present in the preprint of ref. 12, but it is necessary to make our result usable with the anomalous dimension of order $\overline{g}^4$ calculated in Ref. 10 (see Eq. (3.10) of the last paper in Ref. 10 and an erratum of the preprint of Ref. 12).

$$\begin{aligned}
 \bar{C}_{qG}(x_1, x_2) &= \bar{W}(x_1, x_2) - \delta(1-x_1) A_G^{\psi}(x_2) \\
 &= \frac{\bar{g}^2}{16\pi^2} \frac{1}{2} \left[\delta(1-x_1) \{ 4x_2 - 4x_2^2 \right. \\
 &\quad \left. + (2 - 4x_2 + 4x_2^2) \ln(1-x_2) \} + 2 \frac{x_1 \{ x_2^2 + (x_1 + x_2 - 2x_1 x_2)^2 \}}{(1-x_1)_+ (x_1 + x_2 - x_1 x_2)} \right] \quad (2.28)
 \end{aligned}$$

and can be called the singlet coefficient function. Similarly we obtain $\bar{C}_{\bar{q}G} = \bar{C}_{qG}$. If one wants to eliminate the dependence on renormalization prescription and the gluon gauge from the "parton distributions", it is most useful to define¹³⁾ a new q^2 dependent parton distribution f_2 as a convolution of our f and the coefficient function C_2 for the deep inelastic structure function νW_2 . If we replace f by f_2 , we should replace the coefficient function C in Eq. (2.10) by a renormalization-prescription-independent function C' . Using the result and the notation of Ref. 12 for the deep inelastic coefficient function C_2 , we obtain C' as

$$\begin{aligned}
 \bar{C}'_{q\bar{q}}(x_1, x_2) &= \bar{C}_{q\bar{q}}(x_1, x_2) - \delta(1-x_1) C_2^{NS}(x_2) - C_2^{NS}(x_1) \delta(1-x_2) \\
 &= \frac{\bar{g}^2 C_2(R)}{16\pi^2} \left\{ \left(\frac{4\pi^2}{3} + 2 \right) \delta(1-x_1) \delta(1-x_2) \right. \\
 &\quad \left. + \delta(1-x_1) \left(\frac{-1-2x_2+6x_2^2}{(1-x_2)_+} + 2 \frac{1+x_2^2}{1-x_2} \ln x_2 \right) + (1 \leftrightarrow 2) \right. \\
 &\quad \left. + 2 \frac{x_1^2 + x_2^2}{(1-x_1)_+ (1-x_2)_+} \right\}, \quad (2.29)
 \end{aligned}$$

and

$$\begin{aligned}
 \bar{C}'_{qG}(x_1, x_2) &= \bar{C}_{qG}(x_1, x_2) - \delta(1-x_1) C_2^G(x_2) / 4 T(R) \\
 &= \frac{\bar{g}^2}{16\pi^2} \frac{1}{2} \left[\delta(1-x_1) \{ 2 - 12 x_2 (1-x_2) \right. \\
 &\quad \left. + 2 (1-2x_2+2x_2^2) \ln x_2 \} + 2 \frac{x_1 \{ x_2^2 + (x_1+x_2-2x_1x_2)^2 \}}{(1-x_1)_+ (x_1+x_2-x_1x_2)} \right] \quad (2.30)
 \end{aligned}$$

where $T(R) = N_F/2$ and N_F is the number of flavors.

One can obtain predictions for the fully differential cross section (2.1) by using the perturbative QCD formula (2.10) and coefficient functions (2.24) and (2.28) once the q^2 dependent parton distribution function f is known, for instance, from deep-inelastic scattering. For very small q_T^2/q^2 , however, one finds a pathological behavior $d\sigma/dq_T^2 \propto 1/q_T^2$. The apparent logarithmic divergence of the integration over this q_T^2 distribution was sometimes regarded as the origin of mass singularities $\ln q^2/p^2$. However, we have already extracted these mass singularities and interpreted them as the anomalous dimension of the q^2 dependent parton distribution function. Therefore, by integrating over our q_T^2 distribution of Eqs. (2.10), (2.24) and (2.28), we no longer encounter mass singularities. This is achieved by the presence of delta-functions at $q_T^2 = 0$ with negative coefficient similarly to Eq. (2.17). On the other hand, this clearly indicates that our perturbative QCD approach requires "parton primordial p_T "¹⁹⁾ if one wants to make meaningful predictions for $q_T \simeq 0$. Precisely speaking, the perturbative QCD should not be applicable for such a region because it is based on the analysis in the limit of $q^2 \rightarrow \infty$ with $q_T^2/q^2 = X_{12} - 1$ fixed. To discuss the q_T^2 distribution even in such a region (which is substantial at presently available energies), we need to introduce more assumptions on the parton primordial p_T ¹⁹⁾. In a separate paper²⁰⁾ we have proposed a method to incorporate the parton primordial p_T in perturbative QCD and compared it with other works²¹⁾.

For completeness we also present, in Appendix C, the coefficient functions for three other independent structure functions contained in Eq. (2.2) which can be determined by measuring lepton angular distributions^{22), 23), 24)}.

3. MASS SPECTRUM AND COMPARISON TO OTHER WORKS

3.1 Mass spectrum

If we integrate not only q_T but also the longitudinal momentum q_z in the fully differential cross section (2.1), we obtain a simple formula for the lepton-pair mass spectrum

$$\frac{d\sigma}{dq^2} = \frac{4\pi\alpha^2}{3s q^2} W(\tau, q^2) \quad (3.1)$$

where $\tau = q^2/s$. The perturbative QCD formula (2.10) becomes

$$W(\tau, q^2) = \frac{1}{3} \sum_{i,j} \int_0^1 \frac{dx_1}{x_1} f_i(x_1, q^2) \int_{\frac{\tau}{x_1}}^1 \frac{dx_2}{x_2} f_j(x_2, q^2) C_{ij}\left(\frac{\tau}{x_1 x_2}\right) \quad (3.2)$$

where the coefficient function for the mass spectrum can be calculated from that for the differential cross section

$$C_{ij}(\tau) = \tau \int_0^1 \frac{dx_1}{x_1^2} \int_0^1 \frac{dx_2}{x_2^2} C_{ij}\left(x_1, x_2, \frac{\tau}{x_1 x_2}\right). \quad (3.3)$$

We can obtain the coefficient function of the quark-antiquark annihilation from Eqs. (2.24) and (2.22)

$$\begin{aligned} C_{q\bar{q}}(\tau) &= C_{\bar{q}q}(\tau) \\ &= \frac{\bar{g}^2 C_2(R)}{16\pi^2} \left\{ -4 \frac{1+\tau^2}{1-\tau} \ln \tau + 8 \frac{1+\tau^2}{(1-\tau)_+} \ln(1-\tau) - \left(\frac{2\pi^2}{3} + 16 \right) \delta(1-\tau) \right\}. \end{aligned} \quad (3.4)$$

From Eq. (2.28) we obtain the coefficient function of the quark gluon collision

$$\begin{aligned} C_{qG}(\tau) &= C_{\bar{q}G}(\tau) = C_{Gq}(\tau) = C_{G\bar{q}}(\tau) \\ &= \frac{\bar{g}^2}{16\pi^2} \frac{1}{2} \left[(1-\tau)^2 - 2\{\tau^2 + (1-\tau)^2\} \{\ln \tau - 2 \ln(1-\tau)\} \right] \end{aligned} \quad (3.5)$$

3.2 Comparison to other works

Earlier exploratory works calculated only the logarithmic terms (mass singularities) and did not try to find the resulting finite terms precisely^{4),19)}. Although lepton-pair production to order $\bar{g}^2$ has been calculated in perturbative QCD by a number of groups¹³⁾⁻¹⁵⁾, the dependence on the renormalization prescription has not been clearly mentioned except in Ref. 13). Unfortunately, however, they apparently used the renormalization scheme which was different from the minimal subtraction scheme and they calculated only the mass spectrum. On the other hand they presented their result in a renormalization-prescription-independent way as the difference between the coefficient function of lepton-pair production and the corresponding coefficient function of deep inelastic lepton-hadron scattering. We can also calculate the same quantity from our result by using the result in Ref. 12) where deep inelastic lepton-hadron scattering is calculated in the minimal subtraction scheme. We agree with the result in Ref. 13) for the sub-process $q + \text{gluon} \rightarrow q + \gamma$, but we disagree^{*)} for the sub-process $q + \bar{q} \rightarrow \gamma + \text{anything}$:

$$\begin{aligned} C'_{q\bar{q}}(\tau) &\equiv C_{q\bar{q}}(\tau) - 2C_2^{NS}(\tau) \\ &= \frac{\bar{g}^2 C_2(R)}{16\pi^2} \left\{ \left(\frac{2\pi^2}{3} + 2 \right) \delta(1-\tau) \right. \\ &\quad \left. - 2 \frac{3-2\tau-4\tau^2}{(1-\tau)_+} + 4 \frac{1+\tau^2}{(1-\tau)_+} \ln(1-\tau) \right\}, \end{aligned} \quad (3.6)$$

and

*) The Eq. (93) in Ref. 13) says $2\pi^2/3 - 2$ instead of $2\pi^2/3 + 2$ in our equation (3.6).

$$C'_{qG}(\tau) \equiv C_{qG}(\tau) - C_2^G(\tau) / 4T(R) \quad (3.7)$$

$$= \frac{\bar{g}^2}{16\pi^2} \frac{1}{2} \left\{ 3 - 10\tau + 9\tau^2 + 2(\tau^2 + (1-\tau)^2) \ln(1-\tau) \right\}.$$

To check the deep inelastic coefficient function we have explicitly confirmed the result of Ref.12) by the same method as in Ref. 13) (i.e. by calculating the cross section instead of the virtual Compton amplitude). By changing the subtraction scheme we can compare the result of Ref. 12) with that of Ref.13) and find a disagreement by the amount necessary to bring their result in agreement with ours^{*)}. We have also observed:

1) If the subtraction is made such that all the operator matrix elements vanish, renormalization constants do not satisfy the Ward-Takahashi identity^{**)} except in the Landau gauge ($Z_1 = Z_2 \propto \alpha \neq 0$). Conversely, if one insists on $Z_1 = Z_2$ and subtracts the self-energy part as in Ref.13), one must subtract the non-vanishing operator matrix element to obtain the coefficient function at least for the first moment (they used the Feynman gauge).

2) If we want to recover the result on the virtual Compton amplitude by calculating the cross section (i.e., imaginary parts of the virtual Compton amplitude), we should assign infinitesimal masses to regularize mass-singularities in a consistent manner such as shown in Fig. 9 (thick lines represent quarks with an infinitesimal mass m). If one always assigns a mass for the cut quark line, for example, one obtains a wrong result by a finite amount.

A rough comparison was made in Ref.13) between the naive Drell-Yan model ($\bar{g}^0$ order) and the perturbative QCD prediction to order $\bar{g}^2$. Our new results imply, however, that the prediction of perturbative QCD should be increased from that in Ref.13) by about 9% for $\bar{g}^2/4\pi \simeq 0.2$ for processes dominated by the sub-process $q + \bar{q} \rightarrow \gamma + X$ such as the proton-antiproton collisions.

*) We understand the subtraction scheme of Ref. 13) to be: $\Sigma(p) = 0$, $Z_1 = Z_2$, and $\langle p | O^{\mu_1 \dots \mu_n} | p \rangle = 0$ for $n > 1$ at $p^2 = -\mu^2$.

***) This point was first brought to our attention by T. Muta.

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APPENDIX A

Structure function of $q + \bar{q} \rightarrow \gamma + \text{anything}$ in the Feynman gauge

It is convenient first to take moments

$$\bar{w}_{mn} = \int_0^1 dx_1 x_1^{m-1} \int_0^1 dx_2 x_2^{n-1} \bar{w}(x_1, x_2). \quad (\text{A.1})$$

to extract mass singularities by taking careful account of corrections to the limit of integrations in x_1 and x_2 due to off-shell partons (2.14). In the Feynman gauge for gluons it is sufficient to use the linear approximation for the phase space boundary

$$0 \leq x_1 \leq 1 + \omega_1 - x_2 \omega_2 \quad (\text{A.2})$$

$$0 \leq x_2 \leq 1 + \omega_2 - x_1 \omega_1$$

After separating mass singularities we can invert the moment to obtain $\bar{w}(x_1, x_2)$.

1) Real gluon production in Fig. 4:

Before the extraction of mass singularities, we obtain dropping terms of order ω_i

$$\begin{aligned} \bar{w}(x_1, x_2) \simeq \frac{g^2 C_2(R)}{16\pi^2} 2 \left[\left\{ \frac{x_1^2}{(1-x_1+\omega_1)(1-x_2+\omega_2)} - \frac{\omega_2 x_2}{(1-x_1+\omega_1)^2} \right\} \right. \\ \left. + \{ 1 \leftrightarrow 2 \} \right]. \quad (\text{A.3}) \end{aligned}$$

This becomes

$$\begin{aligned} \frac{\bar{g}^2 C_2(R)}{16\pi^2} 2 \left[\left\{ \ln \frac{1}{\omega_1} \ln \frac{1}{\omega_2} \delta(1-x_1) \delta(1-x_2) + \ln \frac{1}{\omega_2} \delta(1-x_1) \frac{1+x_2^2}{(1-x_2)_+} \right. \right. \\ \left. \left. - \delta(1-x_1) \left(1 + \frac{1+x_2^2}{1-x_2} \ln x_2 \right) + \frac{x_1^2}{(1-x_1)_+ (1-x_2)_+} \right\} \right. \\ \left. + \left\{ 1 \leftrightarrow 2 \right\} \right]. \end{aligned} \quad (A.4)$$

2) Virtual corrections in Fig. 5:

$$\begin{aligned} \overline{W}(x_1, x_2) = \frac{\bar{g}^2 C_2(R)}{16\pi^2} \delta(1-x_1) \delta(1-x_2) \cdot \\ \cdot \left\{ -4 \ln \frac{1}{\omega_1} \ln \frac{1}{\omega_2} + 3 \ln \frac{1}{\omega_1} + 3 \ln \frac{1}{\omega_2} - 2 - \frac{4\pi^2}{3} \right\}. \end{aligned} \quad (A.5)$$

3) The sum of (A.4) and (A.5) gives Eq. (2.16) except terms proportional to $1 - \alpha$.

4) After subtracting the operator matrix element, we obtain the coefficient function in Eq. (2.24) whose moments are given by

$$\begin{aligned} \overline{C}_{mn} = \frac{\bar{g}^2 C_2(R)}{16\pi^2} \left\{ -16 + \frac{2}{m(m+1)} + \frac{2}{(m+1)^2} \right. \\ - \frac{2}{m(m+1)} \sum_{j=1}^m \frac{1}{j} + 4 \sum_{s=1}^m \frac{1}{s} \sum_{j=1}^s \frac{1}{j} \\ + \frac{2}{n(n+1)} + \frac{2}{(n+1)^2} - \frac{2}{n(n+1)} \sum_{j=1}^n \frac{1}{j} + 4 \sum_{s=1}^n \frac{1}{s} \sum_{j=1}^s \frac{1}{j} \\ - \frac{2}{m(m+1)} \sum_{j=1}^n \frac{1}{j} - \frac{2}{n(n+1)} \sum_{j=1}^m \frac{1}{j} + 4 \sum_{j=1}^m \frac{1}{j} \sum_{k=1}^n \frac{1}{k} \\ \left. - \frac{2}{m(n+1)} - \frac{2}{(m+1)n} \right\}. \end{aligned} \quad (A.6)$$

APPENDIX B

Gauge term of $q + \bar{q} \rightarrow \gamma + \text{anything}$

1) To regularize the infrared divergence, we have to introduce an infinitesimal mass λ for the gluon. In an arbitrary covariant gauge we should use the gluon propagator

$$D^{\mu\nu}(k) = \frac{1}{k^2 - \lambda^2} \left(-g^{\mu\nu} + \frac{1-\alpha}{k^2 - \alpha\lambda^2} k^\mu k^\nu \right) \quad (\text{B.1})$$

but we replace it by a simpler propagator

$$D^{\mu\nu}(k) = \frac{-g^{\mu\nu}}{k^2 - \lambda^2} + \frac{1-\alpha}{(k^2 - \lambda^2)^2} k^\mu k^\nu \quad (\text{B.2})$$

because we can easily show that these propagators give results which differ only in orders of λ^2 provided we calculate a set of diagrams which is infrared finite. Therefore, we take the gluon polarization sum as

$$\sum \epsilon^{\mu} \epsilon^{\nu} \delta(k^2 - \lambda^2) = \left(-g^{\mu\nu} + (1-\alpha) k^\mu k^\nu \frac{\partial}{\partial \lambda^2} \right) \delta(k^2 - \lambda^2). \quad (\text{B.3})$$

We define

$$\omega_3 = -\lambda^2 / q^2 < 0 \quad (\text{B.4})$$

2) After an algebraic manipulation (Ward identities), we obtain the moment of the gauge term for the real gluon production (Fig. 4)

$$\overline{w}_{mn} = (1-\alpha) \frac{\bar{g}^2 C_2(R)}{16\pi^2} \frac{\partial}{\partial \omega_3} I_{mn} \quad (\text{B.5})$$

$$I_{mn} = \int dx_1 x_1^{m-1} \int dx_2 x_2^{n-1} (A+B+C) \quad (\text{B.6})$$

$$A = \omega_2 \left\{ \frac{1-x_2-x_2\omega_2}{1-x_1+x_1\omega_1} x_1^2 + \frac{x_1^3 x_2 (1+\omega_1)(-\omega_2+\omega_3)}{(1-x_1+x_1\omega_1)^2} \right\} \quad (\text{B.7})$$

$$B = A (1 \leftrightarrow 2) \quad (\text{B.8})$$

$$C = \frac{x_1^2 x_2^2 2 \omega_1 \omega_2 (1+\omega_1+\omega_2-\omega_3)}{(1-x_1+x_1\omega_1)(1-x_2+x_2\omega_2)} \quad (\text{B.9})$$

The ω_3 dependence of I_{mn} comes from the limit of integration in x_1 and x_2 in addition to the explicit ω_3 dependence of the integrand.

3) The differentiation of terms explicitly dependent on ω_3 in the integrand gives contributions to $\bar{w}_{mn}$

$$(1-\alpha) \frac{\bar{g}^2 C_2(R)}{16\pi^2} \left(\frac{1}{m} + \frac{1}{n} \right). \quad (\text{B.10})$$

Hereafter we can neglect ω_3 in the integrand.

4) The limit of integration in x_1 and x_2 is given for $\omega_1, \omega_2 > 0$ and $\omega_3 < 0$ as

$$0 \leq x_1 \leq (1-\omega_1)^{-1} \quad (\text{B.11})$$

$$0 \leq x_2 \leq x_2^{\max}(x_1)$$

and the phase space boundary $x_2^{\max}(x_1)$ depends on ω_3 . Therefore we then obtain

$$\begin{aligned} & \frac{\partial}{\partial \omega_3} \int_0^{(1-\omega_1)^{-1}} dx_1 \int_0^{x_2^{\max}} dx_2 (A + B + C) \\ &= \int_0^{(1-\omega_1)^{-1}} dx_1 \left[\frac{\partial x_2^{\max}}{\partial \omega_3} \cdot (A + B + C) \right]_{x_2 = x_2^{\max}} \end{aligned} \quad (\text{B.12})$$

Hereafter we will denote $x_2^{\max}(x_1)$ as x_2 for simplicity. In the limit $\omega_3 \rightarrow 0$ the derivative $\partial x_2^{\max} / \partial \omega_3$ diverges at $x_1 = (1 - \omega_1 + \omega_2)^{-1}$. Therefore we should retain ω_3 and divide the integration region there. In the region

$$0 < x_1 < (1 - \omega_1 + \omega_2)^{-1} \quad (\text{B.13})$$

we can approximate

$$\begin{aligned} 2 \frac{\partial x_2}{\partial \omega_3} &\simeq \frac{1}{x_1^{-1} - 1 + \omega_1} - 1 + \frac{1}{x_1} \left(1 + \frac{\omega_2}{x_1^{-1} - 1 + \omega_1} \right) \cdot \\ &\cdot \left\{ (x_1^{-1} - 1 + \omega_1 - \omega_2)^2 - 4\omega_2\omega_3 \right\}^{-\frac{1}{2}} \end{aligned} \quad (\text{B.14})$$

In the region

$$(1 - \omega_1 + \omega_2)^{-1} < x_1 < (1 - \omega_1)^{-1} \quad (\text{B.15})$$

we can approximate

$$\begin{aligned} 2 \frac{\partial x_2}{\partial \omega_3} &\simeq \frac{1}{\omega_2^2} \left(\frac{1}{x_1} - 1 + \omega_1 \right) + \frac{x_1^{-1} - 1 + \omega_1}{x_1 \omega_2^2} \cdot \\ &\cdot \left(\frac{1}{x_1} - 1 + \omega_1 + \omega_2 \right) \left\{ \left(\frac{1}{x_1} - 1 + \omega_1 - \omega_2 \right)^2 - 4\omega_2\omega_3 \right\}^{-\frac{1}{2}} \end{aligned} \quad (\text{B.16})$$

5) Neglecting the explicit ω_3 dependence we obtain contributions from the term A in Eq. (B.7)

$$\bar{w}_{mn} = (1-\alpha) \frac{\bar{g}^2 C_2(R)}{16\pi^2} \left(-\ln \frac{1}{-\omega_3} + \ln \frac{1}{\omega_2} + \sum_{j=1}^m \frac{1}{j} + 1 \right) \quad (B.17)$$

6) From the term C in Eq. (B.9) we obtain contributions to $\bar{w}_{mn}$ from the integration region (B.13)

$$(1-\alpha) \frac{\bar{g}^2 C_2(R)}{16\pi^2} \left(\ln \frac{1}{-\omega_3} - \ln \frac{1}{\omega_2} \right) \quad (B.18)$$

from the integration region (B.15)

$$(1-\alpha) \frac{\bar{g}^2 C_2(R)}{16\pi^2} \left(\ln \frac{1}{-\omega_3} - \ln \frac{1}{\omega_2} - 2 \ln \frac{1}{\omega_1} \right). \quad (B.19)$$

7) Similarly we obtain the contribution to $\bar{w}_{mn}$ from the term B in Eq. (B.8) neglecting the explicit dependence on ω_3 in B

$$(1-\alpha) \frac{\bar{g}^2 C_2(R)}{16\pi^2} \left(-\ln \frac{1}{-\omega_3} + \ln \frac{1}{\omega_1} + \sum_{j=1}^n \frac{1}{j} + 1 \right) \quad (B.20)$$

8) Combining (B.10) with (B.17), (B.18), (B.19) and (B.20) we obtain for the real gluon production

$$\bar{w}_{mn} = (1-\alpha) \frac{\bar{g}^2 C_2(R)}{16\pi^2} \left(-\ln \frac{1}{\omega_1} - \ln \frac{1}{\omega_2} + 2 + \sum_{j=1}^m \frac{1}{j} + \sum_{j=1}^n \frac{1}{j} + \frac{1}{m} + \frac{1}{n} \right). \quad (B.21)$$

9) For virtual gluon corrections we obtain the contribution to $\bar{w}_{mn}$ from diagrams with the vertex correction

$$(1-\alpha) \frac{\bar{g}^2 C_2(R)}{16\pi^2} \left(2 \ln \frac{q^2}{4\pi\mu^2} + 2\gamma_E - 4 \right) \quad (B.22)$$

from diagrams with self-energy corrections

$$(1-\alpha) \frac{\bar{g}^2 C_2(R)}{16 \pi^2} \left(-2 \ln \frac{q^2}{4\pi \mu^2} + \ln \frac{1}{\omega_1} + \ln \frac{1}{\omega_2} - 2\gamma_E + 2 \right) \quad (\text{B.23})$$

10) Combining (B.21), (B.22) and (B.23) we obtain

$$\overline{w}_{mn} = (1-\alpha) \frac{\bar{g}^2 C_2(R)}{16 \pi^2} \left(\sum_{j=1}^m \frac{1}{j} + \frac{1}{m} + \sum_{j=1}^n \frac{1}{j} + \frac{1}{n} \right) \quad (\text{B.24})$$

which gives the gauge term in Eq. (2.16).

APPENDIX C

Other structure functions

The structure tensor Eq. (2.2) has four independent invariant structure functions which can be separated by measuring lepton angular distributions²²⁾⁻²⁴⁾.

We define

$$\tilde{g}^{\mu\nu} = g^{\mu\nu} - q^\mu q^\nu / q^2 \quad (C.1)$$

$$\tilde{P}_i^\mu = \tilde{g}^{\mu\nu} P_{i\nu} \sqrt{q^2} / P_i q \quad (C.2)$$

$$T_1^{\mu\nu} = -\tilde{g}^{\mu\nu} \quad (C.3)$$

$$T_2^{\mu\nu} = (\tilde{P}_1 - \tilde{P}_2)^\mu (\tilde{P}_1 - \tilde{P}_2)^\nu \quad (C.4)$$

$$T_3^{\mu\nu} = (\tilde{P}_1 + \tilde{P}_2)^\mu (\tilde{P}_1 + \tilde{P}_2)^\nu \quad (C.5)$$

and

$$T_4^{\mu\nu} = \tilde{P}_1^\mu \tilde{P}_1^\nu - \tilde{P}_2^\mu \tilde{P}_2^\nu \quad (C.6)$$

We define projections of $W^{\mu\nu}$ in terms of $T_i^{\mu\nu}$ as

$$(W, W^a, W^b, W^c) = (T_1^{\mu\nu}, T_2^{\mu\nu}, T_3^{\mu\nu}, T_4^{\mu\nu}) \cdot W_{\mu\nu} \quad (C.7)$$

Perturbative QCD formulas (2.10) hold for W^a , W^b and W^c similarly to W with coefficient functions: for the sub-process $q + \bar{q} \rightarrow \gamma + \text{anything}$

$$\begin{aligned}\bar{C}_{q\bar{q}}^a(x_1, x_2) &= \bar{C}_{q\bar{q}}^b(x_1, x_2) = \bar{C}_{\bar{q}q}^a(x_1, x_2) = \bar{C}_{\bar{q}q}^b(x_1, x_2) \\ &= \frac{\bar{g}^2 C_2(R)}{16\pi^2} 4(x_1^2 + x_2^2)(x_1 + x_2 - x_1 x_2).\end{aligned}\quad (0.8)$$

$$\begin{aligned}\bar{C}_{q\bar{q}}^c(x_1, x_2) &= \bar{C}_{\bar{q}q}^c(x_1, x_2) \\ &= \frac{\bar{g}^2 C_2(R)}{16\pi^2} 4(x_1^2 - x_2^2)(x_1 + x_2 - x_1 x_2)\end{aligned}\quad (0.9)$$

for the sub-process $q + \text{gluon} \rightarrow q + \gamma$

$$\begin{aligned}\bar{C}_{qG}^a(x_1, x_2) &= \bar{C}_{\bar{q}G}^a(x_1, x_2) \\ &= \frac{\bar{g}^2}{16\pi^2} \frac{1}{2} 4x_1 \{ (x_1 + x_2)^2 + x_2^2 \} (1 - x_2)\end{aligned}\quad (0.10)$$

$$\begin{aligned}\bar{C}_{qG}^b(x_1, x_2) &= \bar{C}_{\bar{q}G}^b(x_1, x_2) \\ &= \frac{\bar{g}^2}{16\pi^2} \frac{1}{2} 4x_1 \{ (x_1 - x_2)^2 + x_2^2 \} (1 - x_2)\end{aligned}\quad (0.11)$$

$$\begin{aligned}\bar{C}_{qG}^c(x_1, x_2) &= \bar{C}_{\bar{q}G}^c(x_1, x_2) \\ &= \frac{\bar{g}^2}{16\pi^2} \frac{1}{2} 4x_1 (x_1^2 - 2x_2^2) (1 - x_2)\end{aligned}\quad (0.12)$$

One should note that coefficient functions for W^a , W^b and W^c do not depend on both the renormalization prescription and the gluon gauge to order $\bar{g}^2$ because of the absence of the zero-th order contribution in $\bar{g}$ (similarly to the longitudinal structure function in the deep inelastic scattering). In fact, they correspond to photon polarizations in the $\vec{P}_1$ and $\vec{P}_2$ plane in the photon rest frame²³⁾.

At $p_1^\mu + p_2^\mu = q^\mu$ there are only two independent nonvanishing tensors ($T_1^{\mu\nu}$ and $T_2^{\mu\nu}$). Therefore, if we decompose $W^{\mu\nu} = \sum_i T_i^{\mu\nu} W_i$, coefficient

functions are well-defined for only two of them (W_1 and W_2) at $x_1 = x_2 = x_{12} = 1$. After separating mass singularities we obtain for both sub-processes $q + \bar{q} \rightarrow \gamma + \text{anything}$ and $q + \text{gluon} \rightarrow q + \gamma$

$$\bar{c}_1(x_1, x_2) = \frac{1}{2} \bar{c}(x_1, x_2) \quad (C.13)$$

This relation was noticed and interpreted as a manifestation of spin $\frac{1}{2}$ quark although mass singularities were not separated^{23),24)}.

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FIGURE CAPTIONS

- Fig. 1 : Lepton-pair production from the collision of hadrons P_1 and P_2 . The four-momentum of the virtual photon is denoted as q .
- Fig. 2 : A graphical illustration of the factorization in lepton-pair production from the collision of partons p_1 and p_2 . The dashed line represents the discontinuity. The structure function w of the left-hand side is given by a convolution of the coefficient function C_{12} and mass singularity factors A_i .
- Fig. 3 : The Born term of the sub-process $q + \bar{q} \rightarrow \gamma + \text{anything}$. The wavy and the solid line represent the photon and the quark.
- Fig. 4 : The real gluon production in the sub-process quark (p_1) + antiquark (p_2) $\rightarrow$ photon (q) + anything. The curly line indicates gluons.
- Fig. 5 : The virtual gluon corrections to the Born term $q + \bar{q} \rightarrow \gamma$.
- Fig. 6 : Diagrams for the operator matrix element of order $\bar{g}^2$ between quark states. The cross on the top indicates the twist-2 quark operator defined in Eq. (2.19).
- Fig. 7 : The structure function of the sub-process quark (p_1) + gluon (p_2) $\rightarrow$ photon (q) + quark.
- Fig. 8 : Diagrams for the operator matrix element of order $\bar{g}^2$ between gluon states. The cross on the top indicates the twist-2 quark operator defined in Eq. (2.19). One should take only anticlockwise quark loops with single flavor.
- Fig. 9 : An example of a consistent way of assigning an infinitesimal mass m for the intermediate quark line (the thick, solid line) in order to reproduce the imaginary part of the Feynman diagram on the right-hand side.

fig.1

fig. 2

fig. 3

fig. 4

fig.5

fig. 6

fig. 7

fig. 8

fig. 9