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TRIMUON PRODUCTION BY ANTINEUTRINOS

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ABSTRACT

We examine the electromagnetic and hadronic contributions to anti-neutrino trimuon production. The $\mu^+\mu^+\mu^-$ rate and distributions are in reasonable agreement with the experimental values quoted by the CDHS group for their eight events. More data is required before any detailed comparison can be made between theory and experiment.

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The neutrino production of trimuons is now well understood thanks to the analysis of a large sample of events by the CERN-Dortmund-Heidelberg-Saclay (CDHS) group¹⁾. Their data can be explained by a two-component model consisting of electromagnetic and hadronic production of dimuon pairs in regular inclusive neutrino collisions. The electromagnetic rate has been calculated within the framework of the quark parton model and is free of adjustable parameters²⁾. The hadronic production mechanism is based on using data on the reaction $\pi N \rightarrow \mu^+ \mu^- X$ in the neutrino channel. As such, the rate depends sensitively on the input parameters in the calculation^{1),3)}, and the normalization of this component is not based on any theoretical calculation. Other signals, such as heavy lepton or heavy quark production and decay⁴⁾, which were advanced to explain some of the earlier data⁵⁾, are not required and limits can be placed on the masses and/or coupling strengths for such particles. In other words, no new physics, not even charm, is required to explain the neutrino production of trimuons, even though a small signal from associated charm production may exist in the hadronic component⁶⁾.

The CDHS group have just finished the analysis of an antineutrino run in which they found 8 $\mu^+ \mu^+ \mu^-$ events and 10 $\mu^- \mu^- \mu^+$ events⁷⁾. It is important to know if these events can be explained by the same production mechanism. In this paper we take the conservative viewpoint that no new physics is required to explain these events and we present results for the rates and distributions expected from the electromagnetic and hadronic models. Some estimates have already been made for antineutrino trimuon production in the absence of any data^{2),3)}. Now that some events have been measured it is appropriate to publish a detailed comparison between theory and experiment, updating the theoretical input where possible.

We therefore assume that the 8 $\mu^+ \mu^+ \mu^-$ events seen by the CDHS group are genuine antineutrino trimuon events and the 10 $\mu^- \mu^- \mu^+$ events are neutrino trimuon events arising from the neutrino background in the antineutrino beam.

1. HADRONIC PRODUCTION OF TRIMUONS

Let us first of all discuss the hadronic production model^{1),3)} because it is the most important component in the neutrino induced trimuon events. The model assumes that there is a deep inelastic neutrino (or antineutrino) interaction which produces a $\mu^-(\mu^+)$. The vector and axial vector currents produce a hadron shower with a mass W which ranges, in the 350 GeV neutrino beam, from approximately 2 to 18 GeV. Next, one assumes that this hadronic state will populate the channel $\mu^+ \mu^- X$ with a rate and distributions identical to those measured in purely hadronic collisions at the same centre-of-mass energy. For

instance, in a $\bar{\nu}$ collision where a negatively charged boson is exchanged, the dimuon pair distributions in $\bar{\nu}N \rightarrow \mu^+(\mu^+\mu^-)X$ are identical to those measured in the reaction $\pi^+N \rightarrow \mu^+\mu^-X$ at the same centre-of-mass energy with the direction of the pion taken to be along the hadron shower axis. Therefore the model assumes an almost complete factorization between the neutrino inclusive interaction, which is a function of the usual scaling variables x and y , and the subsequent decay of the hadronic system into a dimuon pair, which is a function of the pair mass, m and its longitudinal and perpendicular momenta $p_{||}$ and $p_{\perp}$. The only link between the two processes is through the W dependence of the strong interaction cross-section which is very small because the cross-section is either flat in W or rises logarithmically.

The model has been used by the CDHS group to explain those events in which the dimuon pair is correlated with the hadronic shower axis¹⁾. One can see this component very clearly in a plot of the $\mu^-\mu^+$ events versus the azimuthal angle $\phi_{1,(2+3)}$. This is the angle between the primary muon and the secondary dimuon pair in the plane perpendicular to the neutrino beam. The severe peaking in $\phi_{1,(2+3)}$ near 180° necessitates a model where the dimuon follows the hadron shower direction. Thus this model is satisfactory in explaining the distributions of the majority of the trimuon events. The trimuon rate, however, is very sensitive to the input distributions assumed for the dimuon pair. The cuts on the muon energies (all $E_\mu \geq 4.5$ GeV) reduce the acceptance for the hadronic component to only a few percent and weights heavily the large $x_F = p_{||}/p_{\max}$ region. The two previous analyses of the hadronic component took different viewpoints with respect to the choice of the x_F dependence.

Barger et al.,³⁾ tried to make a good fit to the available data on the reaction $\pi^-N \rightarrow \mu^+\mu^-X$. They assumed the form

$$E \frac{d\sigma}{d^3p dm} = a(m) \exp(-5.7 m_t) (1 - |x_F|)^{C(m)} \quad (1)$$

with E , p and m the energy, momentum and mass of the dimuon pair, in the πN centre-of-mass system and $m_t^2 = p_{\perp}^2 + m^2$. The function $a(m)$ in units of $\mu b/\text{GeV}^3$ includes a smoothly falling background with Breit-Wigner peaks for the (ρ, ω) and ϕ contributions

$$\begin{aligned}
 a(m) &= a_c + a_{\rho\omega} + a_\phi \\
 a_c &= 2.3 \times 10^{-4} \left(\frac{m}{2m_\mu} - 1 \right)^2 \exp \left(-3.2 \frac{m}{2m_\mu} \right) \\
 a_{\rho\omega} &= 2m / \left[(m^2 - m_\rho^2)^2 + m_\rho^2 \Gamma_\rho^2 \right] \\
 a_\phi &= 0.15 m / \left[(m^2 - m_\phi^2)^2 + m_\phi^2 \Delta \Gamma^2 \right]
 \end{aligned} \tag{2}$$

The relative rate for dimuon inclusive production is obtained by dividing by $\sigma_{\text{tot}}(\pi N) = 25\text{mb}$. The parameter m ranges from $2m_\mu$ to 1.2 GeV and

$$C(m) = 1 + \left(\frac{0.5}{m} \right)^2 \tag{3}$$

so the x_F distribution is flatter for larger mass in agreement with the data of Anderson et al.⁸⁾. With this fit, the authors of Ref. 3) found that the predicted trimuon rate was too small to explain the trimuon events at large $\phi_{1,(2+3)}$ so they multiplied by a factor of 2.5 to get agreement between theory and experiment.

The CDHS group took another approach. They used the form

$$E \frac{d\sigma}{d^3p dm} = F(m) y_{\text{max}} \exp \left(-5.13 / y_{\text{max}}^{0.38} \right) \exp \left(-3.5 p_\perp / m^{1/2} \right) x_F (1-x_F) \tag{4}$$

where $y_{\text{max}} = \log[(E_{\text{max}} + p_{\parallel \text{max}})/m]$ and $F(m)$ is given by the m dependence of $\pi N \rightarrow \mu^+ \mu^- X$ as measured by Anderson et al.⁸⁾, at an energy of 150 GeV and $x_F > 0.15$. In particular this fit assumes a rather flat x_F dependence which is independent of the mass and more consistent with data on the electroproduction of the ρ meson⁹⁾. This hadronic parametrization gives a good fit to the neutrino trimuon distributions $\pi N \rightarrow \mu^+ \mu^- X$. In particular if one changes the x_F dependence to $(1-x_F)^{c(m)}$ where $c(m)$ is given in Eq. (3), then the rate calculated from Eq. (4) with a pion energy of 100 GeV and $x_F > 0.15$ goes up by approximately a factor of two. The reason is that the mass spectrum is now coupled to the x_F dependence and the heavy mass pairs have a larger probability to survive the x_F cut. However, if this is the case, the dimuon mass spectrum gets completely distorted and cannot fit the trimuon data. Adjusting the parameters in the x_F (and also $p_\perp$) dependence to fit the neutrino data is fine but how do we possibly get back to a normalization based on πN cross-sections? Clearly then the normalization of the 3μ rate cannot be made within factors of two or three. There is no doubt that the properties of the trimuon events resemble those of dimuons produced in πN collisions but there must be some differences in going from the case of π interactions to W interactions.

If we arbitrarily change the x_F dependence in Eq. (1) then similar effects occur. In going from $(1 - |x_F|)^{c(m)}$ to $x_F(1 - x_F)$ the rate goes down by ~ 50% because fewer high mass pairs survive the cut. The dimuon mass is also distorted and rather than $\langle m \rangle = 0.55$ GeV, we find that $\langle m \rangle = 0.49$ GeV for $E_\pi = 100$ GeV and $x_F > 0.15$. Hence, if we get the best fit to the πN cross-section and then change the x_F dependence, the rate goes down, which is in the wrong direction to fit the trimuon rate. The actual decrease for the trimuon cross-section is not so large because we have a more complicated kinematical acceptance rather than a straightforward cut at $x_F = 0.15$.

Since the two previous analyses were performed there have been some changes which are worth noting. In particular there is now data on p , π^+ and π^- interactions in carbon at 225 GeV ($x_F > 0.07$) which is published in the thesis of Henry¹⁰⁾. He also gives revised values for the cross-sections for p and π^+ interactions on beryllium at 150 GeV ($x_F > 0.15$) and these cross-sections are larger than previously published by the same group⁸⁾ with most of the increase in the lowest mass bin. In particular the cross-section per nucleon extracted from the beryllium data has risen from 900nb to 1300nb for $x_F > 0.15$ and from 2200nb to 6500nb for $x_F > 0$. The increase does not affect the neutrino results very much due to the cuts on the muon energies. In the thesis it is claimed that a reasonable parametrization to the data is given by

$$E \frac{d\sigma}{dp dm} = A(m) \exp(-B(m) p_\perp) \exp(-C(m) x_F) \quad (5)$$

where $B(m)$ decreases with increasing mass while $C(m)$ decreases rapidly to $m = 0.6$ GeV and then remains constant. Therefore this fit also yields an x_F dependence which is flatter for large masses in agreement with the previous fit in Eq. (1). However, if one examines the data in the ρ mass bin where $C \approx 4$ it seems that the x_F dependence is steeper than that seen in ρ inclusive production at lower energies¹¹⁾.

As far as neutrino trimuon rates are concerned, one cannot reach any firm conclusion from this analysis. Probably the safe thing to say is that the bulk of the neutrino trimuon events, namely those at large $\phi_{1,(2+3)}$ arise from some hadronic mechanism similar in nature to the reaction $\pi N \rightarrow \mu^+ \mu^- X$. However, the rate cannot be determined to within a factor of two or three. Hence it is impossible to ask detailed questions about, say, the size of the axial vector component versus the vector component. Hopefully, data from muon scattering experiments will soon be available. Then interesting comparisons can be made between the neutrino and muon trimuon mechanisms.

Now what can one say, therefore, about antineutrino production? If we follow the same philosophy we should parametrize data on the reaction $\pi^- N \rightarrow \mu^+ \mu^- X$, and insert it into the model. Such data are only available at 225 GeV where the cross-section, mass, x_F and $p_\perp$ dependence are similar to those seen in the $\pi^+ N$ reaction¹⁰⁾. Hence a reasonable thing to do is to use identical parametrizations for the inclusive dimuon production by π^+ and π^- mesons. However, to roughly bound the magnitude of the cross-section we allow ourselves the freedom to change the x_F dependence. We then compare the trimuon rates expected for both the neutrino and antineutrino trimuon production, because even if the absolute normalization is uncertain, the relative rates should fit the experimental results.

Up to now we have concentrated on the dimuon distributions. The model also needs values for the structure functions for deep inelastic neutrino scattering. It is now possible to use a fit to these functions which incorporates scaling violations, as measured in the CDHS experiment¹²⁾. To avoid calculating beta functions at every point we chose a simple interpolation in the variable $\bar{s} = \log [(\log Q^2/0.47)/\log (10.64)]$ above $Q^2 = 5 \text{ GeV}^2$ and assumed exact scaling below $Q^2 = 5 \text{ GeV}^2$. Our parametrization is therefore

$$\begin{aligned} x F_3(x, Q^2) &= 3 x^{\eta_1(\bar{s})} (1-x)^{\eta_2(\bar{s})} / D(\bar{s}) \\ F_2(x, Q^2) &= x F_3 + A(\bar{s}) (1-x)^{\beta(\bar{s})} \end{aligned} \quad (6)$$

where

$$\begin{aligned} \eta_1(\bar{s}) &= 0.56 - 0.147 \bar{s} \\ \eta_2(\bar{s}) &= 2.71 + 0.813 \bar{s} \\ D(\bar{s}) &= 0.77 + 0.29 \bar{s} \\ A(\bar{s}) &= 0.99 + 1.71 \bar{s} \end{aligned} \quad (7)$$

and

$$\beta(\bar{s}) = 8.1 + 4.76 \bar{s}$$

With these values the total cross-section for neutrinos on iron is $0.59 \times 10^{-36} \text{ cm}^2$ at 100 GeV and the corresponding number for antineutrinos is $0.29 \times 10^{-36} \text{ cm}^2$ (both with $E_\mu > 4.5 \text{ GeV}$), so $\sigma_{\bar{\nu}}/\sigma_\nu \approx 0.5$. For antineutrinos, the fact that the y distribution behaves like $(1-y)^2$ means that the $\bar{\nu}$ reaction has a lower hadronic mass W , and consequently the $\bar{\nu}$ trimuon production rate is

lower than the ν production rate. This effect persists even if we normalize the rates by dividing by the $\bar{\nu}(\nu)$ inclusive cross-sections. It is magnified considerably if we incorporate cuts on the muons because fewer secondary muons from the $\bar{\nu}$ reaction have energies greater than 4.5 GeV. The qualitative remarks will be made quantitative later on.

2. ELECTROMAGNETIC PRODUCTION OF TRIMUONS

The electromagnetic production of trimuons assumes that muon pairs are radiated from the muon and the quark in the normal inclusive reaction $\nu_{\mu} + N \rightarrow \mu^{-} + X$. We have already published results for this model²⁾ and know qualitatively what to expect when we switch from a neutrino to an antineutrino beam. The rate for $\bar{\nu}$ is approximately one-half of the rate for ν due to the change in the y distribution so if we divide by the inclusive cross-section we expect almost identical trimuon rates for $\bar{\nu}$ and ν . Remember that it is interference terms which dominate in electromagnetic interactions so the normalized rates are not very sensitive to the changes in the y distribution. (This is not the case for the hadronic model.)

The cuts on the final muons affect both rates roughly equally because even though the dimuon pair radiated from the quarks are softer in energy, the muons radiated from the final muon are harder in energy and the two effects counter-balance each other. We therefore expect the $\phi_{1,(2+3)}$ distribution to peak more at small angles for the $\bar{\nu}$ reaction. This effect remains when cuts are incorporated. Thus, at a fixed energy, the distributions change but the rates remain roughly equal.

We have changed our previous calculations by taking a different fit to the parton structure functions. In our model it is the valence quarks which radiate the muon pairs so we only include the valence terms in the nucleon structure functions. Hence we take $F_2(x) = xF_3(x)$ where $xF_3(x)$ is given in Eq. (6). This fit gives a slightly higher rate for the $\mu^{-}\mu^{-}\mu^{+}$ events than was reported earlier.

Before we present our results we should comment on the differences in the spectra between the ν and $\bar{\nu}$ beams. The charged current rates for μ^{+} and μ^{-} production in the 350 and 330 GeV antineutrino run are shown in Fig. 1. One sees that there is a serious contamination in the $\bar{\nu}$ beams because there are more μ^{-} events than μ^{+} events above 100 GeV. For comparison we show the shape (not normalized) of the charged current event rate for the 350 GeV neutrino run where the CDHS group found most of their trimuon events¹⁾. The spectra are found by dividing these event rates by the beam energy. A comparison of the two

neutrino beams shows that the ν spectrum in the $\bar{\nu}$ beam is slightly harder than the old 350 GeV spectrum so we expect a slightly larger $\mu^-\mu^-\mu^+$ rate than measured previously.

3. RESULTS

Let us first examine the total cross-sections for the hadronic and electromagnetic reactions in neutrino and antineutrino beams. The rates relative to the charged current cross-sections are given in Fig. 2 for two cases: (1) without cuts on the muons and (2) with cuts on the muon energies at 4.5 GeV. The hadronic cross-section is calculated from Eqs. (1) and (6). One sees immediately that the cuts drastically reduce the hadronic cross-section. At approximately 50 GeV the acceptance is only 3% for neutrinos and 2% for antineutrinos. Moreover, as we increase the energy the acceptance for $\mu^-\mu^-\mu^+$ events in the ν beam increases more rapidly than the acceptance for $\mu^+\mu^+\mu^-$ events in the $\bar{\nu}$ beam. At 150 GeV the ratios become 13% and 6% respectively. The electromagnetic interaction has a much better acceptance, namely 25% at 50 GeV and 50% at 150 GeV for both neutrinos and antineutrinos. Our results for the cross-sections are calculated by evaluating seven dimensional integrals by Monte Carlo methods. The error involved in this evaluation is approximately 5% when no cuts are applied and 15% when cuts are applied, which is accurate enough for our purposes.

Now let us turn to the actual values for the measured rates when we include the neutrino spectrum. If we take the fluxes above 30 GeV then the average visible energy is higher for the trimuon events than for the muon inclusive cross-sections because the trimuon cross-sections have a steeper E_ν dependence. Note that the E_ν cut near 30 GeV does not affect the numerator because the trimuon rate is small there. The actual onset of the trimuon rate is sensitive to the shape of the neutrino (antineutrino) spectrum in the region near 30 GeV. For instance, the 350 GeV neutrino spectrum is large in that region so that trimuon events can already occur above 20 GeV (and actually 5 of the CDHS $\mu^-\mu^-\mu^+$ events have energies below 30 GeV). The other neutrino spectrum is much flatter near 30 GeV so the trimuon rate is negligible until ~ 33 GeV. The total $\mu^-\mu^-\mu^+$ event rate is not sensitive to the actual position of this cut but remember we are always dividing by the single muon rate and that is sensitive to the location of an energy cut.

Table I gives our results for the trimuon rates separated into electromagnetic and hadronic components. For the hadronic fit we again use Eqs. (1) and (6), with the x_F dependence as is and with the dependence changed to $x_F(1 - x_F)$. In both cases the rates are roughly the same and the total hadronic rate is too small to account for the measured values. Therefore we take an extra factor of

three and compute the total cross-section as electromagnetic plus three times the hadronic. The averages for the measurable quantities are given in Tables II and III computed on the following basis. For the $\bar{\nu}$ reaction, the electromagnetic and total hadronic components are taken to be equal in magnitude whereas for the ν events, the total hadronic component is taken to be three times the electromagnetic. In such case we use Eq. (1) for the hadronic input. The agreement with the experimental values is satisfactory.

We have checked the hadronic trimuon rates with other fits to the pion data and do not find any substantial changes from the results reported here. The actual rates can go up and down by maybe fifty percent with changes in the distributions. However, it seems an inescapable conclusion of this analysis that the hadronic trimuon production rate is not well understood. Probably the model is just too simple to account for the experimental data. We do not want to propose that there is something seriously lacking, for example, the contribution of a heavy quark, because the distributions clearly rule out such effects. Nevertheless, there could be some small component from associated charm production. We need further data, in particular in the $\bar{\nu}$ channel, before such conclusions can be confirmed. It will be interesting to see whether muon scattering experiments, with their higher beam energies, will see similar effects.

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TABLE I

	electro- magnetic component	hadronic component (1-x _F) ^c	hadronic component x _F (1-x _F)	Total I+3*II	Expt.
$\sigma(++-)/\sigma(+)$ in $\bar{\nu}$ beam	0.5	0.2	0.2	1.1	1.8±0.6
$\sigma(--+)/\sigma(-)$ in $\bar{\nu}$ beam	1.0	0.8	0.6	3.4	4.1±1.3
$\sigma(--+)/\sigma(-)$ in 350 GeV ν beam	0.9	0.7	0.6	2.7	3.0±0.4

TABLE II

	Expt.	Electro- magnetic	Hadronic	Total
E _{vis} (GeV)	110±10	107	114	112
E _{had} (GeV)	44±8	34	57	50
P ₁ (GeV/c)	35±7	39	34	35
P ₂ (GeV/c)	14.5±3	16	12	14
P ₃ (GeV/c)	16±2.5	18	12	14
m ₁₂ (GeV)	1.9±0.33	1.7	2.7	2.5
m ₁₃ (GeV)	2.6±0.46	1.8	2.5	2.3
m ₂₃ (GeV)	0.83±0.17	0.7	0.6	0.6
m ₁₂₃ (GeV)	3.4±0.6	2.7	2.8	3.6
ϕ_{12} (degree)	90±20	70	136	120
ϕ_{12} (degree)	98±20	69	138	115
$\phi_{1,(2+3)}$ (degree)	96±20	67	146	120
P _{T2} ^s (GeV/c)	0.7±0.14	3.1	0.4	0.9
P _{T3} ^s (GeV/c)	0.91±0.21	3.8	0.4	0.9
P _{T23} ^s (GeV/c)	1.4±0.4	1.8	0.5	0.8
x	0.21±0.07	0.10	0.25	0.21
y	0.69±0.05	0.85	0.70	0.73

TABLE III

	Expt.	Electro- magnetic	Hadronic	Total
$E_{vis}(\text{GeV})$	90 ± 10	73	88	84
$E_{had}(\text{GeV})$	41 ± 10	15	33	28
$P_1(\text{GeV}/c)$	25 ± 7	30	32	31
$P_2(\text{GeV}/c)$	9.5 ± 1	14	10	12
$P_3(\text{GeV}/c)$	12 ± 2	15	11	13
$m_{12}(\text{GeV})$	1.35 ± 0.15	1.3	2.3	1.8
$m_{13}(\text{GeV})$	1.5 ± 0.24	1.2	2.3	1.8
$m_{23}(\text{GeV})$	1.0 ± 0.15	0.6	0.6	0.6
$m_{123}(\text{GeV})$	2.2 ± 0.28	1.9	3.4	2.5
$\phi_{12}(\text{degree})$	82 ± 20	74	135	95
$\phi_{13}(\text{degree})$	63 ± 20	73	137	95
$\phi_{1,(2+3)}(\text{degree})$	81 ± 30	71	144	110
$P_{T2}^S(\text{GeV}/c)$	0.81 ± 0.16	3.4	0.4	1.5
$P_{T3}^S(\text{GeV}/c)$	1.05 ± 0.19	4.1	0.4	1.6
$P_{T23}^S(\text{GeV}/c)$	1.55 ± 0.25	1.5	0.5	1.1
x	0.15 ± 0.03	0.08	0.18	0.14
y	0.75 ± 0.06	0.6	0.64	0.62

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Table captions

- Table I : Cross-section ratios ($E_\nu > 30$ GeV, $E_\mu > 4.5$ GeV). Units of 10^{-5} .
- Table II : Average values for the $\mu^-\mu^-\mu^+$ events.
- Table III: Average values for the $\mu^+\mu^+\mu^-$ events.

Figure captions

- Fig. 1 : Single muon event rates in the antineutrino run (solid lines) and in
the 350 GeV neutrino run (dashed line).
- Fig. 2 : Trimuon total cross-sections normalized to the single muon rate. The
neutrino results are shown as solid lines and the antineutrino results
are shown as dashed lines. With cuts means all muons have $E_\mu > 4.5$ GeV.

Fig.2.