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EXPERIMENTAL METHOD TO DETECT THE VACUUM BIREFRINGENCE
INDUCED BY A MAGNETIC FIELD

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ABSTRACT

In this letter a method of measuring the birefringence induced in vacuum by a magnetic field is described: this effect is evaluated using the non-linear Euler-Heisenberg-Weisskopf Lagrangian.

The optical apparatus discussed here may detect an induced ellipticity on a laser beam down to 10^{-11} .

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The propagation of light in strong magnetic fields is a very important problem in physics [1,2], and particularly in astrophysics [3] where, near neutron stars for instance, magnetic fields up to 10^{12} G may be present.

In the frame of the classical electromagnetic theory no effects are predicted, because of the linear structure of the Maxwell equations. On the other hand, quantum electrodynamics (QED) admits (because of the vacuum polarization effect) a photon-photon interaction described, to the lowest order, by the Feynman graph shown in fig. 1. In the case of a photon propagating in an external electric or magnetic field ($|\vec{E}| \sim 10^{15}$ V/cm; $|\vec{B}| \ll 10^{13}$ G), the non-linear effects predicted by the Euler-Heisenberg-Weisskopf (EHW) Lagrangian [1,2] (apart from radiative corrections):

$$\mathcal{L}_{\text{EHW}} = \frac{1}{8\pi} (E^2 - B^2) + \frac{A}{4\pi} [(E^2 - B^2)^2 + 7(\vec{E} \cdot \vec{B})^2],$$

where

$$A = \frac{1}{90\pi} \alpha^2 \frac{\lambda_e^3}{m_e c^2} \simeq 1.3 \times 10^{-32} \text{ cm}^3/\text{erg}.$$

Until now, the most important experiments that have shown non-linear effects in QED are

- a) the Delbruck scattering [4];
- b) the $g_{\mu}-2$ experiment [5];
- c) the muonic atom spectroscopy [6], [7].

In this letter we discuss the possibility of observing, by means of classical optical methods, the effect on the polarization state of a laser light beam, caused by its interaction with a very strong magnetic field (10^5 G) in vacuum. Since the graph of fig. 2 is the only contribution (to lowest order) to which we are sensitive, the proposed experiment is a direct test of the photon-photon interaction in the very low γ -energy region: the quantity to be directly measured is the amount of birefringence induced by the magnetic field in the vacuum.

It can be shown [8] that, if $\vec{k}$ is the direction in which the light propagates and $\vec{B}_0$ is the external magnetic field, then the light linearly polarized in the $\vec{k}, \vec{B}_0$ plane ($\parallel$ mode) propagates with a phase velocity different from the light polarized orthogonally ($\perp$ mode). Let us define the two indices of refraction

$$n_{\parallel} = \frac{c}{v_{\parallel}}, \quad n_{\perp} = \frac{c}{v_{\perp}}. \quad (1)$$

According to ref. [8], one has

$$n_{\parallel} = 1 + 7AB_0^2 \sin^2\theta \quad (2)$$

$$n_{\perp} = 1 + 4AB_0^2 \sin^2\theta, \quad (3)$$

where θ is the angle between $\vec{k}$ and $\vec{B}_0$.

The principle of the experiment is therefore to measure directly the difference

$$\Delta n = n_{\parallel} - n_{\perp} = 3AB_0^2 \sin^2\theta. \quad (4)$$

The experimental apparatus is shown in fig. 3. A continuous laser beam is linearly polarized by the first polarizer prism N1. The light beam then passes through a Faraday cell (FC), which rotates the optical vector around the propagation axis by the angle

$$\Phi(t) = \Phi_0 \cos(\Omega_0 t). \quad (5)$$

The Faraday cell is followed by a quarter wave plate (QW), mounted with its slow axis in the z direction.

These three elements are used to introduce a modulate ellipticity at a convenient frequency on the linearly polarized laser light. The modulated light beam is sent into an optical cavity (OC), defined by a system of spherical mirrors. The optical cavity is placed in high vacuum and traversed by a magnetic field

$$\vec{B}_0 = B_0(\cos \xi, 0, \sin \xi), \quad \xi \simeq \pi/4. \quad (6)$$

During its multiple passages through the cavity, the light will acquire an ellipticity Ψ due to the birefringence induced by the photon-photon interaction, given by the expression

$$\Psi = \pi \frac{L}{\lambda} (n_{\parallel} - n_{\perp}) , \quad (7)$$

where L is the length of the beam path inside OC and λ is the light wavelength. This ellipticity adds to the initial, modulated, ellipticity; therefore, after the passage through the analyser prism (N2), the light intensity will contain a component proportional to Ψ at the modulation frequency Ω_0 . The intensity I_{ex} of the extraordinary ray coming from N2, detected by the photomultiplier (PM), is given indeed by the expression*)

$$I_{\text{ex}} = I_0 \{ \Theta + \Phi(t) [\Psi + \phi] + \Phi^2(t) \} , \quad (8)$$

where I_0 is the beam intensity before N2

$$\begin{aligned} \Theta &= \sigma^2 + (\beta + \gamma)^2 + \beta^2 \\ \phi &= -2[\beta + \delta(\beta + \gamma)] \end{aligned} \quad (9)$$

and

β is the mechanical misalignment angle between N1 and QW,

γ is the mechanical misalignment angle between QW and N2,

δ is the complement to $\pi/2$ of the real phase delay introduced by QW,

σ^2 is the extinction factor of the prism polarizers.

Equation (8) shows that the light intensity after N2 is made up of the following components:

- a) a d.c. signal, depending on the extinction factor of the prism polarizers;
- b) a $2\Omega_0$ frequency signal of an amplitude proportional to Φ_0^2 ;

*) The expression of I_{ex} , up to the second order in Ψ , is

$$\begin{aligned} I_{\text{ex}} = I_0 \left\{ \sin^2(\Phi + \gamma) - \frac{(1 + \sin \delta)}{2} \sin(2\Phi - 2\beta) \sin(2\beta + 2\gamma) - \right. \\ \left. - \frac{\Psi}{2} \cos \delta \sin(2\Phi - 2\beta) \sin(2\xi - 2\gamma) + \right. \\ \left. + \frac{\Psi^2}{4} \sin(2\xi - 2\gamma) \sin(2\beta + 2\xi) \cos(2\Phi - 2\beta) \right\} . \end{aligned}$$

- c) a Ω_0 frequency signal proportional to the modulation angle $\Phi(t)$ and to Ψ which is the quantity to be measured;
- d) a Ω_0 frequency signal depending on the misalignment angles between N1, QW, and N2.

The signals (a) and (b) can easily be filtered out. The signal (d) should be minimized by mechanical adjustment and/or by introducing auxiliary Faraday cells for precise polarization angle control. A residual misalignment angle less than 10^{-8} rad is required [9].

The statistical fluctuations in the number of the detected photons (shot-noise) imposes a limit on the sensitivity, which is roughly given by the signal-to-noise ratio R

$$R = \Phi_0 \Psi \sqrt{\frac{2 W_0 \tau}{(\sigma^2 + \Phi_0^2) \hbar \omega}}, \quad (10)$$

where τ is the measuring time, W_0 is the laser power, and $\hbar \omega$ the photon energy.

By assuming in eq. (10)

$$\Phi_0^2 \approx \sigma^2 \quad (11)$$

the measuring time τ needed to achieve a predetermined value of R, using eqs. (7) and (4), is

$$\tau = \frac{hc}{9\pi^2 A^2} R^2 \frac{\lambda}{W_0 f B_0^4 L^2}, \quad (12)$$

where f is the photodetector quantum efficiency.

A realistic choice for the practical values of the parameters discussed above could be:

- a) $W \approx 10 \div 20$ W c.w. produced by an argon laser ($\lambda = 514.5$ nm);
- b) $B_0 \approx 100$ kG of magnetic field 4 m long;
- c) $L \approx 2$ km obtained with 500 reflections in the optical cavity;
- d) $\sigma^2 \approx 10^{-8}$ for the prism polarizers.

The birefringence induced by the magnetic field would, under those circumstances, be

$$\Psi = \pi \frac{L}{\lambda} (n_{\parallel} - n_{\perp}) \approx 0.5 \cdot 10^{-11} \text{ rad} . \quad (13)$$

To observe with a signal-to-noise ratio $R = 1$ this amount of birefringence, an integration time $\tau = 5000 \text{ sec} \approx 1 \text{ h } 30 \text{ min}$ would be needed.

The thermal Nyquist noise on the detector has an amplitude given by the expression

$$\langle V \rangle_{\text{Nyquist}} = \sqrt{\frac{4KT R_a}{\tau}} , \quad (14)$$

where R_a is the detector load resistor ($R_a = 10 \text{ k}\Omega$), T is the resistor temperature, and k the Boltzmann constant.

This amplitude has to be compared with the amplitude of the effect to be measured, which is roughly given by

$$\langle V \rangle = V_0 \frac{\Psi}{\Phi_0} ,$$

where V_0 is the d.c. component of the signal. It turns out that, with $V_0 \approx 1 \text{ V}$ we have

$$\langle V \rangle \approx 10 \text{ nV} ,$$

whereas, at room temperature, the Nyquist noise amounts to

$$\langle V \rangle_{\text{Nyquist}} \approx 0.2 \text{ nV} .$$

Concerning the systematic errors, the most dangerous source is the Cotton-Mouton effect on the residual gas in the optical cavity OC, since it has the same influence on the light polarization as the effect we want to measure. We assume the residual gas to be hydrogen at 3°K of temperature and 10^{-11} Torr of pressure. By taking the Cotton-Mouton constant for water^{*)}, the ellipticity

*) Since we have not been able to find the Cotton-Mouton constant for hydrogen, we have taken the one for water and scaled it with the suitable density factor. The constant at n.t.p. we have used is

$$C \approx 4 \times 10^{-19} \text{ rad} \cdot \text{G}^{-2} \cdot \text{cm}^{-2} .$$

introduced because of the residual gas is

$$\Psi_{\text{cm}} \approx 10^{-14} \text{ rad} ,$$

about 1000 times less than the effect we want to measure.

Another possible source of systematic error to be considered is the Faraday effect in the residual gas, generated because $\vec{B}_0$ cannot be perfectly orthogonal to the direction of the laser beam. The Verdet constant of the hydrogen at n.t.p. is

$$V_{\text{H}_2} \approx 6 \times 10^{-6} \text{ min/G}\cdot\text{cm} .$$

Therefore, by assuming a non-perfect orthogonality ρ of the magnetic field $\vec{B}_0$ of the order of 1% (or less) with respect to the laser beam direction, we obtain

$$\Xi_{\text{Far}} \approx 3 \times 10^{-13} \text{ rad} ,$$

which is about 20 times smaller than the effect to be measured. Moreover Ξ_{Far} is present in eq. (8) to the square power, since it cannot interfere with the modulation $\Phi(t)$ due to the fact that $\Phi(t)$ is a modulation in the ellipticity, whereas Ξ_{Far} is a rotation of the light polarization state.

It is worthwhile to point out that the method we have proposed is insensitive

- a) to the residual gas fluctuations in the optical cavity;
- b) to the variations in the length of the cavity (generated by thermal expansion of the optical bench, microseisms, quantum mechanical indetermination in the mirrors positions, etc.);
- c) to the spread in the laser wavelength.

This is so because what we are measuring is the phase difference between two components of the same laser beam, and not the phase difference between two spatially separated beams.

In addition, the intensity of the ordinary ray escaping from N2 can be detected by a photodiode and used to normalize I_{ex} in such a way that also the laser power fluctuations become uncritical.

We wish finally to mention that a further way to improve the signal-to-noise ratio is to modulate the magnetic field $\vec{B}_0$ at a chosen convenient frequency, for instance to about 10^{-2} Hz might be possible even for such strong fields as the one suggested above.

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REFERENCES

- [1] W. Heisenberg and H. Euler, Z. Phys. 98 (1936) 718.
- [2] V.S. Weisskopf, K. Danske Vidensk. Selsk. Mat.-Fys. Medd. 14 No. 6 (1936).
- [3] R. Novick et al., Astrophys. J. 215 (1977) L117.
- [4] F. Ehlotzky and G.C. Sheppey, Nuovo Cimento 33 (1964) 1185.
- [5] J. Bailey et al., Phys. Lett. 67B (1977) 225.
- [6] G. Carboni et al., Nucl. Phys. A278 (1977) 381.
- [7] T. Dubler et al., Nucl. Phys. A294 (1977) 397.
- [8] S.L. Adler, Ann. Phys. (USA) 87 (1971) 599.
- [9] P.E.G. Baird et al., Phys. Rev. Lett. 39 (1977) 798.

Figure captions

- Fig. 1 : Lowest-order Feynman graph describing the photon-photon interaction.
- Fig. 2 : Lowest-order Feynman graph contributing to the birefringence induced in vacuum by an (external) magnetic field.
- Fig. 3 : Principal elements of the experimental apparatus.

Fig. 1

Fig. 2

Fig. 3