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GAUGE HIERARCHY IN PRESENCE OF DISCRETE SYMMETRY

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ABSTRACT

We discuss the gauge hierarchy problem in theories that possess an additional discrete symmetry. As examples, we consider two models based on the gauge groups $U(1)_L \times U(1)_R$ and $SU(2)_L \times SU(2)_R \times U(1)$, with parity transformation as the discrete symmetry. By employing a minimal choice of Higgs multiplets there is no freedom, in the semi-classical approximation, to arrange for an arbitrary hierarchy. Either one has a left-right symmetric phase (no hierarchy) or a totally asymmetric phase (infinite hierarchy). It is shown that radiative corrections, à la Coleman-Weinberg, do not smooth out the transition region separating the two phases. A finite gauge hierarchy is not realized.

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1. INTRODUCTION

The presence of mass scales varying over several orders of magnitude seems unavoidable in theories that try to unify the strong and the electroweak interactions¹⁾. Attempts to obtain a gauge hierarchy in perturbation theory by employing explicit Higgs scalars are frustrated²⁾ due to large renormalization effects which are not controllable without imposing unnatural constraints³⁾ on the parameters of the theory. It has been pointed out however⁴⁾, that it may be possible for the full theory to display a hierarchy, even though a perturbation expansion fails to provide any hint on its existence. Weinberg⁵⁾ has recently performed an analysis that lends support to this idea.

Although it is important to know that the full theory can display any desired gauge hierarchy that we are likely to need, the information is of not much practical use, since the parameters of the full theory cannot be determined without completely solving the theory, a formidable task indeed! Thus, in order to have a better understanding of the gauge hierarchy problem, it is necessary to search for theories which display a natural hierarchy in some reliable calculable scheme, say in perturbation theory. It has been suggested⁶⁾ that if such a gauge theory exists, then it is likely to possess some additional symmetry.

In this paper we discuss the problem of hierarchical breaking in gauge theories which possess an additional discrete symmetry. For concreteness, we took the discrete symmetry to correspond to parity transformation. The gauge groups are taken to be $U(1)_L \times U(1)_R$ and $G = SU(2)_L \times SU(2)_R \times U(1)$ ⁷⁾. The latter group is a subgroup of the unified gauge group $SO(10)$ ⁸⁾, and it has recently been shown⁹⁾ that the left-right symmetry of the theory is required to undergo a hierarchical breakdown in order that the renormalized value of $\sin^2 \theta_W$ agrees with the experimental numbers^{*}).

It is convenient to parametrize the hierarchy in terms of the ratio of the masses of the gauge bosons associated with the left $[U(1)_L, SU(2)_L]$ and the right $[U(1)_R, SU(2)_R]$ gauge groups. By convention, we take $0 \leq Q = m_L/m_R \leq 1$, where, for the gauge group G , m_L and m_R refer to the masses of the electrically charged gauge bosons associated with $SU(2)_L$ and $SU(2)_R$ respectively.

^{*}) It seems attractive to suppose that parity violation in weak interactions¹⁰⁾ arises as a consequence of spontaneous breakdown¹¹⁾ of a left-right symmetric gauge theory.

A small mass ratio $Q(0 < Q \ll 1)$ can be put in by hand if one introduces sufficiently many Higgs multiplets. For instance, for the gauge group G , one could employ two Higgs doublets transforming non-trivially under $SU(2)_L$ and $SU(2)_R$ respectively, as well as a quadruplet carrying both $SU(2)_L$ and $SU(2)_R$ spinor indices. However, the Higgs potential then contains more than a dozen coupling parameters which makes it somewhat unappealing. Moreover, as indicated earlier on, a hierarchy obtained by suitable adjustments of parameters in the tree approximation has its limitations. Although Q is arranged to be hierarchically small in the tree approximation, it is strongly affected by radiative corrections. In the generic case, Q is expected to be of order $\sqrt{g^2}^{-2}$, where g is a typical gauge coupling.

The idea to be explored in this paper is to employ a Higgs Lagrangian which is so constrained that only an infinite gauge hierarchy is allowed at the tree level. We then investigate whether radiative corrections, à la Coleman-Weinberg¹²⁾, could break the remaining gauge invariance at a scale which is either unrelated or is very small compared to the large mass that characterizes the gauge symmetry breakdown at the tree level.

A Higgs system implementing this idea at the classical level is easily written down. For the gauge group G one employs just two Higgs doublets, transforming non-trivially under $SU(2)_L$ and $SU(2)_R$ respectively. For $U(1)_L \times U(1)_R$, one works with two complex scalar fields which transform non-trivially under $U(1)_L$ and $U(1)_R$ respectively. Thus it is seen that the constrained Higgs system corresponds, in both cases, to a minimal Higgs system which can, in principle, cause a hierarchical symmetry breakdown of the gauge symmetry^{*)}. With the above choice of Higgs system, no freedom is left to arrange for a finite hierarchy at the tree level. The theory may exist in only one of two possible phases, depending on the sign of just one dimensionless coupling. It is either the left-right symmetric phase ($Q = 1$) or the completely asymmetric phase ($Q = 0$).

This picture of the two phases, based as it is on the semi-classical approximation, becomes unreliable in a strip with width of order g^4 , around the phase transition line in the coupling parameter space, because there the radiative corrections due to gauge boson loops become important^{**)} . If the radiative

*) In the case of $U(1)_L \times U(1)_R$, the gauge symmetry can in principle, be completely broken, whereas for $SU(2)_L \times SU(2)_R \times U(1)$, a $U(1)$ symmetry corresponding to electromagnetism would still be leftover.

**) The situation is similar but not identical to pseudo-Goldstone phenomenon.

corrections should smooth out the transition from the symmetric phase ($Q = 1$) to the asymmetric phase ($Q = 0$), then one could choose the mass ratio Q as small as one wanted to, just by fine tuning the coupling parameters of the Higgs potential.

The plan of the paper is as follows. In Section 2, we discuss the model based on the gauge group $U(1)_L \times U(1)_R$. We display explicitly the dependence of the absolute minimum of the effective potential (including one loop corrections) on the coupling parameters of the Lagrangian. Remarkably enough, one finds that the radiative corrections produce only a shift of order g^4 of the phase transition line (in coupling constant space) from its classical value, but otherwise leave intact the form of the phase transition.

In Section 3 we discuss the $SU(2)_L \times SU(2)_R \times U(1)$ model. The dependence of the minimum of the effective Higgs potential (including one loop corrections) on the coupling parameters in this case is not transparent enough to study in complete detail the behaviour of Q in a neighbourhood of order g^4 around the phase transition point. Nevertheless, we are able to show that, as in the $U(1)_L \times U(1)_R$ model, the asymmetric phase ($Q = 0$) is reached by a sudden jump from a phase with a finite mass ratio $\bar{Q}$, where $\bar{Q}$ does not depend on the coupling parameters. Thus, contrary to what one might have expected, a finite gauge hierarchy is not realized. We conclude with some remarks in Section 4.

2. $U(1)_L \times U(1)_R$ Model

We consider a $U(1)_L \times U(1)_R$ gauge theory coupled to a pair of complex Higgs scalars ϕ_L and ϕ_R which transform non-trivially under $U(1)_L$ and $U(1)_R$ respectively. The Lagrangian of the theory is $L \leftrightarrow R$ symmetric:

$$\mathcal{L} = -\frac{1}{4}(\partial^\mu A_L^\nu - \partial^\nu A_L^\mu)^2 + |(\partial^\mu - ig A_L^\mu)\phi_L|^2 + L \leftrightarrow R + \mathcal{U}(|\phi_L|, |\phi_R|) \quad (1)$$

The most general $L \leftrightarrow R$ symmetric potential $\mathcal{U}$ contains three parameters $\mu, \lambda_1, \lambda_2$:

$$\mathcal{U}(|\phi_L|, |\phi_R|) = \frac{\lambda_1}{4!} (|\phi_L|^2 + |\phi_R|^2 - \mu^2)^2 + \lambda_2 |\phi_L|^2 |\phi_R|^2 \quad (2)$$

$$\lambda_1 > 0, \quad \lambda_2 > -\lambda_1/6$$

For any real value of μ , a spontaneous breakdown of the gauge symmetry is guaranteed. The nature of this symmetry breakdown is conveniently characterized by the mass ratio Q of the $U(1)_L$ and the $U(1)_R$ gauge vector bosons or equivalently by the ratio of the vacuum expectation values of ϕ_L and ϕ_R :

$$Q = \frac{\langle |\phi_L| \rangle}{\langle |\phi_R| \rangle} \quad (3)$$

Since the full Lagrangian is $L \leftrightarrow R$ symmetric we can, by convention, restrict Q to lie in the range $0 \leq Q \leq 1$.

In the tree approximation Q depends only on the sign of λ_2 and is just the step function:

$$Q = \theta(-\lambda_2) \quad (4)$$

If λ_2 is negative, U has its minimum at a $L \leftrightarrow R$ symmetric point in the $|\phi_L| - |\phi_R|$ plane. The gauge bosons of $U(1)_L$ and $U(1)_R$ acquire the same mass and the $L \leftrightarrow R$ symmetry survives the total breakdown of the gauge symmetry. For λ_2 positive, on the other hand, U has minima at asymmetric points (i.e., at $|\phi_L| = 0$, $|\phi_R| \neq 0$ and at $L \leftrightarrow R$). Only one factor in the product $U(1)_L \times U(1)_R$ is broken, and the underlying $L \leftrightarrow R$ symmetry appears broken for energies below $g\sqrt{\langle |\phi_L| \rangle^2 + \langle |\phi_R| \rangle^2}$.

It is then clear that, in order to discuss the possibility of a small but finite mass ratio Q , λ_2 must be small enough^{*)} so that the quantum corrections to the classical Higgs potential U become important. We now note that for $\lambda_2 = 0$, the discrete $L \leftrightarrow R$ symmetry of the Higgs potential U is embedded in a larger accidental $U(2)$ symmetry. In this case, the interaction of the scalar fields with the quantum fluctuations of the gauge fields will lift the accidental symmetry of the classical potential and thus select out the true ground state, thereby fixing Q . If the coupling constants λ_1 and λ_2 are normalized at a scale given by the Higgs field expectation value $\sqrt{\langle |\phi_L| \rangle^2 + \langle |\phi_R| \rangle^2}$, then Q depends (up to one loop order) on the $U(2)$ breaking coupling constants as follows:

$$Q = f\left(\frac{\lambda_2}{g^4}\right) \quad (5)$$

^{*)} This remark will be made more quantitative below.

From the semi-classical (tree) approximation (4) ($|\lambda_2| \gg g^4$) we know that f varies from 1 to 0 if λ_2 passes from negative to positive values on a scale given by g^4 . If f would be a continuous interpolation between 1 and 0, we would have the possibility to produce a hierarchy

$$0 < Q \ll 1 \quad (6)$$

by tuning λ_2 on the small scale g^4 . In general, however, the function f is expected to be only piece-wise continuous. Whereas the stationary points of the effective Higgs potential move smoothly with the variation of the parameters in the Lagrangian, the absolute minimum can jump. Typically such a jump occurs if two well separated local minima interchange their roles as true (absolute) and false (local) minima at a certain critical value of the coupling constant.

These considerations are nicely illustrated in the $U(1)_L \times U(1)_R$ model. The range of coupling constants that one is interested in is $\lambda_1 \ll g$ and $\lambda_2 = O(g^4)$, $g \ll 1$. Taking into account the gauge boson loop¹²⁾, the mass ratio Q is determined by the effective Higgs potential V :

$$V(|\phi_L|, |\phi_R|) = U(|\phi_L|, |\phi_R|) + \frac{3g^4}{64\pi^2} \left\{ |\phi_L|^4 \left(\ln 2 \frac{|\phi_L|^2}{\sigma^2} - \frac{25}{6} \right) + |\phi_R|^4 \left(\ln 2 \frac{|\phi_R|^2}{\sigma^2} - \frac{25}{6} \right) \right\} \quad (7)$$

The mass scale σ is chosen so as to fix the following renormalization conditions:

$$\begin{aligned} \left. \frac{\partial^2 V}{\partial |\phi_L|^2} \right|_{|\phi_L|=|\phi_R|=0} &= -\frac{\lambda_1}{6} \mu^2 \\ \left. \frac{\partial^4 V}{\partial |\phi_L|^4} \right|_{|\phi_L|=|\phi_R|=\frac{\sigma}{\sqrt{2}}} &= \lambda_1 \\ \left. \frac{\partial^4 V}{\partial |\phi_L|^2 \partial |\phi_R|^2} \right|_{|\phi_L|=|\phi_R|=\frac{\sigma}{\sqrt{2}}} &= \frac{\lambda_1}{3} + 4\lambda_2 \end{aligned} \quad (8)$$

It is convenient, for discussing the minima of V as a function of the two positive variables $|\phi_L|$, $|\phi_R|$, to pass to polar co-ordinates:

$$\begin{aligned} |\phi_R| &= \rho \cos \alpha \\ |\phi_L| &= \rho \sin \alpha \end{aligned} \quad (9)$$

Since V has a local maximum at $\rho = 0$ and increases for large ρ in all directions α , it has its absolute minimum at a finite distance $\langle \rho \rangle$ from the origin. One is free to identify the renormalization scale σ with the mass $\langle \rho \rangle$. This simplifies the discussion and gives a renormalization group invariant meaning to the parameters λ_1 and λ_2 . Instead of $g, \lambda_1, \lambda_2, \mu$ and σ , we regard g, λ_1, λ_2 and $\langle \rho \rangle$ as the independent parameters that characterize the theory.

At $\rho = \langle \rho \rangle$, the angular dependence of $V(\rho, \alpha)$ is as follows:

$$\begin{aligned} \langle \rho \rangle^4 V(\langle \rho \rangle, \alpha) &= \frac{1}{4} \cos 4\alpha \left[\frac{3g^4}{64\pi^2} (\ln 2 - \frac{25}{6}) - \frac{\lambda_2}{2} \right] \\ &+ \frac{3g^4}{64\pi^2} \left[\cos^4 \alpha \ln \cos^2 \alpha + \sin^4 \alpha \ln \sin^2 \alpha \right] \\ &+ \text{const} \end{aligned} \quad (10)$$

$$0 \leq \alpha \leq \pi/4$$

For all values of λ_2 , the absolute minimum $\langle \alpha \rangle$ of this function, and therefore of $V(\rho, \alpha)$ itself, sits at $\alpha = 0$ or $\alpha = \pi/4$, depending on whether $\lambda_2/g^4 \gtrless 3/16\pi^2 (\ln 2 - 25/12)$. This means that even up to one-loop, $Q = \tan \langle \alpha \rangle$ remains a step function. The only thing that happens in comparison with the tree approximation is a shift of the transition point (separating the symmetric and asymmetric phases) of order g^4 . Thus, taking the gauge field loop into account,

$$Q = \theta \left(-\lambda_2 + \frac{3g^4}{16\pi^2} (\ln 2 - \frac{25}{12}) \right) \quad (11)$$

The mechanism which causes the jump of Q at the critical value $\lambda_2^{\text{crit}} = (3g^4/16\pi^2)(\ln 2 - 25/12)$ has already been described. In a small neighbourhood of λ_2^{crit} , there are two local minima, at $\alpha = 0$ and at $\alpha = \pi/4$, whose levels cross over as λ_2 passes the critical value.

For λ_2 larger than λ_2^{crit} , the $U_L(1) \times U_R(1)$ gauge theory looks effectively like a theory of one massive vector boson coupled to a massive physical Higgs particle plus an unbroken $U(1)$ gauge theory coupled to the charged scalar boson ϕ_L . The mass squared of the massive vector boson, the physical Higgs particle and the charged scalar (to this order) are:

$$\begin{aligned} m^2(V) &= g^2 \langle \rho \rangle^2 \\ m^2(H) &= \langle \rho \rangle^2 \left[\frac{\lambda_1}{3} + \frac{3g^4}{8\pi^2} (\ln 2 - \frac{8}{3}) \right] \\ m^2(\phi_L) &= \langle \rho \rangle^2 \left[2\lambda_2 + \frac{3g^4}{16\pi^2} (\frac{11}{3} - \ln 2) \right] \end{aligned} \quad (12)$$

Note that $m^2(\phi_L)$ has a lower bound:

$$m^2(\phi_L) \geq \langle \rho \rangle^2 \frac{3g^4}{16\pi^2} (-\frac{1}{2} + \ln 2) \quad (13)$$

If λ^2 stays below the transition point λ_2^{crit} , the vector bosons A_μ^L and A_μ^R get a mass which is

$$m^2(V) = \frac{1}{2} g^2 \langle \rho \rangle^2 \quad (14)$$

The mass matrix of the scalar fields diagonalizes for the even and the odd combinations under the $L \rightleftharpoons R$ symmetry

$$\phi_\pm = \frac{1}{\sqrt{2}} (\phi_L \pm \phi_R) \quad (15)$$

The corresponding eigenvalues are

$$\begin{aligned} m_+^2 &= \langle \rho \rangle^2 \left[\frac{\lambda_1}{3} + 2\lambda_2 - \frac{g^4}{2\pi^2} \right] \\ m_-^2 &= \langle \rho \rangle^2 \left[-2\lambda_2 - \frac{g^4}{2\pi^2} \right] \end{aligned} \quad (16)$$

Higher loop effects will add higher order corrections to the mass formulae for the massive fields and to the value of λ_2^{crit} . However, they cannot change the qualitative picture of the symmetry breakdown: either a $L - R$ symmetric total breakdown or a $L - R$ asymmetric partial breakdown. In the $L - R$ symmetric phase the effective theory contains no massless particles and therefore no infra-red divergences which could destabilize the perturbation expansion around

the $L \leftrightarrow R$ symmetric vacuum. In the asymmetric phase with one massless gauge boson, the infra-red singularities are essentially those of massive scalar electrodynamics with a stable self-interaction of the scalars. The perturbative vacuum is believed¹²⁾ to be stable in this case.

To conclude our discussion of the $U(1)_L \times U(1)_R$ model (with minimal Higgs' multiplets) we remark that the model behaves as simply as it could, given the discrete symmetry pattern of the interaction. If there is any spontaneous gauge symmetry breakdown then, from mere symmetry arguments, the full effective Higgs potential has to have stationary points, both on the $|\phi_L|$ and $|\phi_R|$ axes as well as on the symmetry line $|\phi_L| = |\phi_R|$. The theory chooses the absolute minimum of V from among these stationary points, forced on it by the discrete $L \leftrightarrow R$ symmetry.

3. $SU(2)_L \times SU(2)_R \times U(1)$ MODEL

The analysis presented above for the $U(1)_L \times U(1)_R$ toy model may be carried over to the realistic $SU(2)_L \times SU(2)_R \times U(1)$ gauge model. The minimal Higgs' system contains scalar fields ϕ_L and ϕ_R which transform as doublets under $SU(2)_L$ and $SU(2)_R$ respectively. The classical Higgs potential U has the form (2), with the gauge invariants $|\phi_L|^2$ and $|\phi_R|^2$ replaced by $\phi_L^\dagger \phi_L$ and $\phi_R^\dagger \phi_R$. The introduction of the polar variables ρ and α , as defined by Eq. (9), is now suggested by the embedding of the $SU(2)_L \times SU(2)_R \times U(1)$ invariance of the Higgs potential in $U(4)$. The latter group is an accidental symmetry of the classical Higgs potential if λ_2 vanishes. The previous discussion for the $U(1)_L \times U(1)_R$ model applies up to Eq. (10) to the realistic model.

The radiative correction term $g^4 V_1(\rho, \alpha)$ from the gauge boson loop produces in the present case a more complicated α dependence of the effective Higgs potential than that of Eq. (7). Equation (10) gets replaced by

$$V(\langle \rho \rangle, \alpha) = - \langle \rho \rangle^4 \frac{\lambda_2}{8} \cos 4\alpha + g^4 V_1(\rho, \alpha) + \text{const.} \quad (0 \leq \alpha \leq \pi/4) \quad (17)$$

Fortunately, we do not need to know the form of $V(\rho, \alpha)$ in detail^{*)}. A hierarchy in the sense of Eq. (6) exists only if there is a value κ for the ratio λ_2/g^4 of the coupling parameters, such that the absolute minimum $\langle \alpha \rangle$ of $V(\langle \rho \rangle, \alpha)$ (or equivalently $Q = \tan \langle \alpha \rangle$) approaches zero continuously from positive

*) We suppress the inessential dependence of V_1 on the ratio of the $U(1)$ coupling and the $SU(2)_{L,R}$ coupling.

values as λ^2/g^4 tends to κ , either from the right or the left-hand side of κ . Since $V(\langle\rho\rangle, \alpha)$ is an even function of α , this can happen if and only if $V(\langle\rho\rangle, \alpha)$ has a minimum of higher order at $\alpha = 0$ for $\lambda_2/g^4 = \kappa$. This implies

$$\frac{\partial^2}{\partial \alpha^2} V(\langle\rho\rangle, 0) = 0 \quad (18)$$

$$V(\langle\rho\rangle, \alpha) \geq V(\langle\rho\rangle, 0) \quad (19)$$

In physical terms this means that one can choose the parameter λ_2/g^4 such that the theory is in its asymmetric phase (with an unbroken $SU(2)_L \times U(1)$ subgroup) and the $SU(2)_L$ doublet is massless. (A similar condition appears in Weinberg's analysis⁵⁾ of the hierarchy problem). Using Eq. (17), we see that (18) is a linear equation for $\kappa = \lambda_2/g^4$ and so determines this ratio uniquely. The expansion of V for small α then has no quadratic term. It is of the form:

$$V(\langle\rho\rangle, \alpha) = a + b\alpha^4 + c\alpha^4 \ln \alpha^2 + O(\alpha^6 \ln \alpha^2) \quad (20)$$

Clearly, for Eq. (19) to hold, the coefficient c must be negative.

A straightforward but tedious calculation of the effective Higgs potential gives:

$$c = \frac{3}{1024\pi^2} g^4 \langle\rho\rangle^4 \frac{3g^4 + 8g^2g'^2 + 6g'^4}{(g^2 + g'^2)^2} \quad (21)$$

which is positive (g and g' are the gauge coupling constants of $SU(2)_{L,R}$ and $U(1)$ respectively^{*)}. Thus, there is no higher minimum of $V(\langle\rho\rangle, \alpha)$ at $\alpha = 0$. This leads to the conclusion that it is impossible to establish a finite hierarchy [see Eq. (6)] in the $SU(2)_L \times SU(2)_R \times U(1)$ model with just two Higgs' doublets ϕ_L and ϕ_R .

^{*)} The $\alpha^4 \ln \alpha^2$ term in Eq. (20) is produced solely by those gauge bosons that stay massless in the totally asymmetric breakdown ($Q = 0$) of the $L - R$ symmetric theory. Therefore, the coefficient c can alternatively be obtained from the effective Higgs potential of the unbroken $SU(2)_L \times U(1)$ model which has been calculated in Ref. 12). Note that the $SU(2)_L$ and $U(1)$ couplings of the effective $SU(2)_L \times U(1)$ model are g and $gg'/(g^2 + g'^2)^{1/2}$. The quantity $c\alpha^4$ is simply the trace of the square of the gauge boson mass squared matrix of the effective $SU(2)_L \times U(1)$ theory.

For the effects of higher order loop corrections to the effective Higgs potential, we rely on the same kind of reasoning that was employed for the $U(1)_L \times U(1)_R$ model. If $\langle\alpha\rangle$ is finite at the one loop level^{*}), the only massless particle left is a photon, whose infra-red singularities are not capable of altering the ground state. With $\langle\alpha\rangle = 0$ at the one loop level, the infra-red singularities are those of an unbroken $SU(2)_L \times U(1)$ gauge theory coupled to massive Higgs scalars. This remaining gauge symmetry is expected to remain intact in higher order perturbation theory.

4. FINAL REMARKS

It has been suggested⁶⁾ that additional symmetries may be needed in order to obtain a gauge hierarchy in perturbation theory. We have discussed the case when the additional symmetry is a discrete one. Such a symmetry is indeed present in certain unified models, for instance in models based on the gauge groups $SO(10)$, E_6 and $[SU(4)]^4$. It was essential, for the examples considered, to work with the minimal Higgs system in order to avoid the shortcomings of the usual approach. In the present scheme, although one might have expected otherwise, one is still unable to produce a finite gauge hierarchy in perturbation theory. It may be worthwhile to consider gauge theories with more complicated additional symmetries.

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^{*}) Note that a finite $\langle\alpha\rangle$, arbitrarily near $\langle\alpha\rangle = 0$ would correspond to a finite hierarchical solution which is excluded by the preceding discussion.

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