Ref.TH.2643-CERN

GLUEBALL-POMERON IDENTITY AND THE TRANSVERSE STRUCTURE
OF TWO-JET EVENTS

L. González Mestres and P. Aurenche
CERN - Geneva

ABSTRACT

We discuss the transverse structure of two-jet events within a scheme à la Harari-Freund, using Dual Field Theory as a guide. We point out that the situation is quite different from current two-random-walk models based on uncorrelated-jet pictures of multiparticle production, and that it also differs in a natural way from standard f iteration schemes leading to Pomeron f identity. It then follows that recent criticism based on the role of transverse dimensions cannot be applied to the "gluonic" Pomeron of Dual Field Theory.

Ref.TH.2643-CERN
8 March 1979

1. INTRODUCTION

The detailed structure of two-jet events (Fig. 1) in hadronic collisions at small p_T , and its relationship with the Pomeron singularity, remain rather unclear from both theoretical and phenomenological points of view¹⁾⁻⁶⁾. Although many interesting ideas have been proposed, several crucial questions are still unanswered and the present experimental information is not enough to fully understand the nature of the bare Pomeron.

This note concerns the transverse structure of two-jet events^{*)} within a scheme à la Harari-Freund⁷⁾, based on Dual Field Theory (DFT)⁸⁾, which we considered in two previous papers³⁾. Identifying the crossed-loop singularity (Fig. 2) with the "gluonic" Pomeron, one gets a definite prediction for the total transverse momentum of each jet (see Ref. 3)), which we wish to compare with current random-walk pictures^{5),9)} based on an uncorrelated-jet interpretation of multiperipheral dynamics.

We point out that, while the result of DFT is quite consistent with the production of two overlapping chains linked through a net of gluons in the whole overlap rapidity region (Fig. 3a), the usual two-jet random-walk picture corresponds to two jets linked only at the ends (Fig. 3b), very much like the intermediate states of Regge-Regge cuts. We therefore argue that the current impact parameter description of two-jet events should undergo a substantial change if it is to describe the multiparticle intermediate states that build up a Pomeron singularity reminiscent of gluonic "fishnet" dynamics¹⁰⁾. This, in particular, invalidates recent objections against the Harari-Freund scheme based on the role of transverse dimensions¹¹⁾, since the "gluonic" Pomeron of DFT has nothing to do with two independent transverse random walks.

2. GLUEBALL EXCHANGE AND CORRELATION BETWEEN JETS

It is commonly assumed^{5),6),9)} that particles belonging to two different jets are uncorrelated in rapidity. This can be a rather reasonable simplification since the materialization of gluons into hadrons takes place at the borders of the diagram (forgetting about flavour loop corrections to Fig. 3a), and the two jets are separated by the whole net of gluons. The two-random-walk hypothesis^{5),11)} assumes that there are two flows of transverse momentum, one on each border, which circulate along the vertical direction--rapidity axis-- of the diagram. But it excludes

^{*)} In what follows it is understood that we work in the centre-of-mass system and choose the longitudinal axis in such a way that the two incident particles carry no transverse momentum.

the possibility of a "flux" of p_T going from one border to another horizontally (transverse correlations at the same rapidity).

In Fig. 3a we have used the same description as in Ref. 3). The two produced jets are the decay products of heavy resonances with squared masses $M_1^2 \sim e^{Y_1}$, $Y_1 = y_0 + y_1$ and $M_2^2 \sim e^{Y_2}$, $Y_2 = y_0 + y_2$, where y_0 is the rapidity overlap of the two heavy resonances considered as composite or decaying objects. The squared centre-of-mass energy is $s \sim e^Y = e^{y_0 + y_1 + y_2}$ for the whole system in the direct channel. Then, at the leading order in the $1/N_c$ expansion, DFT tells us that the transverse momentum k_{T3} of each jet follows a Gaussian distribution and its averaged square is³⁾:

$$\langle k_T^2 \rangle \simeq \lambda y_0 \simeq \lambda \ln \frac{M_1^2 M_2^2}{s} \quad (1)$$

where λ is a constant that can be calculated.

Equation (1) implies the existence of correlations in transverse momentum between the two jets. More precisely, the dependence on y_0 indicates that this phenomenon is characteristic of the overlap region, as can be seen in the following intuitive model. The constant density of secondaries (rapidity plateau) and usual assumptions about the hadronization of constituents lead to a uniform density of horizontal gluon lines in the rapidity space for constituent fishnet diagrams. Then, there will be a number of horizontal gluon lines proportional to y_0 in the overlap region of Fig. 3a. In terms of secondaries, equation (1) can be obtained assuming that, in the overlap region, the transverse momentum of each produced particle carries a component which is compensated with particles from the other jet produced roughly at the same rapidity. Then, inside each jet, we get a non-compensated transverse momentum which, under statistical hypothesis, will be proportional to the number of particles produced from this jet in the overlap region, i.e., proportional to y_0 . Away from the overlap region, short-range order forbids this phenomenon and local compensation can happen only between secondaries of the same jet. In terms of gluons, we can assume that each horizontal gluon line carries a cut-off p_T and that this gives rise to a flux of p_T local in rapidity between both jets. Again, a statistical model for all the flux lines exchanged in the overlap region leads to expression (1). Away from the overlap region, there is no transfer of p_T between the two jets: for instance, in

the upper region of rapidity size y_1 the p_T flow coming from the right just "bounces" at the quark line on the left and comes back to the original jet (local compensation inside the same jet). The situation is illustrated in Fig. 4, and has two consequences: i) since there is a transfer of p_T , there will be transverse correlations between particles belonging to different jets in the overlap rapidity region; ii) these correlations will somehow involve the rapidity variables, since it is only in the overlap region that a transfer of p_T between the two jets can take place^{*)}.

Apart from the kinematical result (1), the need of transverse correlations between the secondaries of both jets can also be understood on general theoretical grounds. As we explained in Ref. 3), the "gluonic" closed-string exchange of DFT is generated in the direct channel by the production of pairs of heavy resonances which carry in common some dual harmonic oscillators (the $a_{\mu}^{(n)\dagger}$ 's⁸⁾) with spin-spin coupling. When the heavy resonances decay into light clusters, some of the oscillators become a component of the orbital wave functions of the produced clusters, whereas the remaining oscillators will couple to cluster polarization tensors. Once the jet has become an n pion system, all the oscillators manifest themselves through the orbital n pion wave function. Then the spin-spin coupling between heavy resonances leads to a coupling between the many-pion orbital tensors that describe the angular momentum of both jets, so that we end up with a coupling between the momenta of clusters (or pions) belonging to different jets. Since transverse dimensions play a crucial role in building the Pomeron^{8),12)}, transverse correlations between the two jets turn out to be a basic ingredient of the structure of diffraction in glueball-Pomeron identity schemes.

3. THE TWO-RANDOM-WALK PICTURE

In the uncorrelated two-jet model⁵⁾, the n particle cross-section is given by:

$$\sigma_n^{2jets}(s) = \sum_{\substack{l+m=n \\ l,m \geq 1}} \sigma_{n(l,m)}^{2jets}(s) \quad (2a)$$

*) This is obviously required by short-range order. But, in this paper we will ignore this point and consider factorized amplitudes since we are interested only in quite general qualitative features. It could happen that, after integration over transverse dimensions, the deviation from factorized amplitudes would not be dramatic.

with

$$\begin{aligned}
 \sigma_{n(\ell, m)}^{2 \text{ jets}} &= \\
 &= \int d^2 B \prod_{i=1}^{\ell} \frac{dx_i}{\bar{x}_i} d^2 b_i \prod_{j=1}^m \frac{dx'_j}{\bar{x}'_j} d^2 b'_j \quad (2b) \\
 &\quad \delta\left(\sum_i x_i + \sum_j x'_j\right) \delta\left(2 - \sum_i \bar{x}_i - \sum_j \bar{x}'_j\right) \delta^2(b_1) \\
 &\quad \delta^2\left(B - \sum_i x_i b_i - \sum_j x'_j b_j\right) \left| M_{n(\ell, m)}^{2 \text{ jets}}(x_i, x'_j, b_i, b'_j) \right|^2
 \end{aligned}$$

where $i = 1, 2, \dots, \ell$ labels the particles that belong to one of the jets (let us call it the first jet) and $j = 1, 2, \dots, m$ stands for those belonging to the other jet (we call it the second one). The x_i and x'_j denote the Feynman variables on each jet, $\bar{x}_i$ and $\bar{x}'_j$ are for the energy variables and b_i and b'_j are the impact parameters associated to the produced $\ell + m$ particles. B is the impact parameter of the initial two-particle state, $B = b_a - b_b$, a and b being the two incident particles. The $\delta^2(B - \sum_i x_i b_i - \sum_j x'_j b_j)$ is reminiscent of angular momentum conservation¹³⁾ and the $\delta^2(b_1)$ is needed because of translational invariance. Finally, the two-random-walk scattering amplitude $M_{n(\ell, m)}^{2 \text{ jets}}$ is usually taken to be:

$$\begin{aligned}
 M_{n(\ell, m)}^{2 \text{ jets}} &= \\
 &= \psi_2(b_1 - b'_1) \psi_b(b_\ell - b'_m) h(x_1, x_2, \dots, x_\ell) \quad (2c) \\
 &\quad h(x'_1, x'_2, \dots, x'_m) \prod_{i=2}^{\ell} e^{-\frac{1}{2a}(b_i - b_{i-1})^2} \prod_{j=2}^m e^{-\frac{1}{2a}(b'_j - b'_{j-1})^2}
 \end{aligned}$$

where ψ_a and ψ_b are wave functions associated to jet emission by the incident hadrons a and b . They are normally assumed to span a finite range in impact parameter. In realistic models, the $\ell + m$ produced particles should be resonances or clusters rather than pions. As usual in soft processes, we shall neglect the dependence of the $\bar{x}$'s on transverse momenta.

The integration over d^2B in (2a) can be trivially performed and one gets:

$$\sigma_{n(\ell,m)}^{2jets}(s) = \Phi_{n(\ell,m)}^L(s) \Phi_{n(\ell,m)}^T \quad (3a)$$

with:

$$\begin{aligned} \Phi_{n(\ell,m)}^L(s) &= \\ &= \int \prod_{i=1}^{\ell} \frac{dx_i}{\bar{x}_i} \prod_{j=1}^m \frac{dx'_j}{\bar{x}'_j} \end{aligned} \quad (3b)$$

$$\begin{aligned} &|h(x_1, x_2, \dots, x_m) h(x'_1, x'_2, \dots, x'_m)|^2 \\ &\delta(\sum_i x_i + \sum_j x'_j) \delta(2 - \sum_i \bar{x}_i - \sum_j \bar{x}'_j) \end{aligned}$$

$$\begin{aligned} \Phi_{n(\ell,m)}^T &= \int \prod_{i=1}^{\ell} d^2b_i \prod_{j=1}^m d^2b'_j \\ &\delta^2(b_1) |\psi_a(b_1 - b'_1) \psi_b(b_\ell - b'_m)|^2 \\ &\prod_{i=2}^{\ell} e^{-\frac{1}{2\alpha}(b_i - b_{i-1})^2} \prod_{j=2}^m e^{-\frac{1}{2\alpha}(b'_j - b'_{j-1})^2} \end{aligned} \quad (3c)$$

so that $\Phi_n^T(\ell, m)$ carries all transverse dynamics and is independent of s and of any other longitudinal variable. Choosing $|\psi_a(z)|^2 = |\psi_b(z)|^2 = \delta^2(z)$, we get $b_1 = b'_1 = 0$ and $b_\ell = b'_m = b$ which gives:

$$\Phi_n^T(\ell, m) = \int d^2b \phi_n(\ell, m)(b) \quad (4a)$$

with:

$$\phi_n(\ell, m)(b) = P_\ell(b) P_m(b) \quad (4b)$$

and:

$$P_\kappa(b) = \int \prod_{i=1}^{\kappa} d^2b_i \delta^2(b_1) \delta^2(b_\kappa - b) \prod_{j=2}^{\kappa} e^{-\frac{1}{2}(b_j - b_{j-1})^2} \quad (4c)$$

which is just the random walk probability for going from $b_1 = 0$ to b in $\kappa-1$ steps. We then have exactly recovered the ansatz used by C.I. Tan¹¹⁾ for transverse dimensions to discuss the consistency of Veneziano's two-jet picture⁹⁾ of the Pomeron.

The first thing to notice is that, since the integrand of (3c) is independent of the energy variables, the total $\langle k_T^2 \rangle$ of each jet cannot depend on y_0 at fixed ℓ and m . It is only through an increasing of the average multiplicity that we can eventually produce an asymptotic dependence of $\langle k_T^2 \rangle$ on the longitudinal variables. But we shall see in what follows that also this is excluded. If we Fourier-transform the Gaussians in impact parameters¹³⁾, the conjugate variable of $b_i - b_{i-1}$ is q_i , the transverse momentum exchanged between particles $i-1$ and i (Fig. 3b) on the first jet. On the second jet, we have in a similar way q'_j for $b'_j - b'_{j-1}$. We then can express our $\sigma_{n(\ell, m)}^{2 \text{ jets}}$ in terms of the Fourier transform of $M_{n(\ell, m)}^{2 \text{ jets}}(x_1, x'_j, b_1, b'_j)$. More precisely, we have always the factorized expression (3a) and the longitudinal dependence (3b). But now the transverse factor is given by (Fig. 5a):

$$\begin{aligned} \bar{\Phi}_n^T(l, m) &= \\ &= \int d^2 \bar{q}_{11} d^2 \bar{q}_{lm} \prod_{i=2}^l d^2 q_i \prod_{j=2}^m d^2 q'_j \Gamma(\bar{q}_{11}, \bar{q}_{lm}, q_i, q'_j) \end{aligned} \quad (5a)$$

where $\bar{q}_{11}$ is the transfer in transverse momentum from particle 1 of the second jet to particle 1 of the first jet, and $\bar{q}_{lm}$ is the transfer between particle l (first jet) and particle m (second jet), as indicated in Fig. 3b. Γ is given by:

$$\begin{aligned} \Gamma(\bar{q}_{11}, \bar{q}_{lm}, q_i, q'_j) &= \\ &= \int d^2 \Delta \rho_a(\bar{q}_{11}) \rho_a^*(\bar{q}_{11}^*) \rho_b(\bar{q}_{lm}) \\ &\quad \rho_b^*(\bar{q}_{lm}^*) \prod_{i=2}^l e^{-\frac{a}{2}(q_i^2 + q_i^{*2})} \prod_{j=2}^m e^{-\frac{a}{2}(q_j'^2 + q_j'^{*2})} \end{aligned} \quad (5b)$$

with:

$$\begin{aligned} \bar{q}_{11}^* &= \bar{q}_{11} + \Delta, & \bar{q}_{lm}^* &= \bar{q}_{lm} + \Delta \\ q_i^* &= q_i + \Delta, & q_j'^* &= q_j' + \Delta \end{aligned} \quad (5c)$$

ρ_a, ρ_b being the Fourier transform of ψ_a, ψ_b and ρ_a^*, ρ_b^* their complex conjugates. In (5a) and (5b) we ignore irrelevant normalization factors.

The appearance of the Δ variable is due to the invariance of the $(l + m)$ particle state under the transformation:

$$\begin{aligned} \bar{q}_{11} &\rightarrow \bar{q}_{11} + \delta & , & & \bar{q}_{em} &\rightarrow \bar{q}_{em} + \delta \\ q_i &\rightarrow q_i + \delta & , & & q'_j &\rightarrow q'_j + \delta \end{aligned}$$

which, in terms of the process of Fig. 3b, correspond to a flux of transverse momentum δ circulating all around the internal loop as in Fig. 5b. Note that, although the internal lines from particles a and b to the jets are not physical items, they behave mathematically as exchanged objects carrying the transverse momenta $\bar{q}_{11}$ and $\bar{q}_{em}$. For our practical purposes, it is useful to consider the density distribution:

$$\begin{aligned} \Omega(\bar{q}_{11}, \bar{q}_{em}) &= \\ &= \int \prod_{i=2}^l d^2 q_i \prod_{j=2}^m d^2 q'_j \Gamma(\bar{q}_{11}, \bar{q}_{em}, q_i, q'_j) \end{aligned} \quad (6)$$

which will allow us to compute the average k_T^2 of each jet since, if k_T is the total transverse momentum of the first jet, one has:

$$k_T = \bar{q}_{em} - \bar{q}_{11} \quad (7)$$

We get (again up to irrelevant factors):

$$\begin{aligned} \Omega(\bar{q}_{11}, \bar{q}_{em}) &= \int d^2 \Delta \\ &e^{-\frac{a}{4}(n-2)\Delta^2} \rho_a(\bar{q}_{11}) \rho_a^*(\bar{q}_{11} + \Delta) \rho_b(\bar{q}_{em}) \rho_b^*(\bar{q}_{em} + \Delta) \end{aligned} \quad (8a)$$

In the limit $n \rightarrow \infty$, we have $e^{-\frac{a}{4}(n-2)\Delta^2} \rightarrow \frac{4\pi}{a(n-2)} \delta^2(\Delta)$ so that one simply gets:

$$\Omega(\bar{q}_{11}, \bar{q}_{lm}) \underset{n \gg 1}{\simeq} \quad (8b)$$

$$\simeq \frac{4\pi}{a(n-2)} |\rho_a(\bar{q}_{11})|^2 |\rho_b(\bar{q}_{lm})|^2$$

so that, at large n , $\langle k_T^2 \rangle$ is independent of both ℓ and m and has no chance of growing like the rapidity variable y_0 . It then follows that uncorrelated jet models cannot describe the transverse structure of gluonic Pomeron exchange in the overlap region of the rapidity ordered s channel jets.

Note also, although this is a minor point, that the choice made implicitly by C.I. Tan ($b_1 = b'_1$, $b_\ell = b'_m$ identically¹¹⁾) is not a physical one since it implies unboundedness of transverse momentum for the leading particles (1 and ℓ in the first jet and 1 and m in the second one). In that case both $|\rho_a|^2$ and $|\rho_b|^2$ are constant and normalized to $1/\delta(0)$, so that $\langle k_T^2 \rangle$ is unbounded independently of y_0 , ℓ and m ^{*)}. A Gaussian dependence for ψ_a and ψ_b would be a better choice, but our analysis does not depend on the particular form of the wave functions. Even if $\bar{q}_{11}$ and $\bar{q}_{lm}$ are unbounded, at large n we still have:

$$\langle (q_\ell - q_2)^2 \rangle \simeq \langle (q'_m - q'_2)^2 \rangle \simeq 4a \quad (8c)$$

which is enough for our purposes, since one has:

$$q_2 - q_\ell = p_{T2} + p_{T3} + \dots + p_{T\ell-1} \quad (8d)$$

where p_{Ti} is the transverse momentum of the i^{th} particle on the first jet, and analogously:

*) When $|\rho_a|^2$ and $|\rho_b|^2$ are constants, (8b) holds identically for all n , with an equal sign instead of $\simeq$. In this way we exactly recover C.I. Tan's expression for $\Phi_n^T(\ell, m)$.

$$q'_m - q'_2 = p'_{T2} + p'_{T3} + \dots + p'_{Tm-1} \quad (8e)$$

p'_{Tj} being the transverse momentum of the j^{th} particle on the second jet.

4. THE ROLE OF TRANSVERSE CORRELATIONS IN TWO-JET EVENTS

As was discussed in Ref. 11), it is the factor $1/(n-2)$ in (8b) which leads to $1/\ln s$ factors in the total cross-section and produces violations of short-range order if the leading singularity is assumed to be generated by a two-random-walk picture. The factor $1/(n-2)$ [$1/(n+2)$ in C.I. Tan's notation] comes, in our description, from the integral over the Δ variable in (8a). But, in terms of this variable, there is a much more familiar way of handling the calculation. We can split the diagram of Fig. 5a in pieces, calculate separately the "amplitudes" associated to Figs. 6a and 6b and finally make a convolution weighted with ρ_a, ρ_b, ρ_a^* and ρ_b^* . This is trivial to do using the factorization properties of the integrands in (3b), (5a) and (5b). If we are interested in calculating $\sigma^2 \text{ jets} = \sum_n \sigma_n^2 \text{ jets}$, we can use (2a) and (3a), as well as the fact that the sums over i and j factorize. Then, after calculating the "amplitude" associated to Fig. 6a (including the longitudinal dependence), we can immediately sum over all ℓ and we get a one-jet Regge amplitude corresponding to a process with squared total centre-of-mass energy $s_1 = M_1^2$ (squared mass of the first jet) and momentum transfer $t_1 = \Delta^2$. Such an amplitude carries a factor $s_1^{\alpha(t_1)}$ (α = Regge trajectory) which introduces a Gaussian cut-off in Δ . Then the phase space integral for Δ in the full amplitude will give a factor $1/\ln s_1$, which leads in a natural way to a factor $1/\ln s$ in $\sigma^2 \text{ jets}(s)$. As in standard Regge cut calculations, the $1/\ln s$ is just the average momentum transfer of individual Regge amplitudes. Going back to Fig. 5b, we can say that, if we shift $\bar{q}_{11}$ to $\bar{q}_{11} + \delta$, we must equally shift $\bar{q}_{\ell m}$ to $\bar{q}_{\ell m} + \delta$ in order not to change the n particle intermediate state. This kinematical long-range correlation is the origin of the factor $1/\ln s_1 \simeq 1/y_1$, y_1 being the distance in rapidity covered by the transfer δ of transverse momentum. The simplest way to destroy such an unwanted feature is to allow for direct transfers of transverse momentum between the two jets away from the ends of the two chains. For instance, in Fig. 7a, allowing the internal wavy lines $1, 2, \dots, p$ to carry finite transverse momenta p_{Tk}'' we destroy the kinematical correlation between $\bar{q}_{11}$ and $\bar{q}_{\ell m}$. The ambiguity there, are the loops of

transverse momenta δ_k , (Fig. 7b) which now cover only finite rapidity intervals. In this way the cut-off proportional to the whole rapidity interval y_1 disappears automatically. This is the role of the net of gluons (Fig. 3a) in DFT.

Although we cannot give precise formulae - since it would require a better understanding of many yet unsolved problems of QCD - we can illustrate with a rough example how the two-random-walk picture will be modified when gluon exchanges are allowed between the two jets. Assume (Fig. 8) that we start from two uncorrelated chains and we introduce the exchange of an object - the wavy line - between the internal lines of both jets somewhere in the overlap rapidity region. We take the nearest particles in rapidity to be particles λ and $\lambda + 1$ in the first jet and μ and $\mu + 1$ in the second jet. Let us call β and β' the impact parameters of the vertices where the new object is coupled to the multiperipheral chains, and assume that the propagation from β to β' is governed by the same kind of gaussian:

$$\psi(\beta, \beta') = e^{-\frac{1}{2a} (\beta - \beta')^2} \quad (9a)$$

as for the internal lines of the chains, with similar propagators from b_λ to β , from β to $b_{\lambda+1}$, from b'_μ to β' and from β' to $b'_{\mu+1}$. We then are led to consider a four-point function of the form:

$$\begin{aligned} \psi(b_\lambda, b_{\lambda+1}, b'_\mu, b'_{\mu+1}) &= \\ &= \int d^2\beta d^2\beta' \psi(\beta, \beta') \psi(\beta, b_\lambda) \psi(\beta, b_{\lambda+1}) \psi(\beta', b'_\mu) \psi(\beta', b'_{\mu+1}) \end{aligned} \quad (9b)$$

which, after calculation, turns out to behave like (up to constant factors):

$$\begin{aligned} \psi(b_\lambda, b_{\lambda+1}, b'_\mu, b'_{\mu+1}) &= \\ &= \exp \left\{ \frac{3}{8} \cdot \frac{1}{2a} \left[(b_\lambda - b_{\lambda+1})^2 + (b'_\mu - b'_{\mu+1})^2 \right] + \right. \\ &\quad \left. + \frac{1}{8} \cdot \frac{1}{2a} \left[(b_\lambda - b'_\mu)^2 + (b_\lambda - b'_{\mu+1})^2 + (b_{\lambda+1} - b'_\mu)^2 + (b_{\lambda+1} - b'_{\mu+1})^2 \right] \right\} \end{aligned} \quad (10)$$

giving rise to gaussian p_T transfers between particles belonging to different jets: correlations inside the same jet are weakened and transverse correlations from one jet to another appear.

An interesting feature of formula (10) is that it does not allow for naive factorized iterations. For instance, assume that particles $\lambda, \lambda + 1, \mu$ and $\mu + 1$ of Fig. 8 are clusters ordered in rapidity in such a way that they correspond to planar clusters of f iteration schemes^{14), 15)}. Then, if $x_\lambda > x'_\mu > x_{\lambda+1} > x'_{\mu+1}$, the Regge pole iteration would be:

$$A_\lambda(J) C_{\lambda\mu} A_\mu(J) C_{\mu,\lambda+1} A_{\lambda+1}(J) C_{\lambda+1,\mu+1} A_{\mu+1}(J) \quad (11)$$

so that there is no direct correlation between, for instance, clusters λ and $\mu+1$ or between clusters λ and $\lambda+1$. (J = angular momentum, A = cluster sum rule, C = cylinder multiperipheral kernel for Reggeon renormalization). On the contrary, expression (10) introduces new correlations which, without going against short-range order, avoid naive factorized expressions. For instance, there is a direct transfer of p_T between clusters λ and $\lambda+1$ due to the jet structure of the process whereas the transfer of p_T between the two jets gives rise to direct correlations between cluster λ and clusters μ and $\mu+1$. Roughly speaking, a cluster will have direct transverse correlations with four or five neighbours in the longitudinal space, instead of with only two (e.g., cluster μ with clusters λ and $\lambda+1$) as iteration schemes seem to suggest. This can possibly give us an idea of how the decay products of the gluonic two-jet system would avoid Pomeron f identity if the glueball singularity really dominates. In fact, when clusters λ and μ are close in rapidity, there can be a priori many kinds of correlations and there is no reason why one should be able to write a Reggeon sum rule for each cluster separately.

5. CONCLUSION AND COMMENTS

We have seen that the two-random-walk schemes must undergo some crucial modifications in order to reasonably describe the transverse properties of realistic two-jet events. In particular, the transverse structure of such events appears to be strictly incompatible with that of a Regge cut. Our analysis of the role of gluonic fishnet diagrams suggests a natural way to escape recent criticism¹¹⁾ of Veneziano's two-jet scheme

provided that this scheme allows for transverse correlations between the two jets. This can be relevant to the Harari-Freund picture since the situation looks quite different from conventional f iteration schemes.

Of course, we have not presented a proof of the validity of the Harari-Freund hypothesis. Our goal was just to point out that no serious argument exists on general grounds against such a scheme. Whether Nature has chosen Pomeron f identity, glueball-Pomeron identity or a mixture of both, can be elucidated only through further theoretical and phenomenological work. A better theoretical understanding of the hadron spectrum - including some mastery of the confinement mechanism - as well as more detailed experimental information on high energy, high multiplicity events - allowing for a deeper phenomenological analysis - would certainly help to reach a more profound comprehension of the nature of diffraction.

ACKNOWLEDGEMENTS

We are grateful to G. Veneziano for discussions and for a critical reading of the manuscript. Discussions with A. Capella, G.F. Chew and J. Dash are also acknowledged.

APPENDIX

In this appendix we want to clarify some points concerning the relation between the glueball decay products and the conventional multiperipheral model which appear to have led to confusion^{11),15)} (see also J. Dash in Ref. 2))^{*)}.

Let us consider the multiperipheral equation for multi-particle production in terms of planar clusters and crossed gaps^{2),11),15)} in the J plane:

$$A = R + RKA \quad (A-1)$$

where A is the total planar plus cylinder amplitude, R is the planar cluster sum rule and K is the kernel made of pairs of crossed Regge exchanges. We take K to be non-singular. The explicit solution of (A-1) reads:

$$\begin{aligned} A &= R + RKR + RKRKR + \dots = \\ &= \sqrt{R} \frac{1}{1 - \sqrt{R} K \sqrt{R}} \sqrt{R} \end{aligned} \quad (A-2)$$

If there is only one Reggeon in the scheme, the R and K are one-dimensional (up to the coupling to external particles), and so is Equation (A-2). In that case, the output leading singularity will be the promoted planar Reggeon. A similar situation will occur¹⁵⁾ if there are daughter trajectories but they couple all coherently with the leading one in such a way that the one-dimensionality of R and K is preserved in the iteration.

In general, however, this does not need to be the case. Therefore, since there is an infinity of daughter trajectories, R and K will be infinite-dimensional matrices and Equation (A-2) will be far from trivial to handle. In order to find the output poles one should diagonalize the operator $\sqrt{R} K \sqrt{R}$ in the space of the planar Reggeons which appear in the cluster sum rules, and then require these eigenvalues to be equal to one.

That P - f identity is no longer unavoidable can be seen from a simple two-dimensional example. Assume we consider a planar sum rule with two Reggeons, f and d where f is the leading trajectory and d some daughter, so that we have:

*) We thank J. Dash for having led us to discuss this question in detail.

$$\mathcal{R} = \begin{bmatrix} \frac{1}{J-\alpha_f} & 0 \\ 0 & \frac{1}{J-\alpha_d} \end{bmatrix} \quad (\text{A-3})$$

and we take K to be:

$$K = \begin{bmatrix} \Delta_f & \epsilon \\ \epsilon & \Delta_d \end{bmatrix} \quad (\text{A-4})$$

where Δ_f, Δ_d and ϵ are real functions of J and t which depend on the details of the dynamics. Then the output poles are the solutions of the equation:

$$(J-\alpha_f - \Delta_f)(J-\alpha_d - \Delta_d) = \epsilon^2 \quad (\text{A-5})$$

which has always real solutions. In the limit $\epsilon \rightarrow 0$, we have poles at $\alpha_f + \Delta_f$ (promoted f) and $\alpha_d + \Delta_d$ (promoted d). Therefore, if $\alpha_d + \Delta_d$ lies higher than $\alpha_f + \Delta_f$, then Pomeron f identity no longer follows since the leading singularity is the promoted d . Taking $\epsilon \neq 0$ will introduce some mixing between the promoted f and the promoted d , thus leading to the sort of scheme we discussed in the appendix of Ref. 3) (Phys. Rev. paper). It can be readily verified from this explicit example that a leading singularity different from the promoted f can be introduced without spoiling short-range order. Furthermore, if we split K in $K_1 + K_2$, with:

$$K_1 = \begin{bmatrix} \Delta_f & \epsilon \\ \epsilon & 0 \end{bmatrix} \quad (\text{A-6})$$

$$K_2 = \begin{bmatrix} 0 & 0 \\ 0 & \Delta_d \end{bmatrix} \quad (\text{A-7})$$

and we first sum over all powers of K_2 , we obviously get an equation where the f dipole is coupled to a singular kernel which has the promoted d pole, just as in Ref. 3) (Phys. Rev. paper). In other words, we can

start from a non-singular kernel and, after rearranging, obtain a singular one for the matrix elements which are coupled to the f dipole. This is possible because we are dealing with matrices instead of numbers and the situation is quite different from naïve one-dimensional schemes.

Of course, this is just a mathematical example to point out that short-range order does not compel glueball-dominated multi-particle states to be governed by trivial iterations of the planar Reggeon sum rule leading to Pomeron- f identity. In practice, we expect the mathematics to be much more complicated and the output "glueball" pole to have little to do with individual promoted Reggeons. If we want to discuss the internal structure of gluonium states, we have to consider DFT Feynman diagrams and not multi-cluster cross sections, so that physically it does not make any sense to say that the glueball is a "promoted daughter". In fact, assuming that a glueball singularity does really exist, it is likely that its properties will be much simpler in the language of Dual Field Theory, soft parton distributions and -- presumably -- jet amplitudes, than in that of multi-particle states described by the iteration of planar cluster sum rules. Things which are simple in a given language can look complicated in another one, and conversely. This is why it is important to realize³⁾ that zero width amplitudes seem to already account for many important features of the real world, such as Reggeization, limited transverse momenta and short-range order, and that these properties can be understood in terms of internal constituents without waiting for the irruption of multi-particle states. It is only with the help of a true theory of hadrons that we can really discuss the relationship between zero width amplitudes and the details of hadron dynamics at N_c (number of colours) = 3, and try to elucidate whether one should expect a leading closed-string exchange to be actually present in diffractive amplitudes. In particular, if one succeeds in solving QCD at $N_c = \infty$, the possibility that zero width amplitudes would already be close to the real world (up to widths and coupling constants) is worth exploring³⁾.

Finally, notice that the transverse correlations discussed in Section 4 are of pure dynamical origin and survive to very crude kinematical approximations. They are not the same as the kinematical correlations which are usually invoked¹⁶⁾ to reconcile correlated-link cluster models with standard multi-Regge amplitudes. Since in practice the average gap is not large, both kinds of correlations will be present.

REFERENCES

- 1) Two-jet models for the Pomeron were first considered by: Huan Lee, Phys. Rev. Lett. 30 (1973) 719; G. Veneziano, Phys. Lett. 43B (1973) 413. For a comprehensive review see G.F. Chew and C. Rosenzweig, Physics Reports 41C (1978) 263, and references therein.
- 2) J.W. Dash, E. Manesis and S.T. Jones, Phys. Rev. D18 (1978) 303.
J. W. Dash, Talk at the XIII Moriond Meeting (1978), ed. J. Tran Thanh Van.
- 3) P. Aurenche and L. González Mestres, Phys. Rev. D18 (1978) 2995 and Zeitschrift für Physik C to appear.
- 4) C.B. Chiu and S. Matsuda, Nuclear Phys. B134 (1978) 463.
- 5) H. Minakata, Tokyo University preprint TMUP-HEL-807 (1978).
- 6) A. Capella, U. Sukhatme, C.I. Tan and Tran Thanh Van, Phys. Lett. 81B (1979) 68.
- 7) H. Harari, Phys. Rev. Lett. 20 (1968) 1395.
P.G.O. Freund, Phys. Rev. Lett. 20 (1968) 235.
- 8) For a comprehensive review see M. Jacob, Dual Theory, North Holland/American Elsevier, Amsterdam (1974).
- 9) G. Veneziano, Talk at the XII Moriond Meeting (1977), ed. J. Tran Thanh Van.
- 10) See, for instance, P. Olesen, Nuclear Phys. B29 (1971) 77.
- 11) C.I. Tan, "Dual Topological Unitarization - Phenomenological Aspect", Talk at the 1978 Marseille Conference on Hadron Physics at High Energies, Brown University Preprint HET-370.
- 12) R.C. Brower, J. Ellis, M.G. Schmidt and J.H. Weis, Nuclear Phys. B128 (1977) 175.
M.B. Einhorn and E. Rabinovici, Nuclear Phys. B128 (1977) 421.
- 13) F. Henyey, Talk at the Moriond Meeting (1973).
- 14) C. Rosenzweig and G.F. Chew, Phys. Lett. 58B (1975) 93.
- 15) J.W. Dash and Alex Martin, Marseille preprint 77/P.964 and revised version (1978), published in the proceedings of the 1978 Marseille Conference on Hadron Physics at High Energies, eds. C. Bourrely, J. Dash and J. Soffer.
- 16) See, for instance, M. Le Bellac, Lectures given in the Academic Training Programme of CERN 1975-76, CERN report 76-14 and references therein.

Figure captions

- Fig. 1 : Two-body collision (incoming particles a and b) producing a two-jet structure (in terms of quark lines).
- Fig. 2 : Dual diagram associated to the shadow of two-jet events.
- Fig. 3 : a) Rapidity - ordered representation of a two-jet event in terms of quarks and gluons (solid lines = quarks, wavy lines = gluons). The vertical direction stands for the rapidity axis.
b) A two-jet event in the two-random-walk model.
- Fig. 4 : Intuitive graphical representation of the p_T transfer between the two jets in the rapidity-ordered constituent picture.
- Fig. 5 : a) The shadow of diagram 3b in terms of the relevant integration variables.
b) The transverse momentum ambiguity for internal lines of diagram 3b, in terms of the arbitrary loop δ of transverse momentum.
- Fig. 6 : Reggeized components of diagram 5a: a) First-jet shadow, b) Second-jet shadow.
- Fig. 7 : Two-random-walk picture modified by local (in rapidity) exchanges between both jets, giving rise to: a) local transfers of p_T (the p_{Tk}''), b) local loop ambiguities for the transverse momenta of internal lines (the δ_k loops).
- Fig. 8 : Local modification of the two-random-walk picture through the exchange of an internal (wavy) line.

FIG.1

FIG.2

FIG.3

FIG.4

(a)

(b)

FIG.5

FIG.6

FIG.7

FIG.8