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BIREFRINGENCE INDUCED BY GRAVITATIONAL WAVES:

A SUGGESTION FOR A NEW DETECTOR

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ABSTRACT

We show that the stress caused in an isotropic medium by the gravitational waves induces a birefringence which can be used for their detection. The apparatus we suggest is similar to the one which has been proposed for measuring the birefringence produced in vacuum by a magnetic field.

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In a previous paper [1] we have shown that the effect of a gravitational wave (G.W.) on an electromagnetic (E.M.) field in vacuum can be described in terms of a model flat space with an anisotropic, time-dependent dielectric tensor ϵ equal to the magnetic permeability μ . The susceptibility tensor $\epsilon - I$ is proportional to the amplitude h of the gravitational wave and its time variation gives rise to a power exchange between the electromagnetic and the gravitational fields. In ref.[2] we have suggested a G.W. detector based on this effect.

In this letter we want to suggest a different detection method based on the change of the optical properties of a material medium induced by the G.W.. We will show that the birefringence generated in an isotropic medium by the G.W. depends upon the susceptibility tensor and not on its derivatives; as did the effects discussed in [1] and [2] .

Let us consider a material medium and let ϵ_M and μ_M be its dielectric and magnetic permeability tensors. When a G.W. impinges on the medium, ϵ_M and μ_M are modified because of two effects: (i) the metrical structure of the space itself is changed; (ii) the properties of the medium are anisotropically altered by the stresses induced by the G.W. It can be shown [3] that the relation between the vectors $\vec{D}$ and $\vec{E}$, $\vec{B}$ and $\vec{H}$ can be described, in the transverse traceless gauge, assuming the medium to be at rest, by the usual relations:

$$D^i = \epsilon^{ij} E_j , \quad B^i = \mu^{ij} H_j . \quad (1)$$

Here ϵ and μ can be written in the form:

$$\begin{aligned}\epsilon^{ij} &= -(\tilde{\epsilon}_M)^i_k g^{kj}, \\ \mu^{ij} &= -g^{ik} (\tilde{\mu}_M)^j_k,\end{aligned}\tag{2}$$

where g_{ij} are the space components of the metric tensor^(*), and $\tilde{\epsilon}_M$ and $\tilde{\mu}_M$ are the dielectric and magnetic permeability tensors as modified by the G.W.

Equations (2) reduce to the expressions given in ref.1 in the case of the vacuum, i.e. when $\tilde{\epsilon}_M = \tilde{\mu}_M = I$.

Let us now consider a monochromatic E.M. wave with polarization vector $\vec{e}$, propagation vector $\vec{k}$, and frequency ω much larger than the frequency Ω of the G.W.

From Maxwell's equations one deduces in the usual way an eigenvalue equation (dispersion relation) for the polarization vector $\vec{e}$ of the form

$$A(k, \epsilon_M, \mu_M, g) \vec{e} = \omega^2 \vec{e}, \tag{3}$$

where the 3×3 matrix A is given by:

$$A(k, \epsilon_M, \mu_M, g) = -k_b k^n \epsilon^{abc} (\tilde{\mu}^{-1})^d_c \epsilon_{dnl} (\tilde{\epsilon}^{-1})^l_m. \tag{4}$$

Here ϵ_{dnl} , ϵ^{abc} are the covariant and contravariant completely antisymmetrical tensors.

(*) We use the signature $(1, -1, -1, -1)$; greek indices run from 0 to 3, latin indices from 1 to 3. Summation over repeated indices is understood.

To determine $\tilde{\epsilon}_M$ and $\tilde{\mu}_M$ we remark that the G.W. induces in the medium a quadrupole stress field $\sigma_{ij}(\vec{x}, t)$ which depends linearly on $h_{ij} = g_{ij} - \eta_{ij}$, where η_{ij} is the Minkowski tensor (for a discussion of this point see [3]).

For the sake of simplicity from now on we shall consider an isotropic medium with $\epsilon_M = \hat{\epsilon}I$, $\mu_M = \hat{\mu}I$. In this case we can write:

$$\sigma_{ij}(\underline{x}) = f(\underline{x}, t) h_{ij}(\underline{x}, t), \quad (5)$$

where f is a dimensionless function depending upon the coupling between the matter and the G.W. This stress field in turn makes the medium optically anisotropic and therefore birefringent. The material dielectric and magnetic tensors, as modified by the G.W., become:

$$\begin{aligned} (\tilde{\epsilon}_M)_{ij} &= \hat{\epsilon} \delta_{ij} + a f h_{ij}, \\ (\tilde{\mu}_M)_{ij} &= \hat{\mu} \delta_{ij} + b f h_{ij}, \end{aligned} \quad (6)$$

where a and b are the elasto-optical strain constants of the medium [4]. (In most cases $b=0$).

As a measure of the birefringence of the medium we shall take the quantity

$$\Delta k(\omega) = \{ |k_{\perp}(\omega, \vec{e}) k_{\perp}^1(\omega, \vec{e}) - k_{\perp}(\omega, \vec{e}_{\perp}) k_{\perp}^1(\omega, \vec{e}_{\perp})| \}^{\frac{1}{2}}, \quad (7)$$

where $k_{\perp}(\omega, \vec{e})$ and $k_{\perp}(\omega, \vec{e}_{\perp})$ are the wave vectors of two orthogonally polarized rays with the same frequency and propagating in the same direction. From Eqs. (3) and (4) it follows that $\Delta k(\omega)$ depends upon $\tilde{\epsilon}_M$ and $\tilde{\mu}_M$ and therefore upon the stress induced by the G.W. and not upon the change of the metric tensor.

When $\sigma_{ij} = 0$, Δk vanishes, a result that can be directly derived from the equivalence principle when the time and space variations of the metric tensor are neglected [3] .

We shall consider in particular an E.M. wave propagating parallel to the G.W. It can be shown [3] that in this case one obtains:

$$\Delta k \cong h \langle f \rangle \left| a(\hat{\mu}\hat{\epsilon}^{-1})^{\frac{1}{2}} - b(\hat{\epsilon}\hat{\mu}^{-1})^{\frac{1}{2}} \right| k , \quad (8)$$

where h is the amplitude of the G.W. and $\langle f \rangle$ is an average value of f .

This gravity-induced birefringence Δk can be observed by means of an apparatus similar to the one suggested to measure the birefringence induced in vacuum by a magnetic field [5] .

To simplify the discussion let us consider a G.W. polarized along the x and y axes propagating along the z -axis. We suppose that a light beam travels along the same axis with a polarization vector $\vec{e}$ forming a 45° angle with the x -axis. After travelling a distance L under the influence of the G.W. the light acquires an ellipticity

$$\phi(t) = \pi L \Delta k(t) = \phi_0 \cos \Omega t . \quad (9)$$

Using Eq. (8) we find (assuming $b=0$)

$$\phi = \pi h L k a \langle f \rangle (\hat{\mu}/\hat{\epsilon})^{\frac{1}{2}} . \quad (10)$$

This ellipticity can be measured by observing the beat that arises in the intensity of the beam when one adds to it an

externally modulated ellipticity $\psi(t) = \psi_0 \cos \omega_0 t$. The experimental apparatus is sketched in Fig. 1.

The output current i from the photomultiplier is [5]

$$i = i_0 \{ \sigma^2 - 2[\phi(t) + \theta] \psi(t) + \psi(t)^2 \} \quad (11)$$

where σ^2 is the extinction factor of the two prism polarizers ($\sigma^2 \sim 10^{-7} - 10^{-8}$), and θ is the error in the alignments of their optical axes. We will assume $\theta \sim 10^{-8}$ rad.

The signal i contains, besides a constant term, a large Fourier component at twice the modulating frequency, $2\omega_0$, whose intensity can be easily filtered out; a component at frequency ω_0 arising from the misalignment θ ; two components at frequencies $\omega_0 \pm \Omega$. Assuming the minimum observable value of ϕ_0 to be 10^{-11} rad [5], the ratio between the two terms in the square brackets in Eq. (11) is 10^{-3} .

Taking into account only the statistical fluctuations of the number of detected photons (shot noise), the signal to noise ratio is

$$R = \sqrt{\frac{P\tau}{\hbar\omega}} \sqrt{\frac{2\psi_0^2}{\sigma^2 + \psi_0^2}} \phi_0 \quad (12)$$

where P is the laser power, $\hbar\omega$ is the photon energy and τ is the measuring time.

Let us consider a 1 W Argon laser ($\lambda = 5145 \text{ \AA}$, $\omega = 3.665 \times 10^{15} \text{ rad s}^{-1}$) and an optical cavity 1 m long filled with a medium whose elasto-optical constants are $a = 0.3$, $b = 0$. With 500 reflections the total length L is 500 m. We will assume for the measuring time the value $\tau = 10^4 \text{ s}$ and for

the amplitude ψ_0 of the modulating ellipticity the square root of the extinction factor, i.e. $\psi_0 \sim 10^{-4}$. Setting $R = 1$ from Eqs. (10), (12) and with $(\phi_0)_{\min} \sim 10^{-11}$ rad the minimum observable value of h becomes:

$$h_{\min} = \frac{1}{\langle f \rangle} 5 \times 10^{-21}, \quad (13)$$

where for $\langle f \rangle$ one can use the approximate expression:

$$\langle f \rangle = \frac{\Omega^2}{\omega_M^2 - \Omega^2}, \quad (14)$$

ω_M being the mechanical frequency of the medium.

Other sources of noise not included in Eq. (12) have been evaluated and found to lead to limits lower than (13).

If we take ω_M of the order of 1 kHz the values of f vary between 1 for $\Omega \sim 1$ kHz to 10^{-4} for $\Omega \sim 10$ Hz and correspondingly $h_{\min}$ varies from 10^{-16} to 10^{-20} . The sensitivity of this detector is almost unaffected by the fluctuations of the length of the optical path and remains approximately constant ($h_{\min} \sim 10^{-20}$) if $\Omega \gg \omega_M$.

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Figure caption

A continuous laser beam (LR) is linearly polarized by the first prism (N1), and then passes through a Faraday cell (FC) which rotates the optical vector by an angle $\psi(t)$. After the Faraday cell there is a quarter-wave plate (QW) (with its slow axis in the x direction). These three elements are used to modulate the ellipticity of the light.

The light is sent into an optical cavity (OC) consisting of a cubic crystal, 100 cm long and 10 cm thick. This crystal has its two end surfaces treated so as to provide interferential spherical mirrors ($R \cong 3 \text{ m}$), which allow the light to make multiple passages (~ 500) through the cavity before going through the second prism (N2), after which it will be analyzed by a photo-multiplier (PM).

Fig. 1