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INTERACTING MONOPOLES

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ABSTRACT

The Bogomolny equation for interacting monopoles of like charge in the case of vanishing Higgs mass is fulfilled by an asymptotic expansion in powers of the inverse monopole distance. Consequently their interaction energy decreases faster than any power. It is zero if the expansion is summable. The degrees of freedom for the multi-monopole system are determined. The additional degrees of freedom expected in case of a hidden non-zero spin are absent. But there emerges a dual gauge group, whose generator forms together with local translations a multiplet of an outer automorphism group $SU(2)$.

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1. INTRODUCTION

Consider $SU(2)$ monopoles in a classical non-Abelian gauge theory with a massless Higgs field φ in the adjoint representation, such that we can treat it as an additional component of the potential A . The gauge group has been fixed for simplicity, it should not be too difficult to generalize our results.

After quantization, monopoles are supposed to yield the basic field of the dual sector of the theory, for which a perturbation expansion in terms of the inverse coupling constant e^{-1} should exist. The behaviour of the classical monopoles can yield important information for the couplings in that sector¹⁾. Montonen and Olive conjectured that the dual theory looks like the original one, with a new dual gauge group, and monopoles of spin one²⁾. However, this idea only seems to be realizable in a slightly modified way in case of the $O(4)$ supersymmetric model³⁾. In particular, Mottola has argued that monopoles have spin zero, as classical one-monopole solutions only have the positional degrees of freedom⁴⁾. His proof of the latter assertion is not quite complete, furthermore, additional degrees of freedom might show up for multi-monopole systems, as in the case of instantons. Nevertheless, we shall see below that his conclusions are correct. However, the duality between a theory of scalar monopoles and a conventional Yang-Mills theory would be even more interesting than the duality between two Yang-Mills theories, as it would yield a dynamical, non-perturbative way of creating gauge vector bosons out of scalars. In Section 3 we shall see that even a connection with gravity seems to exist.

2. AN ASYMPTOTIC SERIES FULFILLING THE BOGOMOLNY EQUATION

The interaction energy of monopoles of positive magnetic charge

$$E_{int} = \frac{1}{4e^2} \int (F_{ij} + \epsilon_{ijk} \partial_k \varphi)^2 dV \quad (1)$$

can be estimated by finding approximate solutions to the Bogomolny equation

$$F_{ij} + \epsilon_{ijk} \partial_k \varphi = 0. \quad (2)$$

In Ref. 5) such an approximate solution has been constructed out of $N+1$ potentials $A^{(r)}$, $r = 0, \dots, N$, where N is the monopole number. $A^{(0)}$ is defined everywhere apart from the monopole positions $\vec{r}_q$, $q = 1, \dots, N$, and is used within

$$U_0 = \{ \vec{r} \mid |\vec{r} - \vec{r}_q| > s^\epsilon/3 \text{ for } q=1, \dots, N \}. \quad (3)$$

Here s is the minimal monopole distance, and ϵ is a fixed positive number less or equal to one. In our notation, q will always vary between 1 and N , and r between 0 and N .

The $A^{(q)}$ are defined over all space and are used within the regions

$$U_q = \{ \vec{r} \mid |\vec{r} - \vec{r}_q| < s^\epsilon/3 + 1 \}. \quad (4)$$

All the $A^{(r)}$ satisfy Eq. (2) exactly. $A^{(0)}$ is Abelian, as the field strength is everywhere proportional to $\varphi^{(0)}$. This is achieved by putting

$$D_i^{(0)} \hat{\varphi}^{(0)} = 0. \quad (5)$$

We use the notation

$$\hat{\varphi} = \varphi/|\varphi| \quad (6)$$

for vectors in isospace. Within $U_0 \cap U_q$ the difference $A^{(q)} - A^{(0)}$ is of order s^{-2} . After joining the pieces smoothly, one obtains an approximate solution of Eq. (2) over all space, which yields

$$E_{int} = o(s^{-4+2\epsilon}). \quad (7)$$

In this section we shall improve Eq. (7) to

$$E_{int} = o(s^{-n}) \text{ for all } n, \quad (8)$$

by a better choice of the $A^{(r)}$. We shall write them as asymptotic expansions in powers of s^{-1} , which fulfil the Bogomolny equation (2) to all orders. Moreover, $A^{(q)} - A^{(0)}$ will vanish in $U_q \cap U_0$, defined by Eqs. (3)-(4) with $\epsilon = 1$. $A^{(0)}$ will be Abelian as before. In any given order

the pieces may be joined smoothly, consequently the interaction energy decreases faster than any inverse power of s .

At first let us consider the $\hat{\phi}^{(r)}$. We put

$$\hat{\phi}^{(0)}(\vec{r}) = \hat{\phi}^{(q)}(\vec{r}) \text{ for } \vec{r} \in \mathcal{U}_q - \{\vec{r}_q\}. \quad (9)$$

For the $\hat{\phi}^{(q)}$ we use the usual radial hedgehog field

$$\hat{\phi}_a^{(q)}(\vec{r}) = \frac{(\vec{r} - \vec{r}_q)_a}{|\vec{r} - \vec{r}_q|}, \quad (10)$$

for $\hat{\phi}^{(0)}$ a smooth interpolation. Such an interpolation can be constructed by deflecting the radial lines emerging from the $\vec{r}_q$, on which $\hat{\phi}$ is kept constant, into N non-intersecting cones going off to infinity, such that the cone boundaries correspond to the line emerging from the $\vec{r}_q$ into the positive z direction. Between the cones, $\hat{\phi}$ can be kept equal to $(0,0,1)$.

Now let us complete the description of $A^{(0)}$. Equations (2) and (5) show that the Laplacian of $|\phi^{(0)}|$ is zero. Thus we write

$$|\hat{\phi}^{(0)}| = c - \sum_q \left(|\vec{r} - \vec{r}_q|^{-1} + \sum_{K=2}^{\infty} f_{qK} |\vec{r} - \vec{r}_q|^{-K-1} Y_K \left(\frac{\vec{r} - \vec{r}_q}{|\vec{r} - \vec{r}_q|} \right) \right). \quad (11)$$

For simplicity we suppress the summation over K_3 and use real spherical harmonics Y_K with real coefficients. For a single unperturbed monopole at $\vec{r}_q$ the f_{qK} are zero, and this will be the starting point of our iterative solution of the Bogomolny equation to all orders in s^{-1} .

For the space components of $A^{(0)}$, Eq. (5) yields

$$A_i^{(0)} = -\hat{\phi}^{(0)} \times \partial_i \hat{\phi}^{(0)} + \Lambda_i \hat{\phi}^{(0)}. \quad (12)$$

From Eq. (2) we obtain

$$\partial_i (\Lambda_j - \bar{\Lambda}_j) \varepsilon_{ijk} = -\partial_k \sum_q \sum_{K=2}^{\infty} f_{qK} |\vec{r} - \vec{r}_q|^{-K-1} Y_K \left(\frac{\vec{r} - \vec{r}_q}{|\vec{r} - \vec{r}_q|} \right), \quad (13)$$

where

$$\partial_i \bar{\Lambda}_j \varepsilon_{ijk} = \hat{\varphi}^{(0)} (\partial_i \hat{\varphi}^{(0)} \times \partial_j \hat{\varphi}^{(0)}) \varepsilon_{ijk} / 2 - \partial_k \sum_q |\vec{r} - \vec{r}_q|^{-1}. \quad (14)$$

Within the region where $A^{(0)}$ is defined, Eq. (13) is easily solvable order by order, with a $\Lambda - \bar{\Lambda}$ vanishing at infinity. The right-hand side of Eq. (14) also has vanishing divergence, and according to Eq. (10) its singularities at the monopole positions cancel. Thus we can find a globally defined solution $\bar{\Lambda}$. For a $\hat{\varphi}^{(0)}$ which spreads out to infinity in a sufficiently smooth manner, $\bar{\Lambda}$ may also be assumed to vanish at infinity.

Different choices of Λ correspond to gauge transformations given by local rotations of isospace around $\varphi^{(0)}$. We eliminate this freedom to a large extent by imposing the Coulomb gauge

$$\partial_i \bar{\Lambda}_i = \partial_i \Lambda_i = 0. \quad (15)$$

This fixes $\bar{\Lambda}$ completely, but not Λ , as one may add spherical harmonics diverging at $\vec{r}_q$. Within the U_q , $\bar{\Lambda}$ is of order s^{-1} . For non-Abelian potentials the appropriate generalization of the Coulomb gauge is the background gauge, which will be used later.

From now on we use a quaternionic notation for the potential, writing

$$A = -i\sigma_i A_i + \varphi \quad (16)$$

with the usual sigma matrices. From Eqs. (11) and (13) we obtain finally

$$\begin{aligned} A^{(0)} = & i\sigma_i (\hat{\varphi}^{(0)} \times \partial_i \hat{\varphi}^{(0)}) + \hat{\varphi}^{(0)} \left(c - \sum_q |\vec{r} - \vec{r}_q|^{-1} - i\sigma_i \bar{\Lambda}_i \right) \\ & + \hat{\varphi}^{(0)} \sum_q \sum_{K=2}^{\infty} \alpha_{qK} |\vec{r} - \vec{r}_q|^{-K-1} Y_K \left(\frac{\vec{r} - \vec{r}_q}{|\vec{r} - \vec{r}_q|} \right), \end{aligned} \quad (17)$$

with 2×2 matrices α_{qK} . They will be determined by the condition that within $U_q \cap U_o$, $A^{(0)}$ and $A^{(q)}$ coincide to all orders in s^{-1} .

Within U_q we may use Laurent expansions of the right-hand side of Eq. (17), yielding

$$A^{(0)} = i\epsilon_i (\hat{\varphi}^{(0)} \times \partial_i \hat{\varphi}^{(0)}) + \hat{\varphi}^{(0)} (c - |\vec{r} - \vec{r}_q|^{-1}) + \hat{\varphi}^{(0)} \left(\sum_{K=2}^{\infty} a_{qK} |\vec{r} - \vec{r}_q|^{-K-1} Y_K \left(\frac{\vec{r} - \vec{r}_q}{|\vec{r} - \vec{r}_q|} \right) + \pi^{(q)}(\vec{r}) \right), \quad (18)$$

where $\Pi^{(q)}$ contains the positive powers. According to Eqs. (11)-(16) the Laplacian of $\Pi^{(q)}$ vanishes, such that

$$\Delta \pi^{(q)} = \sum_{K=0}^{\infty} b_{qK} |\vec{r} - \vec{r}_q|^K Y_K \left(\frac{\vec{r} - \vec{r}_q}{|\vec{r} - \vec{r}_q|} \right). \quad (19)$$

The b_{qK} also are 2×2 matrices. They are of order s^{-K-1} , with leading contributions from $\bar{A}$ and $|\vec{r} - \vec{r}_m|^{-1}$, $m \neq q$. The contributions from the term multiplying a_{mK} , are suppressed by an additional factor $s^{-K'-1}$. Considering all monopoles together, we note that the $\Pi^{(q)}$ are determined to order s^{-n-3} , once the a_{qK} are known to order s^{-n} . To each order only a finite number of terms contribute.

Now let us describe the $A^{(q)}$. Firstly, they have to satisfy the Bogomolny equation (2). In addition, let $\bar{A}^{(q)}$ be the unperturbed potential of a single monopole at $\vec{r}_q$ and put

$$\Delta A^{(q)} = A^{(q)} - \bar{A}^{(q)}. \quad (20)$$

Then we use Eq. (19) up to infinity and require

$$\Delta A^{(q)} - \pi^{(q)} = O(|\vec{r} - \vec{r}_q|^{-3}). \quad (21)$$

On the right-hand side we use the exponent -3 instead of -2 in order to keep the monopole position at $\vec{r}_q$. More generally, the terms with exponent -2 yield the degrees of freedom of the monopole. They will be discussed in Section 3.

After imposing the background gauge, we shall show that at any given order s^{-n} the boundary condition (21) determines a unique solution of the Bogomolny equation. Within $U_q \cap U_0$ it will be of type (18), as the non-Abelian terms go down faster than $\exp(-c|\vec{r} - \vec{r}_q|)$. After having shown the existence of this solution, our construction proving Eq. (8) will be

finished. To any given order, the solution yields the a_{qK} of $A^{(0)}$ to the same order, as $A^{(0)}$ and $A^{(q)}$ have to be made equal in $U_q \cap U_0$. Thus, while the a_{qK} , taken collectively and known up to order s^{-n} , determine the b_{qK} to order s^{-n-3} , those in turn determine the a_{qK} to order s^{-n-3} . The iteration runs and yields $N + 1$ asymptotic series $A^{(r)}$ with exactly the right behaviour to prove Eq. (8).

When we now deal with the remaining task to solve the Bogomolny equation for a single monopole with given boundary condition, we may suppress the index q and take a monopole located at $\vec{r} = 0$. Put

$$D = -i\sigma_i \bar{\partial}_i - \bar{\partial}_\varphi \quad (22)$$

where the covariant derivatives act on isovectors χ like

$$\bar{\partial}_i \chi = \partial_i \chi + \bar{A}_i \times \chi, \quad (23)$$

$$\bar{\partial}_\varphi \chi = \bar{\varphi} \times \chi. \quad (24)$$

Then the Bogomolny equation together with the background gauge condition

$$\bar{\partial}_i \Delta A_i + \bar{\varphi} \times \Delta \varphi = 0 \quad (25)$$

takes the form

$$D \Delta A = \frac{i}{2} \Delta A^* \times \Delta A. \quad (26)$$

With

$$\Delta A = \sum_{n=1}^{\infty} s^{-n} \Delta A^{(n)} \quad (27)$$

we have

$$D \Delta A^{(n)} = \frac{1}{2} \sum_{m=1}^n \Delta A^{(m)+} \times \Delta A^{(n-m)} \quad (28)$$

which will serve together with the boundary condition (21) to determine $\Delta A^{(n)}$, once the lower orders in s^{-1} are known. The Dirac operator D anticommutes with the parity operation and commutes with the two generalized angular momenta

$$K_i = L_i + T_i + \frac{1}{2} (L(\sigma_i) - R(\sigma_i)) \quad (29)$$

and

$$J_i = K_i + \frac{1}{2} R(\sigma_i). \quad (30)$$

Here

$$L_i = \varepsilon_{ijk} r_j p_k = -i \varepsilon_{ijk} r_j \partial_k \quad (31)$$

is the ordinary orbital angular momentum, T_i is the generator of isospin rotations, $R(\sigma_i)$ and $L(\sigma_i)$ denote the multiplication by σ_i from the right and left. We decompose ΔA according to the eigenvalues of K^2 , K_3 , J^2 , and parity. K takes integer values, J half integer ones. The operator D has been investigated by Jackiw and Rebbi⁶⁾, and we shall use a notation which is essentially the same as theirs, even if it is rather clumsy for our purpose. We write

$$\Delta A = \Delta A_+^1 + \Delta A_-^1 + \Delta A_+^2 + \Delta A_-^2, \quad (32)$$

where

$$J = K + \frac{1}{2} \text{ for } \Delta A_{\pm}^1, \quad (33)$$

$$J = K - \frac{1}{2} \text{ for } \Delta A_{\pm}^2. \quad (34)$$

The lower index $\pm$ denotes the product of parity and $(-)^{J+1/2}$. Furthermore

$$\Delta A_+^{1a} = \sum_{K=0}^{\infty} \left(\hat{r}^a F_{1+,J}^1(r) - i(\hat{r} \times L)^a F_{2+,J}^1(r) + L^a(\sigma \hat{r}) F_{3+,J}^1(r) \right) X_K^1(r), \quad (35)$$

where

$$X_K^1(\hat{r}) = \frac{r^{K+2}}{((K+1)(2K+1))^{1/2}} (-i\sigma \hat{r})(\sigma \rho) r^{-K-1} Y_K(\hat{r}). \quad (36)$$

Again we use real spherical harmonics Y_K and suppress the sum over K_3 .

ΔA_- is obtained from ΔA_+ by replacing F_+ by $(-i\sigma r)F_-$. It is preferable to use $-iF_-$ instead of the F_- of Jackiw and Rebbi to keep all F real. ΔA^2 is obtained from ΔA^1 by replacing F^1 by F^2 and X^1 by

$$X_K^2(\hat{r}) = - \frac{r^{-K+1}}{(K(2K+1))^{1/2}} (\sigma \rho) r^K Y_K(\hat{r}). \quad (37)$$

For a Y_K annihilated by K_3 we have

$$iX_K^1 \sigma_3 = X_{K+1}^2. \quad (38)$$

How ΔA looks like in terms of the F can be seen from Eqs. (A27a-f) in Ref. 6). The F of this reference still contain an upper index $\pm$. In our case the plus sign applies.

To each order in s^{-1} only a finite number of partial waves contributes to ΔA , such that Eq. (28) becomes a finite system of ordinary linear differential equations. Because of Eq. (38) the left-hand side looks identical for F^1 and F^2 , such that we can disregard the upper index for part of the discussion. The coefficient functions are well behaved for $0 < r < \infty$, such that the solubility of Eq. (28) only depends on the behaviour at zero and infinity. This also is listed in Ref. 6).

For $F_{+,J}$, $J > 1/2$, there are two linearly independent solutions regular at the origin. For them

$$F_{1+,J} \sim F_{2+,J} \sim F_{3+,J} \sim r^{J+1/2} \quad (39)$$

and

$$F_{1+,J} \sim F_{2+,J} \sim F_{3+,J} \sim r^{J-3/2}, \quad (40)$$

respectively. At infinity, there are two independent divergent solutions. For the leading one,

$$F_{2+,J} \sim F_{3+,J} \sim r^{-2} e^{cr}, \quad (41)$$

whereas $F_{1+,J}$ behaves like a power.

If this singularity is cancelled, the other one yields

$$F_{1+,J} = c_+ r^{J-1/2} + o(e^{-cr}), \quad (42)$$

$$F_{2+,J} \sim F_{3+,J} = o(e^{-cr}). \quad (43)$$

For $F_{-,J}$, $J > 1/2$, there is only one linearly independent solution regular at the origin, for which

$$F_{1-,J} \sim F_{2-,J} \sim F_{3-,J} \sim r^{J-1/2}. \quad (44)$$

At infinity, there is one linearly independent divergent solution, which is again of type (41). If it is cancelled, one obtains the regular behaviour

$$F_{1-,J} = c_- r^{-J-3/2} + o(e^{-cr}), \quad (45)$$

$$F_{2-,J} \sim F_{3-,J} = o(e^{-cr}). \quad (46)$$

For $J = 1/2$, we can put F_{3-} and F_{2+} to zero. For both $F_{+1/2}$ and $F_{-1/2}$ there is one solution regular at the origin, for which Eqs. (39) and (44) apply respectively. The behaviour at infinity is given by Eqs. (42)-(43) and (45)-(46).

Equations (44)-(46) for $J = 1/2$ characterize the unique normalizable solution of

$$DX = 0 \quad (47)$$

found by Jackiw and Rebbi. For us it yields four solutions, one for ΔA_-^1 , $K = 0$, and three for ΔA_-^2 , $K = 0$. The latter ones are the translation modes. In all the other cases the number of linearly independent solutions is just equal to the number of linearly independent solutions at infinity. Thus, in general, one expects that the homogeneous equation (47) has no further normalizable solutions, whereas the inhomogeneous equation

$$DX = Y \quad (48)$$

can be solved for every smooth Y which is a finite superposition of eigenstates of K^2 and sufficiently well behaved at zero and infinity. To see this, note that Eq. (48) can be integrated between zero and infinity according to the usual theorems on ordinary differential equations. The divergences at zero and infinity are of a type covered by the solutions of the homogeneous equation. Thus they can be cancelled by adding a suitable solution of Eq. (47).

As indicated by Jackiw and Rebbi, such arguments are true "in general". They only fail if by some kind of accident an additional solution of Eq. (47), which is regular at the origin, does not develop a divergence at infinity and cannot be used for cancellation. Then Eq. (48) would not be solvable for general Y , whereas Eq. (47) would have a further normalizable solution. This possibility has to be excluded to complete the proof of the uniqueness of the one-monopole solution used by Mottola to argue for zero monopole spin⁴⁾.

To do this, we shall prove that with one exception all solutions of Eq. (47) can be obtained from infinitesimal gauge transformations

$$X = D^+ \chi, \quad (49)$$

where X is a Lorentz scalar. Of course, all solutions of

$$DD^+ \chi = 0 \quad (50)$$

yield an X solving Eq. (47). For each $K > 0$ we have found six solutions of Eq. (47), two for both F_+^1 and F_+^2 , one for both F_-^1 and F_-^2 . Let us look at which ones are of type (49).

Near the origin, DD^+ can be approximated by p^2 , as the other terms are more regular. Thus for each $K > 0$ there are three linearly independent solutions of Eq. (50) regular at the origin, corresponding to the three values of the isospin. They behave like

$$\chi_a \sim r^{2K+3} p_a r^{-K-1} Y_K(\bar{r}), \quad (51)$$

$$\chi_a \sim p_a r^K Y_K(\bar{r}), \quad (52)$$

and

$$\chi_a \sim L_a r^K Y_K(\bar{r}). \quad (53)$$

The $D^+\chi$ are not eigenstates of L^2 . Projecting those out, one just obtains the six linearly independent solutions mentioned above.

For $K = 0$, Eqs. (52)-(53) yield nothing, and Eq. (51) an eigenstate of L^2 . On the other hand, we have found two solutions of Eq. (47), one for F_+^1 and one for F_-^1 . The first one is the dilation mode

$$X = \partial_c \bar{A} \quad (54)$$

where ∂_c is the derivative with respect to the scale parameter c . It cannot be obtained from a gauge transformation, but anyhow, it is not normalizable. The one for F_-^1 is

$$X = D^+ \bar{\varphi}, \quad (55)$$

which is just the infinitesimal gauge transformation characterized by Eq. (51).

Now, Eq. (50) obviously has no normalizable solution, as the Bogomolny equation yields

$$DD^+ = -\bar{D}_i^2 - \bar{D}_j^2, \quad (56)$$

which is positive definite. This excludes the existence of normalizable solutions of Eq. (47), apart from those which arise from a solution χ of Eq. (50) which is not normalizable itself, but yields a normalizable $D^+\chi$. It is easy to see that this is only possible if asymptotically χ is parallel to $\bar{\varphi}$ and has a constant but non-zero length. There is only one linearly independent solution of this kind, namely $\bar{\varphi}$ itself. Thus we have recovered Eq. (55). The translation modes can be obtained from this χ by multiplying with sigma matrices from the right⁴⁾.

Now we have definitely established that there are no further normalizable solutions of Eq. (47). At the same time we have shown that Eq. (28) has a normalizable solution, if the right-hand side is well behaved. To fulfil the boundary condition (21) to that order, we only have to add a solution of the homogeneous equation of type (42)-(43), and a normalizable solution for $K = 1$. This solution is unique, as we have claimed. In $U_q \cap U_0$ it is indeed of type (18).

The only remaining task is to check the behaviour of the right-hand side of Eq. (28). To order s^{-1} it is zero, thus the iteration can be started. At any higher order it falls off exponentially to infinity, as can be seen from Eqs. (42)-(43) and (45)-(46). We only have to note that the F_1 terms yield contributions proportional to $\bar{\varphi}$, such that their cross product vanishes.

At zero one finds that the contribution of orbital angular momentum L to ΔA behave like r^L order by order. Thus our proof of Eq. (8) is completed.

3. THE DEGREES OF FREEDOM OF A MULTI-MONOPOLE SYSTEM

Suppose now that the Bogomolny equation for multi-monopole systems can be solved exactly, which would, for example, be the case if the series (27) is Borel summable. Then the number of degrees of freedom should be equal to the number of linearly independent normalizable solutions of Eq. (47), where D is now the covariant derivative given by the exact N monopole potential. This number is given by the index of D , as Eq. (54) shows that D^+ has no normalizable zero mode. Unfortunately, D is not a Fredholm operator, as long wavelength excitations of the gauge field components parallel to φ and of the Higgs field can have arbitrarily small energy. This can be remedied by introducing a small mass term. For the modified D applied to a two-dimensional spinor it has been shown that the index is equal to $2N$ ^{7),8)}. In our case, where it is applied to a 2×2 matrix, it is equal to $4N$. The introduction of a mass term

might destroy an accidental solution of Eq. (47) for which the detailed behaviour of the potential at finite distances is responsible. However, this is excluded for monopoles which are sufficiently far apart, as such a solution would have shown up in the asymptotic expansion of Section 2. Thus monopoles far apart have $4N$ degrees of freedom. One may safely assume that this is also true for smaller distances, though I do not yet have a formal proof.

The monopole positions alone yield $3N$ degrees of freedom, but the remaining N degrees are at first sight surprising. One of them is again given by Eq. (55), which describes the unique infinitesimal gauge transformation not excluded by the background gauge.

But let us look at this problem from a different point of view. For monopoles sufficiently far apart it should be possible to use the degrees of freedom of each monopole separately. In particular, the monopoles can be moved separately. The boundary conditions (21) for the potential of the other monopoles will be changed slightly, but we can adjust the solutions by a change of the same order, as done in Section 2. This yields the $3N$ positional degrees of freedom. However, the same procedure works if we use the gauge degree of freedom given by Eq. (55) only locally, for example, in U_1 . In $U_0 \cap U_q$, $q \neq 1$, this yields

$$X \approx D(\hat{\varphi}(c - |\vec{r} - \vec{r}_1|^{-1})) \approx -\hat{\varphi}(\sigma\rho) |\vec{r} - \vec{r}_1|^{-1}, \quad (57)$$

as $D\hat{\varphi}$ falls off like $\exp(-c|\vec{r} - \vec{r}_q|)$. Within U_q , X can be continued in the form of a gauge transformation which is of order s^{-2} and of type ΔA_+^2 , $K = 1$. Thus X can be written piecewise as a gauge transformation, but not globally. It also can be obtained from the X corresponding to an infinitesimal shift of $\vec{r}_1$ into the i -th direction by a multiplication with σ_i from the right. The globally defined gauge transformation of Eq. (55) can be obtained as a sum of the N solutions just discussed. As these N additional degrees of freedom do not change measurable quantities, one might argue that they are artifacts of the background gauge. However, I think that the covariance of Eq. (47) with respect to multiplication by sigma matrices from the right is fundamental.

Indeed, consider what happens after quantization. A solution of the Bogomolny equation with N monopoles at position $\vec{r}_q$ should roughly correspond to a wave function

$$\psi(\vec{r}) \sim \sum_g \delta(\vec{r} - \vec{r}_g)^{1/2}. \quad (58)$$

An Abelian gauge group coupled to the magnetic charge yields one arbitrary phase for each term on the right-hand side of Eq. (58), which just corresponds to our additional degrees of freedom. If all the phases are equal one has a global gauge transformation which corresponds to Eq. (55). We infer that the dual sector of our Yang-Mills-Higgs theory contains a dual gauge group with an unbroken $U(1)$ part.

For a more general wave function ψ , the shifts of the monopole positions generalize to arbitrary diffeomorphisms of R^3 , under which ψ transforms as the square root of a density. This is a very strong hint to take gravity into consideration.

Even more interestingly, we have seen that local translations and local gauge transformations are changed into each other if one multiplies the solutions of Eq. (47) by sigma matrices from the right. This yields a $SU(2)$ group of outer automorphisms under which local translations and local gauge transformations form a real, four component spinor representation. Here we may even observe a new way of unifying gravity and gauge fields.

On the other hand, the question of the monopole spin seems to be decided in favour of scalar monopoles.

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