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EXPERIMENTAL TESTS FOR THE EQUALITY OF GLUON-GLUON
AND QUARK-GLUON COUPLING ON RESONANCE IN e^+e^- ANNIHILATION

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A B S T R A C T

We discuss how observation of charmed multiplicities $\langle N_c \rangle$ and correlations $\langle N_c^2 \rangle$, generated on resonance in e^+e^- annihilation, can give evidence for or against the existence of the gluon self-coupling and also determine its size. The calculations, based on the leading log approach, are performed in (i) QCD, where allowance is also made for fixed point coupling, and (ii) in a quark-gluon theory with no gluon self-coupling. It is stressed that such measurements bear on the question of whether the underlying theory exhibits a non-Abelian gauge structure.

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1. - INTRODUCTION

QCD is currently the most fashionable candidate for a theory of the strong interactions. Dodging for the moment the apparent embarrassment that the existence of free quarks, or gluons, has not been established, one still has to ask himself which are the other salient features of this proposed theory whose observation - although not proving it - can nevertheless lend further credibility to it - a commodity one should not consider lightly.

QCD states that :

- i) besides quarks there is another type of fundamental constituent fields, the gluons, and,
- ii) these gluons possess couplings to each other already on the tree graph level, unlike photons.

These two features can be and are being studied in deep inelastic leptonproduction where scaling violations show that quark partons behave almost, but not quite, like free particles. Furthermore, the pattern of these scaling violations can be and is being used to give evidence for the vector character of gluons and for the phenomenon of asymptotic freedom. On the other hand, this phenomenon can be shown, using standard techniques, to exist in four dimensions only for non-Abelian gauge theories, which exhibit self-coupling of the gauge fields.

Still, it is quite desirable to have further, independent, tests of this scheme of QCD available which might give even more direct evidence for, or against, it. It has been suggested again and again that one should look for gluon jets, in particular in e^+e^- annihilation : if one observes a dominant three-jet structure in the hadronic decays of a heavy vector meson resonance, one might find a rather striking piece of evidence : yet results on the Y decays are not conclusive so far. On the other hand, recent results from PETRA seem to indicate the emergence of a three-jet contribution in the continuum region of e^+e^- annihilation as expected from QCD ¹⁾.

Assuming this indication will survive further scrutiny, one still has to address the question of the gluon self-coupling. Two issues are involved :

- i) a qualitative one : does the gluon self-coupling exist at all ?

- ii) a quantitative one : is this self-coupling, up to trivial group theoretical factors, equal to the quark-gluon coupling, as prescribed by QCD, or is it different ? As pointed out by the authors of Ref. 2), such an equality constitutes a rather important test of the gauge character of the underlying theory.

There are, in principle, two ways to test this equality.

- A) One looks for an effect which can be generated only by the gluon self-coupling and which vanishes otherwise. This procedure was adopted in Ref. 2). Unfortunately the authors found that the effect which they were studying was so uncomfortably small that one could very probably not measure it.
- B) One measures a quantity which receives contributions both from the gluon-quark and the gluon-gluon coupling and tries to separate them. Such a procedure might not be as spectacular as the first one and it certainly requires some more confidence in the theory, but it can nevertheless be handled. Tests of this kind have been suggested for deep inelastic lepton-nucleon scattering and for inclusive hadron spectra in e^+e^- annihilation³⁾.

In this note we want to propose another test, along analogous lines, arguing that such an important feature of the theory requires and deserves as many independent checks as possible.

In a recent paper⁴⁾ we have shown how one can calculate the multiplicity of charm (or any other heavy flavour), generated in the decays of a heavy resonance formed in e^+e^- annihilation. Taking the results obtained there as a starting point for further analysis, we will extend and improve upon them with the explicit aim of providing a laboratory to test, quantitatively, the equality of quark-gluon and gluon-gluon coupling.

The paper will be organized as follows : in Section 2 we will summarize and extend the results of Ref. 4) derived in QCD, considering both the case of an asymptotically free theory and of a theory with a small ultra-violet stable fixed point. In Section 3 we will apply the same procedure to a theory with no gluon self-coupling. Finally, Section 4 contains the conclusions and some comments on a possible extension of the analysis presented here.

2. - CHARM MULTIPLICITY IN QCD

It was discussed in Refs. 4) and 5) how one calculates the charm multiplicity generated in e^+e^- annihilation on- and off-resonance, summing all orders in leading logarithms. The on-resonance case has the considerable advantage that the effect thus calculated is much bigger than in the off-resonance case and that it does not have to compete with a large background.

The triple gluon vertex enters the calculation of the charm multiplicity at second order ^{4),5)} (Fig. 1). Summing to all orders in the leading log approximation, one finds in an asymptotically free QCD, using the modified Bessel function K_0 ⁴⁾ :

$$N_c^I(M_R) \simeq \frac{6}{b z_0} \frac{K_0(\xi)}{K_0(z_0)} \quad (1)$$

with $b \equiv 33 - 2N_f$, $N_f \equiv$ number of active flavours,

$$z_0 \equiv \sqrt{\frac{48 c_g}{b} \log \frac{M_R^2}{\Lambda^2}}, \quad \xi \equiv \sqrt{\frac{48 c_g}{b} \log \frac{4m_c^2}{\Lambda^2}}$$

In a QCD with an ultra-violet stable fixed point α_s one obtains, on the other hand :

$$N_c^I(M_R) \simeq \sqrt{\frac{\alpha_s}{16 c_g \pi}} \left[\exp\left(\sqrt{\frac{\alpha_s c_g}{\pi}} \log \frac{M_R^2}{4m_c^2}\right) - \exp\left(-\sqrt{\frac{\alpha_s c_g}{\pi}} \log \frac{M_R^2}{4m_c^2}\right) \right] \quad (2)$$

One can use the total hadronic width of the resonance to fix Λ or α_s , respectively, which mainly reflects the (heavy) quark-gluon coupling. In principle, one could extract them from other reactions like deep inelastic scattering ; yet this requires some care ⁶⁾ and therefore we prefer to normalize the coupling within one reaction.

Then there are still two parameters left : m_c and c_g . m_c is not necessarily identical to the mass of the charmed quark as it might be suggested by other arguments, like charmonium models, etc. One can easily imagine that non-leading contributions can, to a large degree, be absorbed into a redefinition of m_c , as happens in deep inelastic scattering with the parameter Λ . We treat m_c , therefore, as a free parameter.

If and when one finds another resonance, like toponium ($t\bar{t}$), one can form the rescaled multiplicity $R_c \equiv N_c(t\bar{t})/N_c(Y)$, from which any m_c dependence drops out in the leading log approximation :

$$R_c^I = \frac{z_Y}{z_{t\bar{t}}} \frac{K_0(z_Y)}{K_0(z_{t\bar{t}})} \quad (3)$$

where

$$z_R \equiv \sqrt{\frac{48 c_g}{b} \log \frac{M_R^2}{\Lambda^2}}$$

and for fixed point coupling α_s :

$$R_c^I \approx \left(\frac{M_{t\bar{t}}^2}{M_Y^2} \right)^{\sqrt{\frac{48 c_g}{b}}} \quad (4)$$

Thus only one parameter is left open, namely c_g , describing the triple-gluon vertex. In QCD

$$c_g(\text{QCD}) = 3 \quad (5)$$

In Fig. 2 we have plotted the ratio R_c^I (i.e., for a theory exhibiting asymptotic freedom) for three values of c_g , namely $c_g = 2, 3$ and 4 . One sees that even a 30% deviation from the QCD value leads to clearly different curves. Analogous results hold for R_c^{II} , Eq. (4). Furthermore, one should note the following : even if one can study N_c only on one resonance, like Y , and one has therefore to deal explicitly with m_c , one can reasonably argue that $m_c \sim O(1.5 \text{ GeV})$ and even deviations $\Delta m_c \sim O(500 \text{ MeV})$ do not affect the results in Eqs. (1), (2) very much, since they enter through a logarithm, whereas c_g enters $\sim \sqrt{c_g}$.

3. - CHARM MULTIPLICITY IN "QED_c"

One might wonder how the results are affected if one assumes the gluon self-coupling to vanish, yet considers other higher order corrections, at least in the leading log approach. Colour is thus contained only as a global symmetry in such a model theory, which we will denote by "QED_c".

Assuming only ladder diagrams to contribute and furthermore an ultra-violet stable fixed point α_s to exist - which is probably the best one can hope for in an Abelian gauge theory - one finds :

$$R_c \equiv \frac{N_c^{\pi}(t\bar{t})}{N_c^{\pi}(Y)} \approx \left(\frac{M_{t\bar{t}}^2}{M_Y^2} \right)^{\sqrt{\frac{2\alpha_s}{9\pi}(N_f-1)}} \quad (6)$$

where N_f = number of active flavours.

Expression (6) is quite different from the QCD results, Eqs. (3) and (4). This is due mainly to two reasons :

- i) gluons carry a large colour charge, therefore their self-interaction $g \rightarrow gg$ contains the colour factor $c_g = 3$ for $SU(3)$. On the other hand the analogous factor for $q \rightarrow qg [g \rightarrow q\bar{q}]$ is $c_F = \frac{4}{3} [T_F = \frac{1}{2}]$;
- ii) as discussed in Ref. 4), such an Abelian gauge theory contains fewer large logarithms, which therefore sum up to a smaller power.

Unfortunately, one cannot also show ^{4),5)} that in such a theory only ordinary (uncrossed) ladder diagrams contribute. Thus, in principle, at least, crossed diagrams could greatly off-set the expression (6).

We suggest the following way to tackle this problem : comparing the diagrams which generate $c\bar{c}$ pairs with gluon self-coupling (Fig. 3a) and without (Fig. 3b), one sees immediately that in the former case many gluons and only one $c\bar{c}$ pair is produced, whereas in the latter case many $q\bar{q}$ pairs - $c\bar{c}$ or not - are generated. Thus one sees immediately that if one fixes parameters in such a way to obtain $\langle N_{QCD} \rangle \simeq \langle N_{QED} \rangle$, then the multiplicity distribution is rather different ; in particular $\langle N_{QCD}^2 \rangle \neq \langle N_{QED}^2 \rangle$.

In QCD $\langle N_c \rangle \simeq \langle N_c^2 \rangle$ in the leading log approach, deviations being generated by non-leading terms.

In the Abelian gauge theory "QED_c", discussed as a model here, the situation is different. To calculate $\langle N_c^2 \rangle$ it is more convenient to sum the leading logarithms directly instead of using the approach of Refs. 4) and 5), namely to proceed via an integral equation.

To order $(\alpha_s^2)^{m+1}$, meaning in "QED_c" that there are $(m+1)$ $q\bar{q}$ pairs generated, of which v ($0 \leq v \leq m+1$) are $c\bar{c}$ pairs, one obtains (Fig. 3b) :

$$N_{c,(m+1)} \approx \frac{\alpha_s^2}{2\pi} \frac{\left(\log \frac{M_R^2}{4m_c^2}\right)^{3m+1}}{(3m+1)!} F_m \sum_{v=0}^m K^{3m} (N_f-1)^{m-v} [(v+1)-v] \quad (7a)$$

$$N_{c,(m+1)}^2 \approx \frac{\alpha_s^2}{2\pi} \frac{\left(\log \frac{M_R^2}{4m_c^2}\right)^{3m+1}}{(3m+1)!} F_m \sum_{v=0}^m K^{3m} (N_f-1)^{m-v} [(v+1)^2 - v^2] \quad (7b)$$

where $K^3 = 2\alpha_s^2/9\pi^2$. The coefficients F_m contain possible contributions from crossed diagrams which we do not attempt to calculate here ; yet they have to be the same for N_c and N_c^2 . $F_m = 1$ for uncrossed ladder diagrams and, in any case, $F_0 = 1$.

The subtraction in the last square bracket of Eqs. (7a) and (7b) is due to the virtual loop contributions, as can be most easily derived via the Lee-Kinoshita-Nauenberg theorem on infra-red singularities ⁴⁾.

Diagrams containing two (or even three) "towers" of ladders (Fig. 4) do not contribute in the leading log approximation, when applied strictly, since they are suppressed by $\sqrt{\log(M_R^2/4m_c^2)}$ relative to diagrams à la Fig. 3b (this, by the way, would not be true if one calculates N_c^2 in QCD). It will become apparent later on that including them, in any case, would not change the conclusions.

Summing Eqs. (7a), (7b) finally over m one obtains

$$\frac{\langle N_c^2 \rangle}{\langle N_c \rangle} \approx 1 + \frac{2 \sum_{m=0}^{\infty} \frac{\left(\log \frac{M_R^2}{4m_c^2}\right)^{3m}}{(3m+1)!} K^{3m} F_m \sum_{v=0}^m (N_f-1)^{m-v} v}{\sum_{m=0}^{\infty} \frac{\left(\log \frac{M_R^2}{4m_c^2}\right)^{3m}}{(3m+1)!} K^{3m} F_m \sum_{v=0}^m (N_f-1)^{m-v}} \quad (8)$$

which, mainly due to the coefficients $\sim 1/(3m+1)!$, is a swiftly converging expression, if the "fudge factors" F_m stay bounded or do not increase too rapidly with m .

In such a case one finds

$$\frac{\langle N_c^2 \rangle}{\langle N_c \rangle} \simeq 1 + \frac{\alpha_s^2}{54\pi^2} \left(\log \frac{M_R^2}{4m_c^2} \right)^3 F_1 \quad (9)$$

which, for, e.g., $M_R \sim 30$ GeV, $m_c \sim 1.5$ GeV, $\alpha_s \sim 0.2$, is a number very close to unity : $\langle N_c^2 \rangle / \langle N_c \rangle \sim 1 + 0.008 \times F_1$, as long as $F_1 \lesssim 0(10)$. If F_1 should be much larger, then multiple charm production is rather sizeable. If, on the other hand, no clear sign of multiple charm production is found (which one might expect anyway), one can turn the argument around and conclude that F_1 cannot be large.

If the "fudge factors" F_m should increase drastically with m to make their effect felt in N_c , then this happens even more for N_c^2 and $(\langle N_c^2 \rangle - \langle N_c \rangle^2) / \langle N_c \rangle \sim 0(1)$. Again non-observation of multiple charm production would show this not to be the case.

These, so far, rather qualitative statements are now transformed into more quantitative ones : comparing N_c^{II} , Eq. (2), written as a power expansion, and N_c ("QED_c"),

$$N_c^{II} \simeq \frac{\alpha_s}{2\pi} \log \frac{M_R^2}{4m_c^2} \sum_{m=0}^{\infty} \frac{\left(\frac{3\alpha_s}{\pi} \log^2 \frac{M_R^2}{4m_c^2} \right)^m}{(2m+1)!} \quad (10)$$

$$N_c('QED_c') \simeq \frac{\alpha_s}{2\pi} \log \frac{M_R^2}{4m_c^2} \sum_{m=0}^{\infty} \frac{\left(\frac{2\alpha_s^2}{9\pi^2} \log^2 \left(\frac{M_R^2}{4m_c^2} \right) \right)^m}{(3m+1)!} F_m \sum_{\nu=0}^m (N_f-1)^{m-\nu} \quad (11)$$

one can crudely, yet adequately estimate that

$$F_1 \sim 40 \quad (12)$$

is required to obtain $N_c^{II} \sim N_c$ ("QED_c") for $M_R \sim 30$ GeV, $\alpha_s \sim 0.2$.

Yet then Eq. (12) leads to

$$\frac{\langle N_c^2 \rangle}{\langle N_c \rangle} \sim 1.32$$

If conversely $(\langle N_c^2 \rangle / \langle N_c \rangle) \sim 1$, as might be expected any-way, then

$$\langle N_c^2 \rangle \gg \langle N_c('QED_c') \rangle$$

Thus observation, or more correctly, non-observation of multiple charm production is a rather useful instrument to distinguish between models of the strong interactions.

An obvious word of caution : such a test does not make much sense at the Y resonance where phase space alone almost rules out multiple charm production. Yet for a resonance with $M_R \sim 30$ GeV there should be sufficient phase space available to perform such a test of the dynamics.

4. - CONCLUSIONS

In this letter we have discussed how a measurement of the charm multiplicities $\langle N_c \rangle$ and $\langle N_c^2 \rangle$ in the strong decays of heavy vector meson resonances can make a distinction between various theories of the strong interactions. In particular it can give further evidence for (or against) the existence of the gluon self-coupling and also on its size. This would constitute a rather important test of the gauge character of the underlying theory.

The results were derived in the leading log approach. One might justly wonder about a possible influence of non-leading terms. We have argued that their main contributions might just renormalize m_c , which is thus taken to be a free parameter setting a scale for logarithms which occur. If nature is kind enough to provide us with another resonance, which is even heavier than the Y , one can study the ratio of the charm multiplicities on the two resonances. It was shown that the dependence on m_c drops out from such a ratio.

If nature, on the other hand, behaves towards us in a less benign fashion, i.e., not presenting us with another, even heavier, flavour resonance, then enterprising spirits might and should feel tempted to calculate the non-leading terms also.

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FIGURE CAPTIONS

- Fig. 1 Charm multiplicity in second order QCD.
- Fig. 2 Rescaled charm multiplicity $R_c^I \equiv N_c^I(t\bar{t})/N_c^I(Y)$ versus the rescaled resonance mass $M_{t\bar{t}}/\Lambda$ for three different values of c_g :
a) $c_g = 4$; b) $c_g = 3$, QCD ; c) $c_g = 2$.
- Fig. 3 Higher order contributions to the charm multiplicities in
a) QCD, b) "QED_c".
- Fig. 4 A special non-leading contribution to $\langle N_c^2 \rangle$ in "QED_c".

FIG. 1

FIG. 3a

FIG. 3b

FIG. 4

FIG. 2