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GENERATING QUARK AND GLUON FRAGMENTATION FUNCTIONS FROM QCD

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A B S T R A C T

We calculate quark and gluon fragmentation functions at large Q^2 by means of perturbative QCD starting from "physical" thresholds. Our formalism provides a general framework to generate fragmentation functions (valence, sea and gluon) from threshold valence quark and gluon fragmentation functions. Thresholds of various flavours are taken into account and give rise to natural flavour dependence of fragmentation functions. We incorporate resonance production which has an important effect on the energy (z) spectrum of final hadrons. Inclusive hadron spectrum from quarkonium decay is also predicted.

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Recent data on deep inelastic lepton-hadron scattering have nicely confirmed the Q^2 dependence of parton distribution functions predicted by QCD ¹⁾. The observed x dependence of sea quark and gluon distribution functions at large Q^2 has been reproduced qualitatively by assuming only valence quarks in nucleon at small Q^2 and by using Q^2 evolution equation in leading order of QCD ²⁾. As for parton fragmentation functions into hadrons, a precise determination of z (energy fraction) as well as Q^2 dependence will soon be available from current and forthcoming experiments of neutrino and muon beams and electron-positron colliding beams. It is tempting to explore the possibility of a valence quark limit for fragmentation functions. The purpose of our letter is to calculate quark and gluon fragmentation functions at large Q^2 by means of perturbative QCD starting from a valence quark picture at "physical" threshold. Our formalism provides a general framework to generate fragmentation functions of valence and sea quarks and of gluons from threshold valence quark and gluon fragmentation functions. We assume production of $q\bar{q}$ multiplets of pseudoscalar and vector mesons. Because of different thresholds for various flavours we obtain a natural and interesting flavour dependence of fragmentation functions. Inclusive hadron spectrum from heavy quarkonium decay is also calculated to test our prediction on gluon fragmentation functions.

We illustrate our argument using inclusive single hadron production in electron-positron annihilation and define the multiplicity functions ³⁾

$$N_{e^+e^-}^h(z) = \frac{1}{2\sigma_{tot.}} \frac{d\sigma}{dz}(e^+e^- \rightarrow h + X) \quad (1)$$

where $z = 2P \cdot Q / Q^2$ and P and Q are four momenta of the hadron and virtual photon. In the parton model the multiplicity function is given in terms of fragmentation functions $D_{q_i}^h(z)$ of quark q_i into hadron h ,

$$N_{e^+e^-}^{h^+} + N_{e^+e^-}^{h^-} = \frac{1}{\sum e_i^2} \sum e_i^2 (D_{q_i}^{h^+}(z) + D_{\bar{q}_i}^{h^-}(z)) \quad (2)$$

where e_i is the electric charge of quark i . In order to be realistic and to account for the effect of resonances we shall assume that the hadrons (mesons) are produced in flavour multiplets of pseudoscalar and vector mesons and include flavour thresholds. Below the strangeness threshold we only produce hadrons containing u and d quarks. Let us consider the two u quark fragmentation functions $D_u^{(u\bar{d})}(z)$ and $D_u^{(\bar{u}d)}(z)$. The first

one (valence fragmentation function) describes the production of a hadron ($u\bar{d}$) from a valence quark u , the second one (sea fragmentation function) describes the production from a sea quark. It is plausible to conjecture that the valence quark can more easily produce the hadron by picking up a wee antiquark, while the sea quark requires additional production of hadrons. We therefore assume that at the "physical" two-flavour threshold, $Q_2^2 = (2m_{u\bar{d}})^2$ the sea quark fragmentation function vanishes.

$$D_u^{(d\bar{u})}(z, Q_2^2) = 0 \quad (3)$$

Writing now the u quark isospin and energy sum rules ^{*)}

$$\int_0^1 dz \left(D_u^{(u\bar{d})}(z) - D_u^{(d\bar{u})}(z) \right) = 1/2 \quad (4)$$

$$\int_0^1 dz z \left(D_u^{(u\bar{d})}(z) + D_u^{(d\bar{u})}(z) \right) = 1/2 \quad (5)$$

and inserting assumption (3) into (4) and (5) we obtain the initial condition for the valence quark ^{**) (**))}

$$D_u^{(u\bar{d})}(z, Q_2^2) = 1/2 \delta(1-z) \quad (6)$$

The factor $1/2$ corresponds to equal probability of picking up a $\bar{d}$ or $\bar{u}$ antiquark. The initial conditions (3) and (6) enable us to calculate the Q^2 evolution of non-singlet fragmentation functions in a first attempt by means of leading order perturbative QCD formalism ⁶⁾.

At the strangeness threshold, $Q_3^2 = (2m_{s\bar{u}})^2$, which we approximate by a step function, the number of flavours f changes from 2 to 3. Repeating previous arguments we assume for the new sea fragmentation functions

$$D_s^{(u\bar{s})}(z, Q_3^2) = D_u^{(s\bar{u})} = D_s^{(u\bar{d})} = 0 \quad (7)$$

^{*)} One can show that isospin sum rules are always valid whereas other charge sum rules may not hold ⁴⁾.

^{**) (**))} A similar initial condition has been proposed for heavy quarks ⁵⁾.

and for the new valence fragmentation functions

$$D_s^{(s\bar{u})}(z, Q_3^2) = D_u^{(u\bar{s})} = D_s^{s\bar{s}} = \frac{1}{3} \delta(1-z) \quad (8)$$

where $1/3$ represents the probability that the picked up wee quark be $\bar{u}$, $\bar{d}$ or $\bar{s}$. Since the u quark can now pick up wee s quark $1/3$ of the time, the probability (not cross-section) of producing non-strange mesons from u quark should be only $2/3$ of what it was below threshold, for instance,

$$D_u^{(u\bar{d})}(Q = 2m_{s\bar{u}} + \epsilon) = \frac{2}{3} D_u^{(u\bar{d})}(Q = 2m_{s\bar{u}} - \epsilon) \quad (9)$$

We can now formulate our general prescription for non-singlet QCD evolution.

- 1) At the Q_f threshold the valence fragmentation functions involving the new quark q_f are given by

$$D_{q_f}^{(q_f\bar{q})} = D_q^{(q\bar{q})} = \frac{1}{f} \delta(1-z) \quad (10)$$

the sea fragmentation functions involving the new flavour are set to zero ; the old quark fragmentation functions into old mesons (for $f > 2$) are reduced by a factor $(f-1)/f$:

$$D_{q_i}^{(q_i\bar{q}_k)}(Q = Q_f + \epsilon) = \frac{f-1}{f} D_{q_i}^{(q_i\bar{q}_k)}(Q = Q_f - \epsilon) \quad (11)$$

The physical flavour thresholds are approximated by the average mass of pseudoscalar and vector mesons containing the new quark and the $u(d)$ quark, for example, $Q_2 = (m_\pi + m_\rho)$, $Q_3 = (m_K + m_{K^*})$, etc.

- 2) In the region where f flavours are produced, the non-singlet evolution is described by

$$\langle D_{q_i}^{(q_i\bar{q}_k)} - D_{\bar{q}_i}^{(q_i\bar{q}_k)}(Q) \rangle_n = \langle D_{q_i}^{(q_i\bar{q}_k)} - D_{\bar{q}_i}^{(q_i\bar{q}_k)}(Q_f + \epsilon) \rangle_n \exp[-\gamma_n^{qq} s] \quad (12)$$

where

$$\gamma_n^{qq} = \frac{1}{3\pi} \left[1 - \frac{2}{n(n+1)} + 4 \sum_{j=2}^n \frac{1}{j} \right] \quad (13)$$

and

$$s = \frac{12\pi}{33-2f} \ln \left[\ln(\bar{Q}^2/\Lambda^2) / \ln(Q_1^2/\Lambda^2) \right] \quad (14)$$

For the value of Λ we used $\Lambda = 0.75$.

3) The final distributions contain, in general, directly produced pseudo-scalar mesons and pseudoscalar mesons from the decay of vector meson resonances. We approximate the distribution from a vector meson decay by two-body decay into massless products ^{7),*)}. In terms of moments, $\langle D^{\text{product}} \rangle_n = 2 \langle D^{\text{vector meson}} \rangle_n / n$ (we should replace 2 by 3 if the decay multiplicity is 3). For the ratio of direct production of pseudoscalar mesons to vector mesons we take the spin weight ratio : 1/3.

The Q^2 evolution of the non-singlet fragmentation function $D_u^{\pi^+} - D_u^{\pi^-}$ is shown in Fig. 1a. As we started with a δ function at $z=1$, at the first threshold the various moments have values close to each other (not the same because of resonances). The step function change of moments at various flavour thresholds is an artifact of our approximate treatment of new flavour thresholds and can easily be refined. The Q^2 evolution of non-singlet fragmentation functions for charmed quark production, $D_c^{D^+} - D_c^{D^-}$, is shown in Fig. 1b in comparison with recent SLAC data ^{**) , 8)}. Since the starting point ("physical" threshold) is higher for heavier flavour, heavy quark fragmentation function has been evolved less at any given Q^2 . This effect naturally makes the peak of heavier quark non-singlet fragmentation

) Except for the decay $D^ \rightarrow D\pi$ which we treat as a threshold decay : the momentum fractions carried by D and π are m_D/m_{D^*} and $(m_{D^*}-m_D)/m_{D^*} \simeq m_\pi/m_{D^*}$, respectively.

**) As the energy is very low, 7 GeV, sea quark D^+ fragmentation is excluded and non-singlet evolution gives a good approximation. We estimated the moments in the allowed kinematic region, $0.54 < z < 1$, with an error of the order of 30%, from the published figure, using the relation

$$\langle D_c^{D^+} \rangle_n \simeq \langle D_c^{D^+} - D_c^{D^-} \rangle_n = \left(\frac{1}{\sigma} \frac{d\sigma}{dz} (e^+e^- \rightarrow D^+ X) \right)_n$$

At such a low energy it is presumably only meaningful to compare first few moments rather than a z distribution.

function nearer to $z=1$. In Fig. 2 we plotted the non-singlet $z(D_u^{\pi^+} - D_u^{\pi^-})$ fragmentation function ^{*)} for $Q^2 = 10 \text{ GeV}^2$, and compared it with available data ^{9),10)}.

In order to obtain individual fragmentation functions, we have to calculate the Q^2 evolution of a singlet which mixes with the gluon. Above f flavour threshold Q_f and below the next threshold, we use the mixing equation in leading order of QCD for f flavours ^{6),**)}

$$\frac{\partial}{\partial t} \begin{bmatrix} \langle D_S^h(Q^2) \rangle_n \\ \langle D_G^h(Q^2) \rangle_n \end{bmatrix} = -\alpha(t) \begin{bmatrix} \gamma_n^{qq} & 2f \gamma_n^{Gq} \\ \gamma_n^{qG} & \gamma_n^{GG} \end{bmatrix} \begin{bmatrix} \langle D_S^h(Q^2) \rangle_n \\ \langle D_G^h(Q^2) \rangle_n \end{bmatrix} \quad (15)$$

where the n^{th} moment gluon fragmentation function into hadron h at Q^2 is denoted as $\langle D_G^h(Q^2) \rangle_n$ and singlet quark fragmentation function for f flavour is defined by

$$D_S^h = \sum_{i=1}^f (D_{q_i}^h + D_{\bar{q}_i}^h), \quad (16)$$

anomalous dimensions are given by Eq. (9) and

$$\begin{aligned} 2f \gamma_n^{Gq} &= -\frac{4f}{3\pi} \frac{n^2+n+2}{n(n^2-1)} \\ \gamma_n^{qG} &= -\frac{1}{4\pi} \frac{n^2+n+2}{n(n+1)(n+2)} \\ \gamma_n^{GG} &= \frac{3}{4\pi} \left[\frac{1}{3} - \frac{4}{n(n-1)} - \frac{4}{(n+1)(n+2)} + 4 \sum_{j=2}^n \frac{1}{j} + \frac{2f}{9} \right] \end{aligned} \quad (17)$$

and $t = \ln Q^2$ and $\alpha(t) = 12\pi / [(33-2f)\ln Q^2/\Lambda^2]$. Energy conservation is guaranteed by a vanishing eigenvalue γ_2^- of the anomalous dimension matrix for $n=2$ provided the initial condition satisfies $\langle D_S^h \rangle_2 / \langle D_G^h \rangle_2 = 2f$ corresponding to the eigenvector for γ_2^- ⁶⁾. Moreover our initial conditions for valence and sea quark fragmentation functions give $\langle D_S^{q_i \bar{q}_j} \rangle_2 = 2/f$

*) We used in this paper simple parametrizations for fragmentation functions : $Nz^\alpha(1-z)^\beta$ for valence (non-singlet) and $b \exp(-az)(1-z)$ for sea and gluons.

**) We generally follow the notation of Uematsu in Ref. 6).

independent of flavour indices i and j . Therefore the energy fraction carried by various $q\bar{q}$ mesons from gluon fragmentation should be :

$$\langle D_G^{q_i \bar{q}_j} \rangle_2 = 1/f^2.$$

The gluon carries no flavour quantum numbers, thus the flavour non-singlet $q\bar{q}$ mesons have to be produced at least in pairs. Regarding colour, the gluon carries two colour lines (3 and 3^*), therefore we assume simply that at the starting Q_f threshold the gluon just produces two mesons ($q_i \bar{q}_f$ and $\bar{q}_i q_f$), and write the initial gluon fragmentation function to produce a new flavour f as ^{*)}

$$D_G^{(q_f \bar{q}_i)} = \frac{2}{f^2} \delta(z - 1/2) \quad (18)$$

For the old flavour production we use, similarly to what we did before,

$$D_G^{(q_j \bar{q}_i)}(Q = Q_f + \epsilon) = \frac{(f-1)^2}{f^2} D_G^{(q_j \bar{q}_i)}(Q = Q_f - \epsilon) \quad (19)$$

Flavour singlet pseudoscalar mesons such as η or η' can be produced singly from a gluon as a leading particle by picking up one wee gluon. This possibility can be formulated by assuming an initial condition for the flavour singlet meson, consistent with the energy conservation requirement

$$D_G^{(q_f \bar{q}_f)}(z, Q_f) = \frac{1}{f^2} \delta(1-z) \quad (20)$$

to be compared with Eq. (18) for flavour non-singlet mesons ^{**)} . This ansatz gives a harder energy spectrum of neutral particles, such as η and η' , in the gluon fragmentation which can be observed in quarkonium decays.

^{*)} As another example consistent with $\langle D_G \rangle_2 = 1/f^2$ and $\langle D_G \rangle_1 = 2/f^2$, we have also tried a uniform distribution in z , $D_G(q_f \bar{q}_i) = 2/f^2$ which turned out to give similar results for low moments.

^{**)} The single $(q_i \bar{q}_i)$ production (20) is presumably kinematically favoured compared to pair production (18).

Since we have fixed all the initial conditions for quark and gluon fragmentation the singlet and gluon Q^2 evolution can be calculated. The prescription given above for non-singlet evolution, with the appropriate modifications and extensions, still applies. The initial conditions at f flavour threshold are (10) and (11) for quark fragmentation functions, (18), (19) and (20) for gluon fragmentation function. The Q^2 evolution equation to be used is the integral version of Eq. (15) [See Eq. (15) in Ref. 6]. Resonances are treated as previously.

Individual fragmentation functions are obtained by combining singlet and non-singlet evolution equations. The number of fragmentation functions is reduced significantly not only because of isospin and charge conjugation invariance but also of our simple choice of initial conditions, e.g., $D_u^{su} = D_d^{su} = D_{\bar{d}}^{su} = D_{\bar{s}}^{su}$ and $D_s^{su} = D_{\bar{u}}^{su}$, etc. Fragmentation functions of u quark into π^+ and π^- are displayed and compared with the cascade model parametrization of Field and Feynman ⁷⁾ in Fig. 3a. Our predicted multiplicity function $N_{e^+e^-}^{\pi^+} + N_{e^+e^-}^{\pi^-}$ production in e^+e^- annihilation is shown for $Q^2 = (5.2 \text{ GeV})^2$ and $Q^2 = 400 \text{ GeV}^2$ and is compared with DASP data $(Q = 5.2 \text{ GeV})^2$. In general we obtain qualitative agreement with data. However, our prediction is too concave in the intermediate z region, while data look almost exponential. This implies that the valence quark fragmentation function evolves too slowly.

Gluon fragmentation functions can be measured in heavy quarkonium decay ¹²⁾. For three-gluon decay of ground state (3S_1) $Q\bar{Q}$ bound state, we obtain approximately $\langle D_{Q\bar{Q}}^h \rangle_n = \langle D_G^h \rangle_n \cdot 6/(n+1)$, where $D_{Q\bar{Q}} \equiv 1/\sigma \, d\sigma/dz$. Our prediction of inclusive spectra of π^+ and $\eta' \equiv \frac{1}{2}(u\bar{u} + d\bar{d}) + 1/\sqrt{2} \, s\bar{s}$ for Y (mass $\simeq 9.46 \text{ GeV}$) is shown in Fig. 4.

We have presented a general framework to calculate quark and gluon fragmentation functions starting from "physical" threshold. Results of our simple ansatz are in qualitative agreement with experimental indications, which is encouraging enough to do a more detailed analysis. There are a number of points which can be refined.

- 1) We have approximated a new flavour threshold crudely by a step function increase of the number of flavours. Therefore immediately above threshold a new quark is produced as freely as old ones. We can modify this approximation to incorporate suppression of a new quark flavour even above the

associated threshold. For example if we assume the probability of picking up $\bar{u}$, $\bar{d}$ and $\bar{s}$ wee quarks to be γ , γ and $1-2\gamma$ respectively ⁷⁾ we should take an initial condition $D_s^{su} = \gamma\delta(1-z)$, $D_u^{us} = (1-2\gamma)\delta(1-z)$ instead of Eq. (8), and $D_u^{u\bar{d}}(+e) = 2\gamma.D_u^{u\bar{d}}(-e)$ instead of Eq. (9). We could also incorporate multiple thresholds, e.g., multiple $c\bar{c}$ pair production may occur only at energies much higher than $Q = m_D + m_{D^*}$.

- 2) Initial conditions for gluon fragmentation functions are less obvious compared to quark fragmentation functions. One can try several other reasonable choices in order to see more precisely which quantities are sensitive to these choices.
- 3) We used a rough approximation for the threshold for $q\bar{q}$ mesons as given by the average of pseudoscalar and vector meson masses. The relative abundance of vector and pseudoscalar mesons is fixed by spin weight, but may eventually be determined by experiment. Our analytic approximation of resonance decays can be improved by using a more realistic kinematics.
- 4) Since we are dealing with fragmentation functions even near threshold and the Altarelli-Parisi evolution equation disregards the effect of masses, it is not quite obvious whether the energy fraction z is the correct variable to be used. It might be more reasonable to consider a variable always with a domain of variation from 0 to 1 as required in the evolution equation.
- 5) Since we are starting from small Q^2 , we should better include next order corrections to the QCD Q^2 evolution equation. This requires a calculation of anomalous dimension to order $\bar{g}^4$ for a fragmentation function (time-like cut vertex).
- 6) In a recent work ¹³⁾, Amati and Veneziano described large Q^2 quark jets degrading down to a hadronization point with formation of colour singlet small mass $q\bar{q}$ clusters. Our proposal can be regarded as a model for the hadronization where neighbouring q and $\bar{q}$ hadronize by picking up wee $\bar{q}$ and q , respectively.

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FIGURE CAPTIONS

- Figure 1 Q^2 evolution of moments of non-singlet fragmentation functions. a) $D_u^{\pi^+} - D_u^{\pi^-}$ and b) $D_c^{D^+} - D_c^{D^+}$. Data are taken from Ref. 8).
- Figure 2 The non-singlet fragmentation function $z(D_u^{\pi^+} - D_u^{\pi^-})$ at $Q^2 = 10 \text{ GeV}^2$ as a function of z . Data from Ref. 9) (circles) and from Ref. 10) at $Q^2 = 9.5 \text{ GeV}^2$ (crosses).
- Figure 3 a) Our theoretical prediction of $zD_u^{\pi^+}$ (solid line) and $zD_u^{\pi^-}$ (dashed line) at $Q^2 = 10 \text{ GeV}^2$ compared with the parametrization of Field and Feynman ⁷⁾, $zD_u^{\pi^+}$ (dotted line) and $zD_u^{\pi^-}$ (dash-dotted line).
b) Predictions of multiplicity functions [Eq. (1)] for π^+ and π^- production in e^+e^- collisions at $Q^2 = (5.2 \text{ GeV})^2$ (solid line) and $Q^2 = 400 \text{ GeV}^2$ (dashed line) compared with data ¹¹⁾ at $Q = 5.2 \text{ GeV}$.
- Figure 4 Inclusive π^+ (dashed line) and η' (solid line) spectra $z/\sigma \, d\sigma/dz$ from Υ (mass = 9.46 GeV). For the directly produced η the same spectrum is obtained as for η' .

FIG. 1a

FIG. 1b

FIG. 2

FIG. 3a

FIG. 3b

FIG. 4