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FACTORIZATION BREAKING IN DEEP INELASTIC NEUTRINO
HADRON PRODUCTION IN PERTURBATIVE QCD

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ABSTRACT

In the naive parton model the single hadron inclusive cross-section of a deep inelastic neutrino reaction factorizes $W(X,Z) = f(X)D(Z)$. The next-to-leading order correction in perturbative QCD is calculated and gives the breakdown of factorization. We propose to test the prediction by taking ratios of double moments of structure functions integrated over produced hadron $\vec{p}_T$.

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Recent high-energy neutrino experiments have provided a nice testing ground for perturbative QCD¹⁾. The naive parton model predicts that the structure function $W(X,Z)$ of hadron production is given by a product of the quark distribution function $f(x)$ in the target and the quark fragmentation function $D(Z)$ into hadrons²⁾. Perturbative QCD predicts factorization in leading order and provides corrections to it in powers of the running coupling constant $\bar{g}^2$ as well as the Q^2 dependence to parton distribution and fragmentation functions. The purpose of this letter is to present the next-to-leading order calculation in perturbative QCD and to propose a test of the predicted factorization breaking for single hadron inclusive production in neutrino reactions.

Without measuring transverse momenta of hadrons, we obtain only three invariant structure functions for $\nu + \text{target } (P_1) \rightarrow \mu + \text{hadron } (P_2) + \text{anything}$

$$W^{\mu\nu} = d^{\mu\nu} W_2 + \tilde{g}^{\mu\nu} W_S + i \epsilon^{\mu\nu\alpha\beta} \frac{P_{1\alpha} q_\beta}{P_1 q} W_3 \quad (1)$$

$$d^{\mu\nu} = -\tilde{g}^{\mu\nu} + \tilde{g}^{\mu\sigma} P_{1\sigma} \tilde{g}^{\nu\lambda} P_{1\lambda} / P_1^\alpha \tilde{g}_{\alpha\beta} P_1^\beta \quad (2)$$

where $\tilde{g}^{\mu\nu} = g^{\mu\nu} - q^\mu q^\nu / q^2$, q^μ being the four-momentum of the weak current, and W_i are functions of $X = -q^2 / 2P_1 q$, $Z = 2P_2 q / q^2$ and $q^2 = -Q^2$. We take $Z > 0$ to avoid target fragmentation^{*)}.

In the naive parton model, structure functions of pion production from ν nucleon collisions are given in terms of the a^{th} quark distribution function f^a and the b^{th} quark fragmentation function D_a^π for instance^{**)}

$$W_2^{\nu \rightarrow \pi} = (f^d + f^{\bar{u}}) (D_u^\pi \cos^2 \theta + D_c^\pi \sin^2 \theta) + f^s D_c^\pi \quad (3)$$

*) Another possible longitudinal variable is $P_1 P_2 / P_1 q$. In this case one needs to look into large P_{2T} or azimuthal angular distributions to avoid target fragments.

**) We neglect sea quark contributions multiplied by $\sin^2 \theta$ (Cabibbo angle).

Since pions are produced from a charmed quark only through gluons, we can eliminate D_c by taking the difference of π^+ and π^- , and obtain a factorized form^{*)}

$$W_2^{\nu \rightarrow \pi^+} - W_2^{\nu \rightarrow \pi^-} = (f^d + f^{\bar{u}})(D_u^{\pi^+} - D_u^{\pi^-}) \cos^2 \theta \quad (4)$$

Another way of obtaining a factorized expression is to take the difference between proton and neutron target data (non-singlet in target channel)

$$W_2^{\nu p \rightarrow \pi} - W_2^{\nu n \rightarrow \pi} = (f^d - f^{\bar{d}} + f^{\bar{u}} - f^u)(D_u^{\pi} \cos^2 \theta + D_c^{\pi} \sin^2 \theta) \quad (5)$$

Even if we have primordial p_T associated with quark distribution and fragmentation functions, by using a covariant parton model approach³⁾ we can show that the factorization prediction of the naive parton model is unaffected provided transverse momenta of produced hadrons are integrated: $W_L(X, Z) = W_L^t(X) D(Z)$, where W_L^t are usual structure functions of fully inclusive deep inelastic scattering. By using a parton model interpretation of the operator-product expansion⁴⁾ we have also found^{**)} that target mass corrections can be incorporated by the ξ scaling formula.

It has been shown that mass singularities of inclusive cross-sections with partons as beam, target, or produced particles factor to all orders in perturbation theory of QCD⁵⁾: parton structure function^{***)} w_i^{ab} for the neutrino reaction $W_a \rightarrow bX$, where a, b = quark, antiquark, or gluon, is given by taking moments

$$\begin{aligned} w_{i, nm}^{ab} &\equiv \int_0^1 dx x^{n-1} \int_0^1 dz z^{m-1} w_i^{ab}(x, z) \\ &= A_n^a \left(\frac{Q^2}{p_1^2} \right) \Gamma_m^b \left(\frac{Q^2}{p_2^2} \right) C_{i, nm}^{ab} \end{aligned} \quad (6)$$

*) If one identifies charge, but not particle species reliably (e.g., in many of the bubble chamber experiments), one may still have a small contamination from $D_c^+ - D_c^- \neq 0$.

**) We neglect masses of quarks and produced hadrons.

***) We use small letters for partons and capitals for hadrons.

where A_n^a is the quark operator matrix element between the a^{th} parton states and Γ_m^b is called the (time-like) cut vertex⁶⁾. Following the perturbative QCD recipe we assume that mass singularities in A_n^a and Γ_m^b can be cancelled by multiplying the factors $\bar{f}_n^a$ and $\bar{D}_m^b$

$$f_n^a(Q^2) = \bar{f}_n^a A_n^a(Q^2/p_1^2) \quad (7)$$

$$D_{b,m}(Q^2) = \bar{D}_m^b \Gamma_m^b(Q^2/p_2^2) \quad (8)$$

to give the Q^2 dependent parton distribution function $f^a(x, Q^2)$ and the fragmentation function $D_b(Z, Q^2)$. The hadronic structure function is then given by^{*})

$$W_{i, nm} = \sum_{a,b} f_n^a(Q^2) D_{b,m}(Q^2) C_{i, nm}^{a,b} \quad (9)$$

In leading order we obtain that

$$C_{2, nm}^{qq} = C_{2, nm}^{\bar{q}\bar{q}} = C_{3, nm}^{qq} = C_{3, nm}^{\bar{q}\bar{q}} = 1$$

and all others vanish. We can calculate the "coefficient function" $C_{i, nm}^{ab}$ perturbatively, but to obtain $f_n(Q^2)$ and $D_m(Q^2)$ we have to fit data at some Q^2 .

To do an unambiguous test of perturbative QCD prediction, we propose to study if the structure function $W(X, Z)$ deviates from a product of functions of X and Z . Namely, one should examine deviations from unity of the following ratio of double moments which does not involve f_n and D_m ^{**)}

$$\frac{W_{nm} W_{kl}}{W_{km} W_{nl}} = \frac{C_{nm} C_{kl}}{C_{nl} C_{km}} = 1 + O(\bar{g}^2) \quad (10)$$

*) We neglect effects of primordial p_T and target mass in calculating the next-to-leading order correction.

**) We suppress the suffix $i = 2, 3$ temporarily. We consider the non-singlet case where only $W_q \rightarrow qX$ (or q) contributes. The mixing problem is discussed at the end.

To obtain the coefficient function C_{nm} we use Eq. (6) and subtract contributions of operator matrix elements and cut vertices from the parton structure function w_{nm} . The resulting coefficient function has the general form^{*)}

$$C_{nm} = 1 + \frac{\bar{g}^2}{16\pi^2} (a + b_n + c_m + d_{nm}) \quad (11)$$

$$\simeq \left(1 + \frac{\bar{g}^2}{16\pi^2} \left(\frac{a}{z} + b_n\right)\right) \left(1 + \frac{\bar{g}^2}{16\pi^2} \left(\frac{a}{z} + c_m\right)\right) \left(1 + \frac{\bar{g}^2}{16\pi^2} d_{nm}\right)$$

where $\bar{g}^2/16\pi^2 = (\beta_0 \ln Q^2/\Lambda^2)^{-1}$ with $\beta_0 = 11-2N_f/3$. Equations (6) and (11) show that operator matrix elements and cut vertices affect only a , b_n and a , c_m , respectively^{**)} . Therefore, d_{nm} and b_n can be obtained without calculating cut vertices Γ_m . The ratio (10) is then given by

$$\frac{W_{nm} W_{kl}}{W_{km} W_{nl}} = 1 + \frac{\bar{g}^2}{16\pi^2} d_{nm,kl} \quad (12)$$

$$d_{nm,kl} \equiv d_{nm} - d_{km} + d_{kl} - d_{nl} \quad (13)$$

As a second test we can examine if the parton distribution function in Eq. (9) is the same one that appeared in the fully inclusive deep inelastic structure function $W^t(X)$ by studying the ratio $W_{nm} W_k^t / W_n^t W_{km}^t$. Using the next-to-leading correction B_n to the coefficient function for W_n^t in Ref. 7), we obtain

$$\frac{W_{nm} W_k^t}{W_n^t W_{km}^t} = 1 + \frac{\bar{g}^2}{16\pi^2} (b_n + d_{nm} - B_n - b_k - d_{km} + B_k) \quad (14)$$

Since the cut vertex is common to all three structure functions W_i , we can perform the above tests (12) and (14) by taking ratios between any (possibly different) linear combinations of W_i . In particular, the cross-section itself $d\sigma/dX dy dZ$ or its integral over y can be used instead of W_i ^{***)}.

*) For definiteness we define a , b and c to correspond to terms in $C(x,z)$ with $\delta(1-x)\delta(1-z)$, $\delta(1-x)$ and $\delta(1-z)$, respectively, and $d(x,z)$ to be free of $\delta(1-x)$ and $\delta(1-z)$.

**) Consequently the dependence on renormalization schemes and gluon gauges sits only in a , b_n and c_m .

***) The prediction for W_S can best be tested separately without taking ratios, because it vanishes in leading order and has no renormalization ambiguities to order g^2 .

If one wants to study the next-to-leading correction completely, one needs to calculate the anomalous dimension of order $\bar{g}^4$. Existing calculations for operator matrix elements $A_n^{(8)}$ and for cut vertices $\Gamma_m^{(6)}$ employed different renormalization schemes and should not be combined together. Here we choose the minimal subtraction scheme of Refs 7) and 8) and postpone the calculation of cut vertices to a later paper.

We calculate perturbative QCD corrections by putting partons infinitesimally off-shell to regularize infra-red and mass singularities. Double logarithms due to infra-red singularities are cancelled by virtual gluon corrections. We find that terms a , b_n and c_m are common to W_2 and W_3 whereas W_5 has only d_{nm} . We present d_{nm} and b_n for all combinations of $q, \bar{q}$ gluon and for all three structure functions W_i .

1) $W_q \rightarrow qX$ (see Fig. 1)

$$d_{3nm}^{qq} = \frac{8}{3} \left(\sum_{j=1}^{n-1} \frac{1}{j} \sum_{i=1}^{m-1} \frac{1}{i} + \sum_{j=1}^{n+1} \frac{1}{j} \sum_{i=1}^{m+1} \frac{1}{i} + \frac{2}{(n+2)(m+1)} \right) \quad (15)$$

$$d_{2nm}^{qq} = d_{3nm}^{qq} + \frac{16}{3} \left\{ \frac{3}{(n+2)m} - \frac{3}{(n+3)m(m+1)} - \frac{1}{(n+2)(m+1)} \right\} \quad (16)$$

$$d_{5nm}^{qq} = \frac{32}{3m} \left\{ \frac{1}{n+2} - \frac{1}{(n+3)(m+1)} \right\} \quad (17)$$

$$b_n^{qq} = \frac{8}{3} \left\{ \left(\sum_{j=1}^n \frac{1}{j} \right)^2 - \sum_{j=1}^{n-1} \frac{1}{j^2} - \frac{1}{n(n+1)} \sum_{j=2}^n \frac{1}{j} + \frac{2\pi^2}{3} - \frac{7}{2} \right\} \quad (18)$$

2) $W_q \rightarrow GX$, $m \geq 2$ (see Fig. 2)

$$d_{3nm}^{qG} = \frac{8}{3} \left\{ \frac{1}{(n+2)m} \left(\frac{1}{m+1} - \frac{1}{n+1} \right) + \sum_{j=1}^{n+2} \frac{1}{j} \frac{m+2}{m(m+1)} - (m-1)! \sum_{i=1}^{\infty} \frac{(i-1)!}{(m+i-1)!} \left(\sum_{j=1}^{n+i-1} \frac{1}{j} + \sum_{j=1}^{n+i+1} \frac{1}{j} \right) \right\} \quad (19)$$

$$d_{2nm}^{qG} = d_{3nm}^{qG} + \frac{16}{3m} \left\{ \frac{3}{(n+3)(m+1)} + \frac{1}{(n+1)(n+2)} - \frac{1}{(n+2)(m+1)} \right\} \quad (20)$$

$$d_{5nm}^{qG} = \frac{4}{3} \frac{8}{(n+3)m(m+1)} \quad (21)$$

whereas $b_n^{qG} = 0$.

3) $\underline{WG \rightarrow qX}$ ^{*)} (see Fig. 3)

$$d_{3nm}^{qq} = - \frac{n^2+n+2}{n(n+1)(n+2)} \sum_{j=1}^{m-1} \frac{1}{j} - (m-1)! \sum_{i=1}^{\infty} \frac{(n+i)^2+(n+i)+2}{(n+i)(n+i+1)(n+i+2)} \frac{(i-1)!}{(m+i-1)!} \quad (22)$$

$$d_{2nm}^{Gq} = - d_{3nm}^{Gq} - \frac{2(n^2+n+2)}{n(n+1)(n+2)} \sum_{j=1}^{m-1} \frac{1}{j} + \frac{2}{m} \left(\frac{6}{(n+2)(n+3)} - \frac{1}{n+1} \right) \quad (23)$$

$$d_{snm}^{Gq} = \frac{8}{(n+2)(n+3)m} \quad (24)$$

$$b_n^{Gq} = \frac{1}{n^2} - \frac{n^2+n+2}{n(n+1)(n+2)} \sum_{j=1}^n \frac{1}{j} \quad (25)$$

If q is replaced by $\bar{q}$, $C_3 \rightarrow -C_3$ (relative sign of Born and next-to-leading terms stays the same).

The cleanest test may be done by taking a non-singlet in both target and produced hadron channels (i.e., by taking the differences of charge states). For this non-singlet combination of W_3 we obtain a particularly simple prediction for the factorization breaking Eq. (12) with

$$d_{3nm,kl}^{qq} = \frac{8}{3} \left\{ \sum_{j=k}^{n-1} \frac{1}{j} \sum_{i=l}^{m-1} \frac{1}{i} + \sum_{j=k+2}^{n+1} \frac{1}{j} \sum_{i=l+2}^{m+1} \frac{1}{i} + \frac{2(n-k)(m-l)}{(n+2)(k+2)(m+1)(l+1)} \right\} \quad (26)$$

for $n > k, m > l$. Numerically, if $\bar{g}^2/4\pi \approx 0.5$, the predicted factorization breaking is of the order of 10% or 20% for small n or m , but becomes larger as n or m increases: for instance, $W_{22}W_{11}/W_{21}W_{12} \approx 1.12$ and $W_{32}W_{11}/W_{31}W_{12} \approx 1.18$ for the non-singlet W_3 .

*) It should be noted that W_3 has mixing in the target channel in contrast to the fully inclusive structure function W_3^t . For the operator matrix element we should take only clockwise quark loops with single flavour because we distinguish quarks and antiquarks here.

Even when the non-singlet part is not separated we can analyze the mixing problem in the following way. Let us take, for instance, $\nu p \rightarrow \mu \pi^\pm X$. Perturbative QCD modifies the naive parton model prediction (4) by adding a mixing contribution $2f_n^G(D_{um}^{\pi^+} - D_{um}^{\pi^-}) C_{2,nm}^{Gq}$ where $C_{2,nm}^{Gq}$ is the coefficient function of $WG \rightarrow qX$. The ratio $f_n^G/(f_n^d + f_n^{\bar{u}})$ can be obtained by fitting the Q^2 dependence due to the mixing in the leading logarithmic approximation⁷⁾⁻⁹⁾. For instance, the ratio of W_2 for the difference of $\nu p \rightarrow \mu \pi^\pm X$ is modified from (12) to

$$\begin{aligned} & \frac{(W_{2nm}^{\nu \rightarrow \pi^+} - W_{2nm}^{\nu \rightarrow \pi^-})(W_{2kl}^{\nu \rightarrow \pi^+} - W_{2kl}^{\nu \rightarrow \pi^-})}{(W_{2nl}^{\nu \rightarrow \pi^+} - W_{2nl}^{\nu \rightarrow \pi^-})(W_{2km}^{\nu \rightarrow \pi^+} - W_{2km}^{\nu \rightarrow \pi^-})} - 1 \\ &= \frac{\bar{g}^2}{16\pi^2} \left\{ d_{2nm,kl}^{qq} + \frac{2f_n^G(d_{2nm}^{Gq} - d_{2nl}^{Gq})}{f_n^d + f_n^{\bar{u}}} + \frac{2f_k^G(d_{2kl}^{Gq} - d_{2km}^{Gq})}{f_k^d + f_k^{\bar{u}}} \right\} \quad (27) \end{aligned}$$

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Figure captions

- Fig. 1 : Perturbative QCD diagrams which contribute to $Wq \rightarrow qX$. Wavy, solid, and curly lines represent weak currents, quarks, and gluons.
- Fig. 2 : Perturbative QCD diagrams for $Wq \rightarrow GX$.
- Fig. 3 : Perturbative QCD diagrams for $WG \rightarrow qX$.

FIG. 1

FIG. 2

FIG. 3