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GAUGE HIERARCHIES AND THE UNIFICATION MASS

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A B S T R A C T

It is shown that the existence of gauge hierarchies in unified theories of strong, weak and electromagnetic interactions permits unification of the three forces at energies as high as the Planck mass. As examples, we discuss the $O(10)$, E_6 and $[SU(4)]^4$ models. Implications for proton decay and for present and future weak intermediate boson phenomenology are considered.

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It has often been conjectured that the Planck mass M_p provides the sole intrinsic mass scale for unified gauge theories (UGTS) of strong, weak electromagnetic interactions. Starting from this scale, one could envisage a hierarchy of energy scales which would include the scale of QCD and the masses of the weak intermediate bosons. The precise mechanism which can cause such a hierarchy is not yet understood. Recently, some progress has been made by S. Weinberg ¹⁾. He has argued that a gauge hierarchy is a plausible phenomenon, at least when Higgs bosons are also present in the theory. The work described in this paper depends on the existence of gauge hierarchies but not on the details of the mechanism that cause them.

An important consequence of the unification of the three forces in UGTS is the existence of a class of gauge bosons (we call them X bosons) which carry both colour and flavour charges. The mass of the X bosons, which characterizes the unification scale of the three forces, has been estimated to lie somewhere between 5×10^{14} GeV and 10^{16} GeV ^{2),3)}. A significantly lower mass value would cause the proton to decay much too quickly, whereas the upper bound comes from a comparison of the predicted and the experimentally measured values of $\sin^2 \theta_W$.

In making the above prediction on the X mass, it is explicitly assumed that there are just two mass scales in UGTS, the present energy scale and the superheavy X mass. This assumption is indeed necessary for the simplest UGT model, the $SU(5)$ model of Georgi and Glashow ⁴⁾. However, in view of our remarks on the possible existence of gauge hierarchies, the assumption is unnecessarily restrictive for other UGTS, examples of which have been discussed in the literature. These include models based on $O(10)$ ⁵⁾, E_6 ⁶⁾ and semi-simple groups such as $[SU(4)]^4$ ⁷⁾. The flavour part of these models contains the Weinberg-Salam model ⁸⁾ as a subgroup. Thus, the maximal flavour subgroups of $O(10)$ and E_6 are $SU(2)_L \times SU(2)_R \times U(1)$ and $SU(3)_L \times SU(3)_R$ respectively. It is possible, for instance, that $O(10)$ does not break directly down to $SU(2)_L \times U(1) \times SU(3)_C$, but instead breaks down first to $SU(2)_L \times SU(2)_R \times U(1) \times SU(3)_C$, and in a second step down to $SU(2)_L \times U(1) \times SU(3)_C$.

In this note we allow the possibility that more than two mass scales are present in the UGTS. To keep the discussion simple, we restrict ourselves to three mass scales and will discuss the interplay between them. An interesting consequence is the possibility of having X bosons with masses of the order of $M_p (= 1.22 \times 10^{19} \text{ GeV})$. The proton lifetime would then be predicted to be orders of magnitude larger than the present experimental lower bound of 2×10^{29} years. Some consequences for present and future weak interaction phenomenology are also presented.

For definiteness, let us consider the flavour group $S = SU(2)_L \times SU(2)_R \times U(1)$, embedded in some UGT such as $O(10)$, E_6 or $[SU(4)]^4$. The group generators are $T_{iL,R}$ ($i=1,2,3$) and Y . The electric charge Q is given by

$$Q = T_{3L} + T_{3R} + \frac{1}{2} Y \quad (1)$$

One possible assignment of quarks and leptons in S is

$$\text{Quarks : } \begin{pmatrix} u \\ d' \end{pmatrix}_{L,R} \quad \begin{pmatrix} c \\ s' \end{pmatrix}_{L,R} \quad \begin{pmatrix} t \\ b' \end{pmatrix}_{L,R} \dots \quad \begin{matrix} T_{L,R} = 1/2 \\ Y = 1/3 \end{matrix} \quad (2a)$$

$$\text{Leptons : } \begin{pmatrix} \nu_e \\ e^- \end{pmatrix}_{L,R} \quad \begin{pmatrix} \nu_\mu \\ \mu^- \end{pmatrix}_{L,R} \quad \begin{pmatrix} \nu_\tau \\ \tau^- \end{pmatrix}_{L,R} \dots \quad \begin{matrix} T_{L,R} = 1/2 \\ Y = -1 \end{matrix} \quad (2b)$$

where the primes allow for quark mixing and CP violating phases.

The charged gauge bosons associated with $SU(2)_{L,R}$ are denoted by $W_{L,R}$ and mediate $V-A$ ($V+A$) interactions. It follows from present experiments that W_R are, at least, three times more massive than W_L . Clearly, the mass of W_R (m_R) provides a possible new scale in the theory. The other two scales are M_L and M_X which denote, respectively, the masses of the charged W_L bosons and the X bosons. We shall study the renormalization effects due to the three different mass scales M_L , M_R and M_X on the weak coupling constants measured at present energies.

In left-right symmetric gauge models, such as those based on S , one introduces a weak mixing angle θ_V defined by ⁹⁾

$$e(M_L) = g_L(M_L) \cos \lambda(M_L) \sin \theta_V(M_L) = g_R(M_L) \sin \lambda(M_L) \sin \theta_V(M_L) \quad (3)$$

with
$$\tan \lambda(M_L) = g_L(M_L) / g_R(M_L) .$$

*) The precise assignment of the fermions is not essential for our discussion.

Here $g_L(M_L)$, $g_R(M_L)$, $e(M_L)$ are the effective $SU(2)_L$, $SU(2)_R$ and electromagnetic coupling constants at present energy scale which is given by the mass m_L of the W_L bosons $[\alpha(M_L) = e_L^2(M_L)/4\pi \simeq 1/128.5]$ ³⁾. Left-right symmetry implies $g_L(\mu) = g_R(\mu)$ for $\mu \gtrsim M_R$. At present energies, however, $g_L(\mu)$ and $g_R(\mu)$ differ from each other by finite radiative corrections, since $M_L \neq M_R$.

If the effective flavour group at M_L is $SU(2)_L \times U(1)$, only g_L and e play a rôle in present experiments. All effects explicitly involving g_R are suppressed by, at least, a factor M_L^2/M_R^2 . Furthermore, all processes involving left-handed neutrinos are completely independent of g_R in the local four-fermion coupling approximation. This follows from a theorem of Georgi and Weinberg, further generalized by Gatto and Strocchi ¹⁰⁾. The relevant quantity for comparison with present experiments is therefore ^{*)}

$$\sin^2 \theta_w(M_L) = e^2(M_L) / g_L^2(M_L) \quad (4)$$

With S embedded in a UGT, the mixing angle θ_v at the unification scale M_x is given by

$$\sin^2 \theta_v(M_x) = \frac{\sum_L T_{3L}^{i^2} + \sum_i T_{3R}^{i^2}}{\sum_i Q_i^2} \quad (5)$$

where the sum is over the members of any irreducible representation belonging to UGT. For the $O(10)$, E_6 and $[SU(4)]^4$ groups, one obtains

$$\sin^2 \theta_v(M_x) = 3/4 \quad (6)$$

To determine the value of $\sin^2 \theta_w$ (or equivalently of $\sin^2 \theta_v$, since the two are related) at M_L , we employ the renormalization group (RG) method. The gauge coupling constants g_i satisfy the RG equation

*) The zeroth order relation $\sin^2 \theta_w = \frac{1}{2} \sin^2 \theta_v$ gets modified due to finite radiative corrections.

$$\mu \frac{\partial}{\partial \mu} g_i(\mu) = \beta_i^g \simeq b_i g_i^3 = (b_i^g + b_i^f + b_i^h) g_i^3 \quad (7)$$

where b^g , b^f and b^h denote the contributions from gauge bosons, fermions and Higgs scalars, respectively. We shall need the following equations :

$$e^{-2}(M_R) \simeq \frac{2}{\sin^2 \theta_v(M_x)} g_g^{-2} + 2B \ln(M_x/M_R) \quad (8)$$

$$e^{-2}(M_L) \simeq e^{-2}(M_R) + 2B' \ln(M_R/M_L) \quad (9)$$

$$g_L^{-2}(M_L) \simeq g_g^{-2} + 2B'' \ln(M_x/M_L) \quad (10)$$

where g_g is the single gauge coupling constant at $\mu \simeq M_x$,

$$B = -\frac{44}{48\pi^2} + \frac{1}{18\pi^2} (N_L^f + N_R^f) + b'^h \quad (11a)$$

$$B' = -\frac{22}{48\pi^2} + \frac{1}{18\pi^2} (N_L^f + N_R^f) + b''^h \quad (11b)$$

$$B'' = -\frac{22}{48\pi^2} + \frac{2}{48\pi^2} N_L^f + b_L^h \quad (11c)$$

and $N_{L,R}^f$ denote the number of sequential fermion flavours ^{*)}. Combining (8), (9) and (10) and substituting in (4) gives

$$\begin{aligned} \sin^2 \theta_w(M_L) \simeq & \frac{1}{2} \sin^2 \theta_v(M_x) - e^2(M_L) \left\{ \ln\left(\frac{M_x}{M_L}\right) \left(B \sin^2 \theta_v(M_x) - 2B'' \right) \right. \\ & \left. + \ln\left(\frac{M_R}{M_L}\right) \left(\sin^2 \theta_v(M_x) (B' - B) \right) \right\} \end{aligned} \quad (12)$$

^{*)} We assume, for simplicity, that all fermions have masses $\leq M_L$. In this case their contribution to the final formula (13) vanishes.

Putting $\sin^2 \theta_w(M_x) = 3/4$ and assuming N^h $SU(2)_L$ Higgs doublets (mass $\sim M_L$) and N^h $SU(2)_R$ Higgs doublets (mass $\sim M_R$) leads to the final result ^{*} :

$$\sin^2 \theta_w(M_L) \simeq \frac{3}{8} \left[1 - \frac{\alpha(M_L)}{2\pi} \left\{ \left(\frac{66}{9} - \frac{3}{9} N^h \right) \ln \left(\frac{M_R}{M_L} \right) + \left(\frac{44}{9} + \frac{3}{9} N^h \right) \ln \left(\frac{M_x}{M_L} \right) \right\} \right] \quad (13)$$

Let us examine some features of (13) and the consequences that follow from them.

- 1) The next order logarithmic corrections to (13) are

$$O(\alpha^2 \ln(M_x/M_L)), \quad O(\alpha^2 \ln(M_R/M_L)) \text{ and } O(\alpha).$$

They should not change the result by more than about 2%.

- 2) With $M_R = M_x$, we obtain

$$\sin^2 \theta_w(M_L) \simeq \frac{3}{8} \left[1 - \frac{\alpha(M_L)}{2\pi} \left\{ \frac{110}{9} \ln \left(\frac{M_x}{M_L} \right) - \frac{1}{9} N^h \ln \left(\frac{M_x}{M_L} \right) \right\} \right] \quad (14)$$

This result was recently obtained by Marciano ³⁾ for the case when only two mass scales (M_L and M_x) are present.

- 3) The most interesting consequence of (13) is the fact that, consistent with the presently quoted values of $\sin^2 \theta_w$, the mass of the X bosons can be as large as the Planck mass ^{**}). Let us demonstrate this by taking $M_x = 1.2 \times 10^{19}$ GeV, $M_L = 80$ GeV and plotting $\sin^2 \theta_w(M_L)$ versus M_R/M_L . The results are displayed in the Table [we put $N^h = 1$ in (13)]. For fixed $\sin^2 \theta_w(M_L)$, letting M_x decrease below the Planck mass makes M_R/M_L go up correspondingly. However, it is tempting to conjecture that the unification of strong, weak and electromagnetic interactions takes place at or near the Planck mass.

^{*}) For simplicity we have assumed that the Higgs bosons have only three possible mass scales which are M_L , M_R and M_x .

^{**}) This may be understood qualitatively by noting that the introduction of the intermediate scale M_R makes the electromagnetic coupling constant decrease faster between M_R and M_x compared to the decrease between M_L and M_R [see Eqs. (8), (9) and (11)]. This causes the unification scale to move upwards for a given value of $\sin^2 \theta_w(M_L)$.

- 4) The implication of the above discussion for proton decay is clear. The proton can, if experimentally required, easily be made to live much longer than the present experimental lower bound on the lifetime of 2×10^{29} yr ¹¹⁾. Indeed, if we take M_X to be of the order of 10^{19} GeV, we obtain the following rough, order of magnitude, estimate of the proton lifetime: $\tau_p \approx 10^{42}$ yr, a value well beyond the reach of any foreseeable experiment !
- 5) The present world average of $\sin^2 \theta_w$ is 0.22 ± 0.03 . Comparison with the Table shows that the charged gauge bosons mediating the right-handed (V+A) interactions are predicted to be at least 10^4 times more massive than the usual bosons which mediate the V-A interactions. Moreover, with the minimal choice of the Higgs multiplets, the second neutral gauge boson of the $SU(2)_L \times SU(2)_R \times U(1)$ model also has a mass of order M_R . For the foreseeable future, the effective gauge group for weak interactions will continue to be $SU(2)_L \times U(1)$.

The discussion presented above can be extended to the flavour subgroup $SU(2)_L \times SU(2)_R \times U(1)_L \times U(1)_R$ of E_6 and $[SU(4)]^4$. Now there exists the possibility ¹²⁾ that two neutral gauge bosons [and not just one, as in the $SU(2)_L \times SU(2)_R \times U(1)_L$ model] escape getting a mass of order M_R . (This possibility will be checked by the asymmetry measurements now under way at PETRA.) The effective flavour group at the scale M_L is then $SU(2)_L \times U(1) \times U(1)$, instead of $SU(2)_L \times U(1)$. The model depends on the three couplings $g_L(M_L)$, $g_R(M_L)$ and $e(M_L)$. With a simple choice of Higgs multiplets, all processes involving left-handed neutrinos still depend only on $g_L(M_L)$ and $e(M_L)$ ^{12), 10)}. The relevant quantity for neutrino processes is therefore $\sin^2 \theta_w(M_L)$. However, other effects not involving neutrinos depend explicitly on $g_R(M_L)$.

Taking radiative corrections into account, one obtains

$$\frac{1}{g_R^2(M_L)} \simeq \frac{1}{g_L^2(M_L)} + 2 \left[b_R^2 - b_L^2 + \frac{22}{48\pi^2} \right] \ln \left(\frac{M_R}{M_L} \right) \quad (15)$$

The difference between $g_L(M_L)$ and $g_R(M_L)$ is especially interesting for parity violating neutral current effects (such as in the SLAC experiment and atomic physics experiments). The effective $SU(2)_L \times U(1) \times U(1)$ model gives the same predictions for these effects as the $SU(2)_L \times U(1)$ model

only in the limit $g_L = g_R$ (in the local four-fermion coupling approximation). The radiative corrections in the two models are different and provide a possible test for distinguishing between them ¹³⁾.

In the derivation of Eq. (13) we did not make use of the RG equation for the QCD coupling $g_S(\mu)$. We wanted to avoid introducing somewhat larger uncertainties in the estimates of the mass scales. It is understood, of course, that g_S becomes equal to g_L and g_R at the mass scale M_X . Taking the RG equation for $g_S(\mu)$ into account and combining with (10) gives

$$\alpha_s(\mu) = \frac{g_s^2(\mu)}{4\pi} \simeq \alpha(M_L) \left[\sin^2 \theta_w(M_L) - \frac{\alpha(M_L)}{2\pi} \left\{ \left(\frac{11}{3} - \frac{1}{6} N_f \right) \ln \left(\frac{M_X}{M_L} \right) + \left(11 - \frac{2}{3} N_f \right) \ln \left(\frac{M_L}{\mu} \right) \right\} \right]^{-1} \quad (16)$$

Taking $M_X = 1.2 \times 10^{19}$ GeV, $N_H = 1$, $N_F = 6$ between 20 GeV and 80 GeV and $N_F = 5$ between 10 GeV and 20 GeV (and neglecting threshold effects), we get

$$\alpha_s(\mu = 10 \text{ GeV}) \simeq \alpha(M_L) \left[\sin^2 \theta_w(M_L) - 0.19 \right]^{-1} \quad (17)$$

Values of $\alpha_s(10 \text{ GeV})$ for given values of $\sin^2 \theta_w(M_L)$ are shown in the Table. Of course, for $\sin^2 \theta_w(M_L)$ near 0.19, Eq. (17) is not valid anymore since perturbation theory breaks down.

In conclusion we have seen that the presence of gauge hierarchies can give new complexions to unified gauge theories. We have discussed the simplest extension of the two-scale hypothesis. Clearly, other possibilities may also be considered. For instance, the QCD gauge group and the $SU(2)$ flavour group can be embedded in separate larger groups, which then unify at or near the Planck mass; there may be superheavy fermions and so on. However, strong motivations (experimental and/or theoretical) for such extensions are still lacking at present.

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- Table -

M_R/M_L	$\sin^2 \theta_w$	$\alpha_s (10 \text{ GeV})$
10	0.274	0.093
10^2	0.266	0.102
10^3	0.259	0.113
10^4	0.251	0.128
10^5	0.244	0.144
10^6	0.236	0.169
10^7	0.229	0.199
10^8	0.221	0.251
10^9	0.214	0.328
10^{10}	0.206	0.487
10^{11}	0.199	0.869
10^{12}	0.191	
10^{13}	0.184	

Plots of M_R/M_L versus $\sin^2 \theta_w$ and of $\sin^2 \theta_w$ versus $\alpha_s (10 \text{ GeV})$. (M_x is taken to be $1.2 \times 10^{19} \text{ GeV}$ and $M_L = 80 \text{ GeV}$).

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