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SEARCH FOR THE EFFECTS OF HIGHER ORDER TERMS
IN DEEP INELASTIC LEPTON-HADRON SCATTERING

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A B S T R A C T

We look for those predictions involving higher order QCD corrections for deep inelastic lepton-hadron scattering which are independent of the renormalization scheme used to define α_s . The effects of these corrections are found to be very small, generally smaller than the errors in the existing data.

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During the last few years, we have begun to interpret "hard-scattering processes" in terms of perturbative quantum chromodynamics (QCD). Indeed, the contribution of lowest order in $\alpha_s(Q^2)$, the running coupling constant, has been calculated and compared to the data for many of these processes ^{*)} (e.g., deep inelastic lepton-hadron scattering, the production of lepton pairs with a large invariant mass in hadronic collisions, the production of particles or jets at large transverse momentum in hadronic collisions, jets in e^+e^- annihilation, etc.). Although there is no obvious contradiction between experiment and theory, it is too early to state unequivocally that QCD is the true theory of the strong interactions. The value of $\alpha_s(Q^2)$ is indirectly extracted from the data (using the renormalization group) and one finds that $\alpha_s(10 \text{ GeV}^2) \simeq 0.2$ to 0.4 . This suggests that higher order terms in $\alpha_s(Q^2)$ may be important, and motivated the programme for their calculation, first in the light-cone dominated process of deep inelastic lepton-hadron scattering ³⁾⁻⁵⁾, and more recently in other processes ^{6),7)}.

So far we have written $\alpha_s(Q^2)$ as though it needed no further definition, after all in quantum electrodynamics (QED) $\alpha = (e^2/4\pi)$ is universally accepted as being the coupling constant defined through the "Thompson limit of Compton scattering". Such a limit does not exist in QCD so we have to specify precisely what we mean by $\alpha_s(Q^2)$; we will come back to this point below.

In this letter we will look for effects of the higher order corrections in deep inelastic lepton-hadron scattering, in particular in "non-singlet" structure functions such as F_3^{VA} , where A is an (almost) isoscalar target, or $F_2^{\mu p} - F_2^{\mu n}$. For the Nth moment ($M_N(Q^2)$) of such a structure function the theoretical predictions can be written in the following form :

$$R^N(Q^2, Q_0^2) \equiv \left[\frac{M_N(Q^2)}{M_N(Q_0^2)} \right]^{1/d_N}$$

$$= \frac{\alpha_s(Q^2)}{\alpha_s(Q_0^2)} \left[1 + a^N (\alpha_s(Q^2) - \alpha_s(Q_0^2)) + \dots \right]^{1/d_N} \quad (1)$$

^{*)} There are far too many papers on this subject to make a full list of references possible here. See for example Refs. 1), 2) or the proceedings of any other recent conference on particle physics.

where

$$d_N = \frac{4}{33-2f} \left[1 - \frac{2}{N(N+1)} + 4 \sum_{j=2}^N \frac{1}{j} \right], \quad (2)$$

f is the number of flavours and the a^N are numbers given in Ref. 4). The a^N depend on the renormalization scheme used to define $\alpha_s(Q^2)$ ⁸⁾, and can be written in the form

$$a^N = b^N + \rho d_N \quad (3)$$

where b^N are renormalization scheme independent numbers, whereas ρ is an arbitrary (but N independent) number, the arbitrariness being precisely equivalent to the freedom of choosing any renormalization scheme. It is only when the complete infinite series is considered that the prediction for R^N becomes independent of the renormalization scheme. We would have liked to define α_s in terms of one measurable number and make predictions for other processes in terms of this α_s (as is done in QED), however, in practice this is not possible yet. Below we will look at predictions which depend on the b^N but not on ρ .

To get an idea whether the predicted higher order corrections manifest themselves in the experimental data, we will try to extract " $\alpha_s(Q^2)$ " directly from these data, using our theoretical knowledge of b^N . If the extracted value of α_s is not "reasonable" then the data are not consistent with the higher order corrections predicted from QCD.

On the right-hand side of (1) we have two as yet unknown quantities $\alpha_s(Q^2)$ and $\alpha_s(Q_0^2)$, so that defining R^N and R^T we can eliminate $\alpha(Q_0^2)$. Thus we find

$$\alpha_s(Q^2) = \frac{R^N(Q^2, Q_0^2) - R^T(Q^2, Q_0^2)}{1 - R^N(Q^2, Q_0^2)} \frac{d_N d_T}{a^T d_N - a^N d_T} + O((R^N - R^T)^2) \quad (4)$$

where in principle the higher order terms in $R^N - R^T$ can be calculated. We call the first term on the right-hand side of Eq. (4) $\tilde{\alpha}_s(Q^2)$ ^{*)}.

Asymptotically the theory predicts that $R^N \rightarrow R^T$ so that $R^N - R^T$ is a "measurable" small quantity, in which, hopefully, one can expand perturbatively. $\tilde{\alpha}_s(Q^2)$ is independent of p , and hence of the definition of α_s . This independence is just a statement that whatever renormalization scheme is used to define α_s , when we expand α_s in terms of a power series in $R^N - R^T$, the linear term is always the same. Thus $\tilde{\alpha}_s(Q^2)$ should be a good expansion parameter ^{**)} (analogous to α in QED) and we have attempted to extract it from the Nachtmann moments of xF_3 obtained by the CDHS ⁹⁾ collaboration (these are the most precise data at high Q^2 so far, the typical errors on the moments are of the order of 5%). For purposes of illustration, we take the pairs of moments R^2, R^6 ; R^3, R^6 and R^2, R^5 ; R^3, R^5 and take $Q_0^2 = 6.5 \text{ GeV}^2$ (the lowest available Q^2) and 10 GeV^2 . The results of this analysis for $Q^2 = 21 \text{ GeV}^2$ and 45 GeV^2 are shown in Fig. 1, from which we conclude that the existing data are not precise enough even to determine the sign of the higher order corrections. The errors are large, of course, because we are taking the difference of two almost equal quantities. We would like to point out the following two features: (i) the points in Fig. 1 are not independent and therefore one cannot just average them, and (ii) the value of $\tilde{\alpha}_s$ is at least consistent with the theoretical expectation mentioned above, which is certainly not trivial. The fact that $\tilde{\alpha}_s(Q^2)$ is so poorly determined from the data is a manifestation of the fact that the effects of higher order corrections are smaller than the experimental errors.

There are several other (related) ways of trying to find (p independent) effects of higher order corrections in moments of structure functions. One of these is to check whether the value of the parameter Λ , determined from the data using the lowest order formula

*) For notational simplicity we suppress the dependence on N , T and Q_0^2 . One would expect such a dependence to be very weak for reasonably small values of N and T .

***) Perhaps conceptually it would have been simpler to define an $\tilde{\alpha}_s$ from the e^+e^- total annihilation cross-section. However, in that case it is not possible in practice [because of the large experimental (and theoretical) uncertainties] to determine it from the data.

$$R^N(Q^2, Q_0^2) = \frac{\log Q_0^2/\Lambda^2}{\log Q^2/\Lambda^2} \quad (5a)$$

has the correct N dependence. Strictly speaking one should extract Λ using a slightly more complicated formula than (5a), that is :

$$R^N(Q^2, Q_0^2) = \frac{\log Q_0^2/\Lambda^2}{\log Q^2/\Lambda^2} \times \frac{\left(1 - \frac{\beta_1 \log \log Q^2/\Lambda^2}{\beta_0 \log Q^2/\Lambda^2}\right)}{\left(1 - \frac{\beta_1 \log \log Q_0^2/\Lambda^2}{\beta_0 \log Q_0^2/\Lambda^2}\right)} \quad (5b)$$

where β_0, β_1 are respectively the coefficients of $-g^3/16\pi^2, -g^5/(16\pi^2)^2$ in the β function. Because of the slow variation of the $\log \log Q^2/\Lambda^2$ with Q^2 , at least for Q^2 substantially larger than Λ^2 , the N dependence of the Λ 's obtained in this way should be very nearly the same as these using Eq. (5a). We have checked that this indeed is the case for the existing data. The fact that we neglect the a_N in Eqs. (5a), (5b) means that the higher order corrections will now manifest themselves as an N dependence in Λ . We find

$$\Lambda_N = \Lambda \exp\left(\frac{2\pi a_N}{\beta_0 d_N}\right) \quad (6)$$

A very nice feature of Eq. (6) is that Λ_N/Λ_T is independent of p and therefore is a clean prediction of QCD.

This definition of Λ_N is different from that of Bardeen et al.⁵⁾ (which was subsequently used by Duke and Roberts^{10),*)} ; in their definition only a part of the higher order corrections was absorbed into an N dependent Λ . Using the definition of Bardeen et al., Λ_N must be extracted not by using the lowest order formula (5a) [or (5b)] but a more complicated formula, one which includes the "unabsorbed" higher order terms ; we find

*) From the paper of Duke¹⁰⁾ one might wrongly conclude that Duke and Roberts have the same definition as ours for Λ_N ; in fact this is not so, their Λ_N is defined using the definition of Bardeen et al.⁵⁾. The procedure we are using has previously been proposed by Bacé¹¹⁾.

our definition preferable, not only because it is more simple, but primarily because both the theoretical prediction and the procedure for extracting Λ_N from the data are (in our case only) explicitly independent of ρ .

In the Table we tabulate Λ_N/Λ_2 for the first few values of N assuming there to be four (or six) flavours, the dependence on the number of flavours is very weak. The entries are somewhat different depending on whether one uses F_2 or F_3 to extract the Λ 's, e.g. Λ_2 extracted using F_2 is predicted to be 37% larger than that extracted using F_3 (this is independent of the number of flavours).

N	F_2	F_3
2	1.00 (1)	1.00 (1)
3	1.19 (1.17)	1.41 (1.39)
4	1.33 (1.30)	1.67 (1.63)
5	1.45 (1.41)	1.87 (1.82)
6	1.56 (1.51)	2.04 (1.98)
7	1.66 (1.60)	2.18 (2.11)
8	1.74 (1.68)	2.31 (2.23)
9	1.82 (1.75)	2.43 (2.34)
10	1.89 (1.82)	2.53 (2.43)

Predicted values of Λ_N/Λ_2 , for four flavours, the values are different if one uses F_2 or F_3 to obtain the Λ 's. The values in parentheses are for six flavours.

From the Table we see that the parameter Λ_N should change substantially with N . In Fig. 2 we plot the values of Λ_N extracted from existing neutrino ^{9),12)} and muon ¹³⁾ data. On the same figure we plot the theoretical prediction, this curve may be displaced vertically at will, since the theory only predicts the shape and not its normalization. At first sight the two sets of neutrino data look inconsistent but on closer examination ¹⁴⁾ it has been found that systematic uncertainties in the analysis (such as the number of flavours used in the formulae, whether or not to include the pseudoelastics and the correction for Fermi motion) account for about half the discrepancy, so that the final discrepancy in the data is not very significant. Moreover, the discrepancy does not stem from the different ranges of Q^2 in the two experiments ¹⁴⁾. Bearing in mind that these systematic uncertainties are not included in the errors, it is clear that the neutrino data are not yet good enough to confirm or disprove the theoretical predictions. The values of Λ_N have also been

extracted from the moments of $F_2^{\mu p} - F_2^{\mu n}$ (13), they show an increase of Λ_N with N in remarkable agreement with the predictions of the Table (see Fig. 2b). This striking agreement is somewhat surprising in view of the fact that these values of Λ_N are determined mostly by the low Q^2 region. In this region one might expect other (higher twist) non-scaling contributions (especially for the higher moments), to be still relevant, and there is no reason to expect these to reproduce Fig. 2b. We leave the reader to interpret the significance of Fig. 2b, and we look forward to the publication of neutrino and muon data which is more accurate, and in the muon case at higher values of Q^2 .

A related ρ independent prediction is that $R^N(Q^2, Q_0^2)$ should decrease with N , this can be easily seen from Eq. (1) which leads to

$$R^N(Q^2, Q_0^2) - R^T(Q^2, Q_0^2) \approx \frac{\alpha_s(Q^2)}{\alpha_s(Q_0^2)} (\alpha_s(Q^2) - \alpha_s(Q_0^2)) \left[\frac{a^N}{d_N} - \frac{a^T}{d_T} \right] \quad (7)$$

where for $N > T$, the expression in the square brackets is positive and independent of ρ . Moreover taking $\alpha(Q^2)$ to lie in a range corresponding to $0.3 < \Lambda < 0.7$, we can estimate the quantitative behaviour of R^N with N . In Fig. 3 we show the behaviour of $R^N(45 \text{ GeV}^2, 6.5 \text{ GeV}^2)$ and $R^N(45 \text{ GeV}^2, 10 \text{ GeV}^2)$ with N , and the theoretical predictions corresponding to the three values of Λ , $\Lambda = 0.3, 0.5, 0.7$. We see that the experimental errors are too large, even to check whether R^N falls or rises with N , so that the data are certainly not inconsistent with the theoretical predictions. To determine that R^N is indeed falling with N [or equivalently that $\tilde{\alpha}_s(Q^2)$ is positive, for all combinations of moments] (see Fig. 4), we have calculated that the moments of the structure functions have to be measured to at least 2% accuracy in a Q^2 range from 3 to 70 GeV^2 . This seems to be beyond experimental possibilities in the nearest future. Equation (7) and Fig. 3 are analogous to looking for deviations from a straight line behaviour in the famous plots of $\log M_N$ vs $\log M_T$ [see, for example, the talk of W.G. Scott in Ref. 2].

We summarize the above discussion by repeating that the predicted (ρ independent) effects of higher order corrections in deep inelastic scattering are very small ^{*)}, smaller than the experimental uncertainties (except perhaps at low Q^2 , where there are theoretical uncertainties). In a particular renormalization scheme the a_N [of Eq. (1)] may be large [this was the case for the minimal subtraction scheme used in Ref. 3], however, much of the corrections can then be absorbed into a redefinition of the coupling constant. Although we have not studied in detail the effects of higher order corrections on singlet quantities, the numbers analogous to the a^N exist ³⁾, and there is nothing to suggest any qualitative change of our conclusions.

Throughout the discussion we have neglected "higher twist" effects, i.e., terms which fall with Q^2 like a power of Q^2 . It is certainly possible ^{15),16)} that these effects account for most of the scaling violations seen at present values of Q^2 (in that case the agreement of the data with the theoretical curve in Fig. 2b is an accident), so that the real value of Λ is much smaller. In this case we would have trouble seeing even the effects of leading order in QCD in deep inelastic lepton-hadron scattering experiments. It is clearly very important to get a handle on these higher twist terms.

*) It has recently been found that the higher order corrections to the Drell-Yan formula for massive lepton pair production are very large ⁷⁾.

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FIGURE CAPTIONS

- Fig. 1 Values of $\tilde{\alpha}_s(Q^2)$ for $Q^2 = 21 \text{ GeV}^2$ and 45 GeV^2 . The values of N , T and Q_0^2 used to extract $\tilde{\alpha}_s$ are indicated.
- Fig. 2 a) Values of Λ_N extracted from the F_3 structure function by the BEBC 12) and CDHS 9) neutrino experiments, using the lowest order formula (5a) and also formula (5b) which include fully the higher order corrections, compared with the QCD prediction. The theoretical curve may be displaced vertically, its normalization is not predicted (see text).
b) Values of Λ_N extracted in Ref. 13) using the moments of $F_2^{\mu p} - F_2^{\mu n}$ and formula (5a).
- Fig. 3 Values of R^N vs N derived from CDHS data 9) compared with the expected variation corresponding to different Λ 's.
- Fig. 4 Comparison of the expected variation of R^N with N and with the hypothetical values of R^N coming from moments measured to 2% and showing no N dependence.

FIG.1

FIG.2.a

FIG. 2.b

- $\Lambda = 0.3$
- - $\Lambda = 0.5$
- · - $\Lambda = 0.7$

FIG 3

- $\Lambda = 0.3$
- - $\Lambda = 0.5$
- · - $\Lambda = 0.7$

FIG 4