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THE PRODUCTION OF NEUTRAL VECTOR MESONS BY BREMSSTRAHLUNG
IN ELECTRON - POSITRON COLLIDING BEAMS

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A B S T R A C T

We study the bremsstrahlung production of the ρ meson in the reaction $e^+e^- \rightarrow e^+e^-\rho (\rightarrow e^+e^-\pi^+\pi^-)$. This reaction gives a $C = -1$ background which complicates the study of $C = +1$ two-photon processes at the new colliding beam facilities. The cross-section for the reaction rises from approximately 0.3 nb to 0.7 nb as the beam energy increases from 2 GeV to 15 GeV. From a study of the distributions of the final leptons and pions, we find a suitable choice of cuts which will reduce the event rate down to a small fraction of R. It is not possible to attribute the three-prong events seen at DELCO and PLUTO to this particular production mechanism.

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1. INTRODUCTION

With the introduction of high-energy machines such as PETRA at DESY [1] and PEP at SLAC [2], some new and interesting features of e^+e^- physics are about to be investigated. It is well known that the cross-section for two-photon production processes dominates over the cross-section for one-photon annihilation at high energies. Hence two-photon physics [3-7] will be extensively investigated by various experimental groups. The importance of such a study of $C = +1$ processes necessitates a careful investigation of possible backgrounds. For instance, the production via bremsstrahlung of $C = -1$ states yields cross-sections which also increase logarithmically with s , the square of the c.m. energy. This problem has been examined in detail in the first paper in ref. 4, where it was found that the cross-section for $\mu^+\mu^-$ production in normal two-photon reactions is much larger (over most of the allowed kinematic region) than the cross-section for the bremsstrahlung of $\mu^+\mu^-$ pairs. However, the corresponding result is not necessarily true for the production of hadrons because resonances can be produced in the $C = -1$ state and enhance the bremsstrahlung cross-section relative to the $C = +1$ non-resonant cross-section. Previous authors [8] have already investigated the bremsstrahlung of ρ mesons analytically, and found an asymptotic cross-section (in the narrow width approximation) of the form

$$\sigma(s) = \frac{4\alpha^4}{9m_\rho^2} \left(\frac{m_\rho}{\Gamma_\rho} \right) \left(1 - \frac{4m_\pi^2}{m_\rho^2} \right)^{\frac{3}{2}} \ln \left(\frac{m_\rho}{m_e} \right) \ln \left(\frac{s^2}{m_e m_\rho^3} \right) \quad (1.1)$$

where the notation is self-evident. Clearly analogous results are valid for the production of the ω , ϕ , and J/ψ states. Owing to the mass scale in the above equation, these other cross-sections are generally smaller. The ω is an exception but it leads to states with $\pi^+\pi^-\pi^0$ and these are difficult to reconstruct. In this paper we concentrate on the ρ production, i.e. we calculate the cross-section for $e^+e^- \rightarrow e^+e^-\rho \rightarrow e^+e^-\pi^+\pi^-$ in the energy range up to beam energies $E = 15$ GeV and then study the spectra of the final particles with a view to separating $C = -1$ and $C = +1$ processes.

We find that the ρ production cross-section is smaller than the cross-section expected for $\pi^+\pi^-$ production in two-photon reactions. As noted earlier [5,6], this particular background constitutes a negligible fraction of the two-photon cross-section in the particular kinematical region where both the electrons are detected at small forward angles ($\theta < \sqrt{m_e/E}$). Nevertheless it can become a dangerous background [9] when we depart from this kinematic situation; for example, consider the case where one electron is scattered at a large angle and the other one remains close to the beam pipe. We are forced to consider such cases for the following reasons:

- i) There are theoretical arguments for being interested in this particular region [7].
- ii) There are experiments where the detection of electrons in the forward region is not possible; hence the large-angle electron can be measured but the other electron may possibly have to be reconstructed. If there are contributions from both C-odd and C-even states, this may be a difficult task.
- iii) With the advent of new detectors with forward tagging systems, we will probably wish to analyse all events and therefore we need to know the contribution from the bremsstrahlung cross-section. After all, the cross-section is increasing with beam energy. Furthermore, the increased accuracy of the measurement of $R = \sigma(e^+e^- \rightarrow X)/\sigma(e^+e^- \rightarrow \mu^+\mu^-)$ makes it necessary to know all the possible backgrounds.
- iv) An earlier study of the reaction $e^+e^- \rightarrow e^+e^-\rho$ at a beam energy of 3 GeV found that this reaction could be a source of serious background in the analysis of two-photon final states [9].

In connection with point (ii) we should mention that DELCO has observed some three-prong events with an e^- or e^+e^- in the final state [10]. Since the detector covers only the large-angle range, we need to know if these particular events can be explained by the bremsstrahlung reaction. Connected with point (iii) there has recently been a preliminary report [11] on their two-photon candidates from

the group working with the PLUTO detector at PETRA. When the $\pi^+\pi^-$ invariant mass is reconstructed they have evidence for an enhancement in the region of the ρ . It is therefore imperative to know whether the signal is from the reaction we are investigating or whether it is from the inclusive production of ρ mesons in photon-photon collisions.

All the above reasons have prompted us to complete a calculation of the basic reaction in the energy region from 2 GeV to 15 GeV. This calculation is a natural extension of previous work done on the electromagnetic production of ρ mesons in neutrino collisions [12]. In the latter case the hadronic production of the ρ is clearly more important, but in the present case the electromagnetic mechanism is the only possible $C = -1$ production process.

In the next section we present the calculation of the bremsstrahlung production process for neutral vector mesons at energies corresponding to SPEAR, DORIS, CESR, PEP, and PETRA. We compare our result for the energy dependence of the total cross-section with the asymptotic results obtained earlier. Then, in Section 3, we add on the decay $\rho^0 \rightarrow \pi^+\pi^-$ and present the differential distributions in the variables of interest. We examine in detail the strong angular dependence between the final electron and positron. Since we do all the integrations numerically, we can easily incorporate experimental cuts. We examine the cross-section expected for the DELCO and PLUTO experiments mentioned above. In the final section we summarize our results and describe possible characteristic signatures that can be used to identify the ρ^0 bremsstrahlung reaction.

2. TOTAL CROSS-SECTION

In this section we present the details and the results of the calculation of the total cross-section for the reaction

$$e^-(l_1) + e^+(p_1) \rightarrow e^-(l_2) + e^+(p_2) + \gamma(k) \quad (2.1)$$

The diagrams that need to be considered are shown in fig. 1 (the scattering diagrams) and fig. 2 (the annihilation diagrams). Since the cross-section for the s-channel annihilation process decreases as s^{-1} we neglect the diagrams in fig. 2.

For convenience we define the variables

$$p_y = p_I - p_F, \quad p_x = p_I + p_F, \quad l_y = l_1 - l_2 \quad \text{and} \quad l_x = l_1 + l_2.$$

The sum of the diagrams (1a + 1b) and (1c + 1d) are separately gauge invariant.

The matrix element can be written as $\mathcal{M} = \mathcal{M}_I + \mathcal{M}_{II}$, where $\mathcal{M}_I = \mathcal{M}_a + \mathcal{M}_b$ is the contribution to the matrix element of the diagrams 1a and 1b, and $\mathcal{M}_{II} = \mathcal{M}_c + \mathcal{M}_d$ is the contribution from the diagrams 1c and 1d. The expressions for the matrix elements are (in the Pauli metric)

$$m_I = \frac{e^4 F_V}{M_V^2 p_y^2} \bar{v}(p_I) \gamma_\mu v(p_F) \bar{u}(l_2) \left[\frac{(2l_{2\alpha} + \gamma_\alpha \not{k})}{k^2 + 2l_2 \cdot k} \gamma_\mu + \gamma_\mu \frac{(2l_{1\alpha} - \not{k} \gamma_\alpha)}{k^2 - 2k \cdot l_1} \right] u(l_1) \varepsilon_\alpha^V \quad (2.2)$$

and

$$m_{II} = -\frac{e^4 F_V}{M_V^2 l_y^2} \bar{u}(l_2) \gamma_\mu u(l_1) \bar{v}(p_I) \left[\gamma_\mu \frac{(\not{k} \gamma_\alpha + 2p_F \gamma_\alpha)}{k^2 + 2k \cdot p_F} + \frac{(2p_{I\alpha} - \gamma_\alpha \not{k})}{k^2 - 2p_I \cdot k} \gamma_\mu \right] v(p_F) \varepsilon_\alpha^V \quad (2.3)$$

where ε_α^V is the polarization vector for the vector meson. The Lagrangian for the γ -V interaction is taken to be $\mathcal{L}^{\text{int}} = e F_V \varepsilon_\mu^V \varepsilon_\mu^\gamma$. Obviously the γ -V coupling is not gauge invariant, but it reproduces the pole term from the gauge-invariant interaction $F_{\mu\nu}^V F_{\mu\nu}^\gamma$. The vector meson coupling to the photon can be expressed in terms of the decay width $\Gamma_{V \rightarrow e^+ e^-}$ via [13]

$$\Gamma_{V \rightarrow e^+ e^-} = \frac{4\pi\alpha^2}{3} \frac{F_V^2}{M_V^3} \quad (2.4)$$

where α is the fine structure constant and M_V is the vector meson mass. The rest of the calculation is straightforward. The total cross-section is given by the standard formula

$$\sigma = \frac{1}{|v_1 - v_2|} \frac{1}{2E_1} \frac{1}{2E_2} \frac{1}{(2\pi)^5} \int \frac{ds_1 ds_2 dt_1 dt_2}{\sqrt{-\Delta^4(l_1, p_I, l_2, p_F)}} \frac{\pi}{16\lambda^{\frac{1}{2}}(s, m_1^2, m_2^2)} |\mathcal{M}|^2 \quad (2.5)$$

where $s_1 = (p_F + k)^2$, $s_2 = (\ell_2 + k)^2$, $t_1 = (p_F - p_I)^2$, $t_2 = (\ell_2 - \ell_1)^2$ are the usual invariants [14]. We calculated $\overline{|\mathcal{M}|^2} = |\mathcal{M}_I + \mathcal{M}_{II}|^2/4$, where the bar denotes spin averaging, using the algebraic computer program SCHOONSCHIP [15]. There is no point in giving the result here. Now we need to perform the four-dimensional integral for the cross-section.

The dominant term in the square of the matrix element is of the type $c_1 t_1^{-2} + c_2 t_2^{-2}$. The sharp peaking in t_1 and t_2 is due to the small mass of the electron and makes it difficult to carry out normal Monte Carlo integrations. A further complication is that there is double peaking in both the variables s_1 and s_2 . Hence, while calculating the total cross-section, we exploited the symmetry of the process and rearranged the square of the matrix element into two parts. In each part the dominant term was proportional to either t_1^{-2} or t_2^{-2} so we could use the set $\ln t_1$, $\ln s_2$, t_2 , and s_1 , or $\ln t_2$, $\ln s_1$, t_1 , and s_2 , respectively, as integration variables. Then there was no problem with the Monte Carlo integration. The over-all error in the calculation is about 2% at low energies and 5% at high energies.

We present our results below for the ρ meson, which is the easiest vector meson to identify experimentally via the decay mode $\rho \rightarrow \pi^+ \pi^-$. The production cross-sections for the ω , ϕ , and J/ψ can be estimated by scaling our results according to eq. (1.1). However, near threshold there will be a further suppression due to the larger masses of the particles involved. Actually it is possible to calculate the production of all $C = -1$ hadron states with arbitrary mass M_X , in terms of the cross-section for $e^+ e^- \rightarrow X$, and such a calculation is under way. However, the vector mesons will certainly give the dominant contributions.

We have calculated the cross-section at beam energies of 2, 5, 8, and 15 GeV. The results are presented in the first column of table 1. The results for the asymptotic calculations in ref. 8, are given for comparison in the second and third columns, respectively. The first set of numbers in the third column is the result of a calculation in the zero width approximation, and the second set reflects the effect of the inclusion of the contribution from the threshold region.

As can be seen from table 1, the asymptotic expression overestimates the cross-section at lower energies, but at higher energies, results for the zero width approximation agree with our numerical calculation. We also compare our results with the cross-section for the reaction $e^+e^- \rightarrow \mu^+\mu^-$ and with the cross-section for a representative two-photon process, namely $e^+e^- \rightarrow e^+e^-\pi^+\pi^-$ (see ref. 4) in table 2 and fig. 3.

Suppose that $R = \sigma(e^+e^- \rightarrow X)/\sigma(e^+e^- \rightarrow \mu^+\mu^-) < 10$ in this energy region. Then fig. 3 demonstrates that a knowledge of both $C = +1$ two-photon cross-sections and $C = -1$ bremsstrahlung cross-sections are probably necessary if R is to be measured to an accuracy of 1%. We say 'probably' because, as mentioned in the introduction, the bremsstrahlung production is only very large in certain kinematical regions. For that purpose we need to study the effect of various acceptance cuts on the total cross-section. We present these results in the next section.

3. DISTRIBUTIONS

Studying only the production process is of rather academic interest so we have added the decay $\rho^0 \rightarrow \pi^+\pi^-$ and examined the lepton and pion spectra. Since only one of the two photons is close to its mass shell in the case of bremsstrahlung production, one of the two final leptons can come out at a 'large' angle. In the two-photon reaction both photons can be close to their respective mass shells, which is the case when the final e^- and e^+ emerge close to the beam pipe. We first present the exact spectra -- no cuts have been made. Hence we have carefully added the contributions from the two parts of the matrix element. Later we will discuss the effects due to the energy and angle cuts. In that case it is generally possible to integrate the cross-section in one piece because the really large contributions have been cut out. When we talk about directions we define the z -axis to be along the direction of the incident electron.

In fig. 4 we give the differential spectra in the angle of the electron for beam energies of $E = 2, 8, \text{ and } 15 \text{ GeV}$. The curve for $E = 5 \text{ GeV}$ has been omitted to make the figure more legible. However, the average angles are all given in

table 3. We see that the electron comes out closer to the beam pipe with increasing beam energy. The backward peak, which is caused by the ρ^0 bremsstrahlung from the electron, decreases with increasing beam energy. The angular distribution for the positron is just the mirror image of that of the electron. There is a small peak in the forward direction which diminishes when the energy increases. Most of the positrons therefore come out at $\theta \simeq 180^\circ$, i.e. along the direction of the incident positron beam. Very few electrons or positrons are emitted at large angles, say between 45° and 135° .

Next we show in fig. 5 the distributions in the electron energy at $E = 2, 5, 8$, and 15 GeV. Depending upon which particle radiates, the electron either carries very little energy or retains all its energy. As a result, on the average, it carries one half the beam energy, as can also be seen in table 3. The positron energy distribution is identical to that of the electron but inverted, i.e. when the e^- is slow, the e^+ is fast.

We do not give explicitly the energy and angular distributions for the ρ meson because only the decay pions are identified. However, from the above discussion there are several qualitative features which are worth mentioning. First of all the large mass of the ρ means that in general it tends to travel along the beam direction. Hence if the e^- is produced with a large energy and comes out at $\theta \simeq 0^\circ$, then the ρ comes out at $\theta \simeq 180^\circ$ and the slow e^+ recoils against it to emerge at $\theta \simeq 0^\circ$. To show this more clearly, we give the differential distribution in the e^+e^- opening angle in fig. 6. This distribution has a peak at 0° but, with increasing beam energy, the distribution also develops a peak at 180° . This is clearly due to the availability of more c.m. energy, so the e^+ or e^- does not need to recoil against the ρ but can travel along with it.

The transverse momentum spectrum of the ρ , shown in fig. 7 for $E = 2$ and 15 GeV, demonstrates that the k_T is very limited. This is the transverse momentum along the beam direction. Of course the distribution has a larger tail at the higher beam energy because more phase space is available. Thus we expect that

$\langle E_\rho \rangle \approx \frac{1}{2}\sqrt{s} \approx E$ and, since the k_T is so limited, this means that $k_z \approx E$. As a result of this the pions tend to have energy $\sim E/2$ and also travel predominantly near the directions $\theta = 0^\circ$ and $\theta = 180^\circ$.

In fig. 8 we present the distributions in the angle of one of the pions at the four energies mentioned previously. The peaks in these distributions become narrower as the beam energy increases. At $E = 2$ GeV, approximately 15% of the pions are produced in the range $45^\circ < \theta_\pi < 135^\circ$, while at $E = 15$ GeV only 2% of the pions emerge in the region $25^\circ < \theta_\pi < 155^\circ$. Almost all the cross-section at this energy comes from the region $\theta_\pi < 10^\circ$ or $\theta_\pi > 170^\circ$. At $E = 2$ GeV the distribution in θ_π is peaked at finite angles. This feature persists at $E = 5$ GeV and then disappears at the higher energies. The effect is purely kinematical and demonstrates that the ρ is produced almost at rest at $E = 2$ GeV. When more energy is available the ρ and the decay pions tend to travel along the beam axis. The energy spectra of the pions are rather flat over the range $m_\pi < E_\pi < E/2$ except for the cut-off at the boundaries. We do not give the distributions but list the average p_π in table 3.

In fig. 9 we give the distribution in ψ , the non-coplanarity angle, defined by

$$\cos \psi = \frac{(\vec{q}_1 \times \vec{p}_1) \cdot (\vec{q}_2 \times \vec{p}_2)}{|\vec{q}_1 \times \vec{p}_1| |\vec{q}_2 \times \vec{p}_2|}$$

where $\vec{q}_1$ and $\vec{q}_2$ are the direction vectors of the outgoing pions. This is the angle between the planes containing the beam axis and the pions [5]. For the two-photon process $e^+e^- \rightarrow e^+e^-\pi^+\pi^-$ this distribution has a tail extending to large ψ so that approximately one half of all pion pairs have a coplanarity angle $> 12^\circ$. The distribution for the bremsstrahlung reaction has also a large contribution at large ψ . We only show this distribution at $E = 2$ GeV because it is not very sensitive to an increase in the beam energy.

To understand more fully the correlation between the variables, we have drawn up two scatter plots. In figs. 10 and 11 we give the scatter plot of θ_{e^-} versus

θ_{e+} and θ_{e-} versus E_{e-} , respectively, at a beam energy of $E = 2$ GeV. An increase in beam energy only pushes more points into the corners of the scatter plots, and we do not need to present them here. The scatter plot of θ_{e-} versus θ_{e+} , which is symmetrical about the line from $\theta_{e-} = 180^\circ$ to $\theta_{e+} = 180^\circ$, shows that about 20% of the total cross-section comes from the region where both the electron and the positron emerge within 15° of the beam pipe in the same direction. At $E = 15$ GeV this fraction is reduced to about 10%. The scatter plot also shows that about 30% of the cross-section comes from the region where the electron is scattered within 15° of the beam pipe and the positron is in the region $15^\circ < \theta < 165^\circ$. This fraction remains roughly constant with increasing beam energy. The scatter plot in fig. 11 confirms what we have seen previously. All the high-energy electrons get scattered close to the beam pipe. About 12% of the low-energy electrons $E < 0.4$ GeV get scattered in the region $35^\circ < \theta < 145^\circ$. Since $e^\pm$ identification is not easy at these low energies this is a possible problem. The special signature for the reaction is that most of the total cross-section comes from the region where a low-energy $e^\pm$ is scattered into a large angle and the other particle is emitted at a very small angle.

Before we discuss the signals for the DELCO and PLUTO detectors, let us present the electron energy spectra when there is a cut in the angle of the electron. In fig. 12 we present such distributions for a beam energy of 2 GeV and angular cuts such that $1^\circ < \theta_e < 179^\circ$, $15^\circ < \theta_e < 165^\circ$, and $35^\circ < \theta_e < 145^\circ$. The corresponding distributions for a beam energy of 15 GeV are given in fig. 13. As we can anticipate from our earlier discussion, all the high-energy electrons are steadily removed by these cuts, leaving only the soft electrons. The fraction of the cross-section lost by such cuts is given in table 4. A reasonable fraction survives, but the remaining electrons have very low energies.

In actual experiments both the electron and positron remain undetected if they emerge in a small angle about the beam direction. If we impose both cuts such that, for instance, $1^\circ < \theta_e < 179^\circ$ etc., then the asymmetrical distributions in figs. 12 and 13 become symmetrical again and the heights of the peaks at 0°

and 180° are much smaller. We can describe in words the other significant changes. Obviously the peaks at 180° in the θ_{e^-} distribution and at 0° in the θ_{e^+} distribution disappear completely because they are associated with the situation where the positron, namely the electron, is the more energetic, while the other particle has its minimum energy. The fact that the momentum transfers are now larger forces the ρ to emerge at a larger angle and therefore both peaks in the pion angular distribution move in slightly towards 90° .

For the DELCO detector, which has an angular acceptance $35^\circ < \theta < 145^\circ$, there is essentially zero probability that both the e^- and the e^+ will be detected. If we take the other situation, that one e^+ (or e^-) is detected and the other one goes into the forward (or backward) region, then, at a beam energy of 2 GeV, we see (fig. 10) that about 20% of the cross-section survives. This yields a cross-section of 60 nb, which is of the order of 0.01 R and comparable to the signal seen at DELCO. However, when further cuts are made to ensure that the pions are also detected, then we require that $45^\circ < \theta_{\pi^\pm} < 135^\circ$, which reduces the cross-section down to ~ 4 pb. Hence the reaction $e^+e^- \rightarrow e^+e^-\rho$ cannot possibly explain the events observed in the DELCO detector.

The PLUTO detector at DESY has a small-angle tagging system (SAT) which covers the region 20-70 mrad, and a large-angle tagging system (LAT) which covers the region 70-250 mrad. In this situation all the $e^\pm$ pairs which emerge at an angle $> 1.15^\circ$ can be identified. Their trigger demands that a particle enter the tagging system with an energy $2 \text{ GeV} < E_{\text{tag}} < E$, when the beam energy is 8.5 GeV, and two charged tracks emerge at large angles. These latter tracks must correspond to one particle with $p_T > 0.125 \text{ GeV}$ inside the region $42^\circ \leq \theta \leq 138^\circ$, while the second particle has $p_T > 0.100 \text{ GeV}$ inside the region $30^\circ \leq \theta \leq 150^\circ$. At this beam energy a possible scenario for these events, if they come from the reaction $e^+e^- \rightarrow e^+e^-\rho$, is as follows. For simplicity we assume the e^- to be the leading particle, although everything is symmetrical in e^- and e^+ . Hence there is a fast e^- with $\theta_{e^-} < 1.15^\circ$, so it is undetected. The secondary e^+ must satisfy $1.15^\circ \leq \theta_{e^+} \leq 178.85^\circ$ and $E_{e^+} > 2 \text{ GeV}$. The cross-section at $E = 8.5 \text{ GeV}$ with only the

angular cuts is ~ 0.25 nb. The resulting positron angular distribution is very flat while its energy distribution is peaked at small energies. The fraction of the cross-section above $E_{e^+} = 2$ GeV is only 10%. Hence the actual cross-section which survives the tagging criterion is only 25 pb (times two for the symmetrical situation) = 50 pb. For those events which survive the above cuts there is only a 10% probability that a pion will emerge in the cone $30^\circ \leq \theta_\pi \leq 150^\circ$, because the fast e^+ 's are associated with directions close to the beam pipe. If we were to allow more low-energy e^+ events then they could come out at larger angles, which would also mean more pions at larger angles. If one pion is accepted then the other one will automatically satisfy the same angular cut because the angular distributions of the pions are essentially identical. Hence the total cross-section is at most ~ 3 pb (times two for symmetry) ~ 6 pb, which is much smaller than the preliminary number of 200 pb reported by the PLUTO Collaboration.

Clearly neither the DELCO nor the PLUTO signals can be explained by the bremsstrahlung mechanism. If the evidence for a ρ signal in the $\pi^+\pi^-$ mass spectrum remains at PLUTO, then the most likely explanation is that of ρ inclusive production in $\gamma\gamma$ collisions.

4. CONCLUSIONS

Our analysis allows us to make the following conclusions.

- 1) The numbers in table 1 from our exact calculation agree with the answers from the asymptotic calculations. Hence there is good agreement on the magnitude of the ρ production cross-section.
- 2) From the cross-sections and the knowledge of the luminosities available at present colliding beam machines, we conclude that some detailed knowledge of the reaction $e^+e^- \rightarrow e^+e^-\rho$ is necessary in the study of one-photon and two-photon physics at high energies.
- 3) A characteristic signature for this reaction is an e^- and e^+ travelling in the same direction, close to the beam pipe, and with the ρ travelling in the opposite direction with energy $\approx E$. To actually measure this cross-section

we note that about 50% of it is in the region where one low-energy electron or positron is scattered at a large angle $> 25^\circ$. Also the coplanarity angle tends to be rather large. The distributions we have presented should be helpful in eliminating the background due to this process in present and future colliding beam experiments.

- 4) After angular and energy cuts have been made, the cross-section is not large enough to explain the events seen in the DELCO and PLUTO detectors, even though the *total* cross-section in this region is quite large.

As has been noted earlier, the background caused by the bremsstrahlung production of $C = -1$ states should be considered in the analysis of multihadron events. For this purpose it is necessary to study the production of $C = -1$ hadron states with arbitrary mass M_X . Such a calculation, using the cross-section of the process $e^+e^- \rightarrow X$, is possible and is in progress.

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Table 1

Production cross-sections for the reaction $e^+e^- \rightarrow e^+e^-\rho$, asymptotic and exact results. σ_A and σ_B denote the cross-section without and with the contribution from the threshold region.

Beam energy (GeV)	Numerical calculation σ_A (nb)	Asymptotic calculation a) σ_B (nb)	Asymptotic calculation b)	
			σ_A (nb)	σ_B (nb)
2	0.34	0.50	0.42	0.59
5	0.51	0.56	0.53	0.69
8	0.58	0.70	0.59	0.73
15	0.67	0.79	0.66	0.78

a) E.A. Choban, Yad. Fiz. 13 (1971) 624.

b) A.M. Altukhov, Yad. Fiz. 14 (1971) 391.

Table 2

Production cross-sections for the one-photon, two-photon, and bremsstrahlung processes

Beam energy (GeV)	$\sigma(e^+e^- \rightarrow \mu^+\mu^-)$ (nb)	$\sigma(e^+e^- \rightarrow e^+e^-\rho)$ (nb)	$\sigma(e^+e^- \rightarrow e^+e^-\pi^+\pi^-)$ (nb)
2	5.43	0.34	5.0
5	0.87	0.51	8.0
8	0.33	0.58	12.5
15	0.097	0.67	15.0

Table 3

The average values of measurable quantities in the reaction
 $e^+e^- \rightarrow e^+e^-\rho$

Beam energy (GeV)	$\langle \theta_{e^-} \rangle$ (deg)	$\langle \theta_{e^+} \rangle$ (deg)	$\langle E_- \rangle$ (GeV)	$\langle \theta_{e^+e^-} \rangle$ (deg)	$\langle k_T \rangle$ (GeV)	$\langle \psi \rangle$ (deg)	$\langle \theta_\pi \rangle$ (deg)	$\langle p_\pi \rangle$ (GeV)
2	60	128	1.0	71	0.17	24	90	1.0
5	47	141	2.6	96	0.26	35	88	2.6
8	40	147	4.1	108	0.35	44	86	4.1
15	18	151	8.0	134	0.61	53	86	7.2

Table 4

The fraction of the cross-section lost by cuts on the
angle of the electron at 1° , 15° , 35° , and 45° . Note
that this means that $1^\circ < \theta < 179^\circ$, etc.

Beam energy (GeV)	Fraction of electrons lost with a cut at			
	1°	15°	35°	45°
2	0.41	0.64	0.75	0.83
5	0.43	0.62	0.75	1.0
8	0.43	0.61	0.75	1.0
15	0.42	0.62	0.76	1.0

Figure captions

- Fig. 1 : Feynman diagrams for bremsstrahlung production of vector mesons in the t-channel scattering process.
- Fig. 2 : Feynman diagrams for the bremsstrahlung production of vector mesons in the s-channel annihilation process.
- Fig. 3 : Production cross-sections for the one-photon, two-photon, and bremsstrahlung processes.
- Fig. 4 : Differential spectra in the angle of the electron at beam energies $E = 2, 5, 8$, and 15 GeV.
- Fig. 5 : Differential spectra in the energy of the electron at beam energies $E = 2, 8$, and 15 GeV.
- Fig. 6 : Distributions in the opening angle $\theta_{e^+e^-}$ of the electron-positron pair at beam energies $E = 2, 5, 8$, and 15 GeV.
- Fig. 7 : Transverse momentum distributions for the ρ meson at beam energies $E = 2$ and 15 GeV.
- Fig. 8 : Differential spectra in the angle of the pion at beam energies $E = 2, 5, 8$, and 15 GeV.
- Fig. 9 : The distribution in the coplanarity angle ψ defined in the text. The beam energy is 2 GeV.
- Fig. 10 : Scatter plot of θ_{e^-} against θ_{e^+} at $E = 2$ GeV. The units are in degrees and the number of events is normalized to approximately one thousand.
- Fig. 11 : Scatter plot of θ_{e^-} against E_{e^-} at $E = 2$ GeV. The units are in degrees and GeV, and the number of events is normalized to approximately one thousand.
- Fig. 12 : The distributions in the electron energy with cuts on the electron angle. The beam energy is 2 GeV.
- Fig. 13 : Same as in fig. 12 at a beam energy of 15 GeV.

Fig. 1

Fig. 2

Fig. 3

Fig. 4

Fig. 5

Fig. 6

Fig. 7

Fig. 8

Fig. 9

Fig. 10

Fig. 11

Fig. 12

Fig. 13