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CHARMONIUM CHI STATES : RADIATIVE AND HADRONIC WIDTHS,

SPIN OF THE GLUON

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A B S T R A C T

The dipole sum rules, used earlier to give upper and lower bounds for the radiative transitions for $\chi_J \rightarrow \gamma \psi$ and so estimate the hadronic widths of the χ states, are re-examined with more general assumptions about the $Q\bar{Q}$ interaction, namely the presence of spin-spin, spin-orbit, and tensor forces. The lower and upper bounds for a specific J value are now explicitly force dependent. In fact, a useful lower bound can be established only for the $J=2$ state. The inferred radiative rates and estimates of hadronic width, based on new data, are similar to those given earlier in Refs. 1) and 5). The improved precision of the data affords a test of the spin of the gluon, within conventional QCD assumptions. The inferred ratio of hadronic widths $\Gamma_h(0^{++})/\Gamma_h(2^{++}) = 5.5^{+14}_{-3}$ is in agreement with the ratio 15:4 expected from spin 1 and in disagreement with the ratio 125:2 for scalar gluons.

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1. Dipole sum rules

Earlier the Thomas-Reiche-Kuhn and Wigner-Kirkwood sum rules of atomic physics were employed to give upper and lower bounds on the radiative transition rates of the charmonium χ_J states to the $\psi(3096)$ and so estimate the hadronic widths of these states.¹⁾ These sum rules have been reconsidered with somewhat more realistic assumptions about the $Q\bar{Q}$ interaction. The Hamiltonian is still nonrelativistic, but with a potential containing spin-spin, spin-orbit, and tensor force contributions, as well as a central part:

$$V_{\text{int}} = V_C(r) + \vec{\sigma}_1 \cdot \vec{\sigma}_2 V_S(r) + \vec{L} \cdot \vec{S} V_{LS}(r) + S_{12} V_T(r) \quad (1)$$

The radiative transitions are assumed to be electric dipole, as before.

1.1 Upper bounds

The Thomas-Reiche-Kuhn sum rule for the 3S_1 ground state leads to a generalization of Eq.(3) of ref. 1 for the upper bounds:

$$\Gamma(\chi_J \rightarrow \gamma_2 \psi) < \frac{2}{3} \frac{\alpha e_Q^2 k_{2J}^2}{\mu} \left(1 + \frac{2}{3} x_s \cdot \begin{Bmatrix} 1 \\ -1 \\ -2 \end{Bmatrix} \right) \quad (2)$$

for $J = 2, 1, 0$, respectively. The quantity x_s is given by

$$x_s = \frac{\mu}{k^2} \langle ^3S_1 | r^2 V_{LS}(r) | ^3S_1 \rangle \quad (3)$$

Neglected in Eq.(2) are contributions from the tiny d-state admixture in the $\psi(3097)$, estimated, in a largely model-independent way in the Appendix, to be less than 2×10^{-3} in intensity relative to the s-state.

1.2 Lower bounds

It should be recalled that the Wigner-Kirkwood sum rule for the oscillator strengths for the transitions with $l \rightarrow l' = l - 1$ is negative.

Applied to the transitions $l = 1 \rightarrow l' = 0$, this sum allowed the setting of a lower bound for the rate of the downward transition $1p \rightarrow 1s$. Simple statistical weight considerations gave results for the different χ_J states, under the assumption that spin-orbit and tensor forces were negligible.

With the Hamiltonian (1) the Wigner-Kirkwood sum for 3P_J (+ 3F for $J = 2$) $\rightarrow {}^3S_1$ (+ 3D_1) is plausibly negative only for the $J = 2$ state. It is for this state alone that one can give a potentially useful lower bound, the generalization of Eq.(5) of ref.1:

$$\Gamma(\chi_2 \rightarrow \gamma_2 \psi) > \frac{\alpha e_Q^2 k_{22}^2}{5\mu} \left(1 + \frac{12}{5} x_p \right) + \frac{3}{5} \cdot \frac{k_{22}^2}{k_{12}^2} \cdot \Gamma(\psi' \rightarrow \gamma_1 \chi_2) \quad (4)$$

where x_p is the analogue of x_s , but for the 3P_2 state. Contributions from a 3F_2 admixture in χ_2 have been neglected in Eq.(4).

1.3 Application of the bounds

The presence of the expectation values x_s and x_p in Eqs.(2) and (4) restrict the power of the bounds significantly. Some general limits can be set on the size of x_s and x_p , but they are not terribly useful. The right side of Eq.(2) must be positive. Thus $-3/2 < x_s < 3/4$. Similarly, with the assumption that no triplet $J = 2$ state with even L lies below the 3P_2 state, it can be inferred that x_p must lie in the range, $-5/2 < x_p < 5/4$. The looseness of this constraint is evident when it is seen that the coefficient $\frac{1}{5} \cdot (1 + \frac{12}{5} x_p)$ of the first term in Eq.(4) ranges from -1 to +4/5.

Can tighter constraints be put on x_s and x_p ? For x_s , the same trick that was used to estimate the admixture of d-state in the $\psi(3097)$ from knowledge of the energy level spectrum can be employed to give a more or less model-independent value or bound. The result is that x_s is positive and close to $x_s = 0.12$. Without experimental information on the spin-orbit energy of the d-states, a corresponding argument cannot be given for x_p . One must appeal to models for guidance. A wide range of plausible potentials (with Coulombic and linear parts having their assumed relativistic origins such that the spin-orbit part of the χ_J splittings is roughly correct) yield $x_p = 0.05$ to 0.15 , and $x_s = 0.10$ to 0.18 . In view of the positive expectation value of V_{LS} in the χ states, it seems implausible that the expectation value of $r^2 V_{LS}$ should be negative. Taking $x_s = 0.12$ and the just stated range of values for x_p , together with $m_c = 2\mu = 1.5$ GeV, one finds upper and lower bounds,

$$\Gamma(\chi_2 \rightarrow \gamma_2 \psi) < 527 (1 + \frac{2}{3}x_s) \simeq 570 \text{ keV}$$

$$\Gamma(\chi_2 \rightarrow \gamma_2 \psi) > 158 (1 + \frac{12}{5}x_p) + 104 \simeq 280 - 320 \text{ keV}$$

The contribution of 104 keV, the second term in Eq.(4), follows from the branching ratio $B_1(\psi' \rightarrow \gamma_1 \chi_2) = 7.5 \pm 1.7 \%^{2)}$ and the total width of the ψ' , $\Gamma_t = 228 \pm 56 \text{ keV}^{3)}$. It thus has a 30 % uncertainty.

The product of branching ratios for the transitions $\psi' \rightarrow \gamma_1 \chi_2$ and $\chi_2 \rightarrow \gamma_2 \psi$, $B_1 B_2 = 1.3 \pm 0.3 \%^{2)}$ and the above value of B_1 yield $B_2 = 0.17 \pm 0.05$ for the $J = 2$ state. The total width of χ_2 therefore lies plausibly between the "extremes" of 1.3 and 4.9 MeV, with the range 1.8 - 3.4 MeV preferred. These values are similar to those of Table II of ref. 1, and somewhat larger than the QCD estimate.⁴⁾

2. Relative hadronic widths for the χ_J states and the spin of the gluon

It has been understood for some time that reliable enough experimental values of the branching ratios for $\psi' \rightarrow \gamma_1 \chi_J (B_1)$ and the product of the branching ratios for $\psi' \rightarrow \gamma_1 \chi_J \rightarrow \gamma_1 \gamma_2 \psi (B_1 B_2)$ can provide evidence on the validity of the estimation of total hadronic widths using QCD at "hadronic" distances, e.g., $\Gamma_h(\chi_2) \simeq \Gamma(c\bar{c} \ ^3P_2 \rightarrow gg)$. Without experimental values or estimates of the second radiative transition rates (see above), the actual magnitudes of the widths cannot be found, but relative values of the total hadronic widths for the different χ_J states follow from data on B_1 and $B_1 B_2$, plus the relatively safe assumption of a k^3 dependence for the radiative rates, with a common electric dipole spatial matrix element for all three $\chi_J \rightarrow \gamma_2 \psi$ transitions.^{5),6)}

The recent data from the "crystal ball" detector at SPEAR²⁾:

Mass	J	B_1 (%)	$B_1 B_2$ (%)
3554	2	7.5 ± 1.7	1.3 ± 0.3
3508	1	7.5 ± 1.7	2.5 ± 0.6
3413	0	7.6 ± 1.7	0.10 ± 0.06

can be used with the formula,

$$\frac{\Gamma_h(J')}{\Gamma_h(J)} = \frac{k_{2J'}^3}{k_{2J}^3} \cdot \frac{\left[\frac{B_1(J')}{B_1 B_2(J')} - 1 \right]}{\left[\frac{B_1(J)}{B_1 B_2(J)} - 1 \right]} \quad (5)$$

to obtain the ratios,

$$\frac{\Gamma_h(0^{++})}{\Gamma_h(1^{++})} = 18 \pm \frac{14}{8} \quad (< 60), \quad \frac{\Gamma_h(2^{++})}{\Gamma_h(1^{++})} = 3.2 \pm 1.0$$

The rather arbitrary error estimates are made with the assumption that at least part of the quoted error on B_1 and $B_1 B_2$ is common systematic or normalization error that cancels in the ratio. The QCD expectations for these ratios are⁴⁾ $\sim 15:1$ and $\sim 4:1$, respectively, with the ratio $\Gamma_h(0^{++})/\Gamma_h(2^{++}) = 15/4$ on firmer theoretical footing.

Though even greater experimental accuracy could be desired, the agreement with QCD predictions is quite satisfying. Consistency is not the ultimate test, of course. Establishment of such basics as the spin of the gluon are important. For heavy quark systems, the study of the angular distributions of the gluonic jets from the χ states and their upsilononic analogs in the decay sequence $\psi'(\tau') \rightarrow \delta_1 \chi$, $\chi \rightarrow$ hadrons has been advocated as a means of establishing the spin of the gluon.^{7),8)} Within the same framework, a much simpler test for the gluon spin was pointed out earlier.⁵⁾ The ratio of the two-gluon decay rates $(c\bar{c} \ ^3P_J \rightarrow gg)$ for $J = 0$ and $J = 2$ is very sensitive to the assumed spin of the gluon. With the QCD assumption that these rates represent the total hadronic decay rates, one has the predictions for various gluon spin choices,

$$\frac{\Gamma_h(0^{++})}{\Gamma_h(2^{++})} \approx \frac{\Gamma(c\bar{c} \ ^3P_0 \rightarrow gg)}{\Gamma(c\bar{c} \ ^3P_2 \rightarrow gg)} = \begin{cases} \frac{15}{4} & (1^-) \\ \frac{125}{2} & (0^+) \\ \frac{5}{2} & (0^-) \end{cases} \quad (6)$$

The "experimental" value of this ratio, deduced from Eq.(5), is

$$\frac{\Gamma_h(0^{++})}{\Gamma_h(2^{++})} = 5.5 \pm \frac{14}{3} \quad (7)$$

The ratio is quite consistent with the expected 1^- assignment for the gluon, as was seen earlier. Scalar gluons are excluded. The implausible pseudoscalar value is at the edge of the error bars, but has no theoretical basis.

Although the central value in Eq.(7) is more than a factor of 10 away from the scalar ratio of 62.5, the "upper limit" is only a factor of 3 away. Because the ratio of k^3 factors is quite large here, the reliability of the inferred ratio of hadronic widths can rightly be questioned. Whatever one's judgment about the firmness of the present "determination" of the gluon's spin, the same test will provide the most

direct means when the analogous branching ratios are determined for the upsilon family. The k^3 hypothesis should be very reliable there. Indeed, the problem will be to resolve experimentally the different χ states.

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APPENDIX

A.1 Estimate of the d-state admixture in the charmonium triplet ground state (3097)

It is known from the splittings of the χ states, with $M_2 = 3554 \pm 5$ MeV, $M_1 = 3508 \pm 4$ MeV, $M_0 = 3413 \pm 5$ MeV, that both tensor and spin-orbit forces contribute. If it is assumed that perturbation theory is valid for these intervals, one can conclude from the data that⁹⁾

$$\langle 1 p | V_T | 1 p \rangle = 10 \text{ MeV} \quad (\text{A.1})$$

$$\langle 1 p | V_{LS} | 1 p \rangle = 35 \text{ MeV} \quad (\text{A.2})$$

in the notation of Eq. (1).

Within the same perturbative framework, the d-state admixture ϵ of 3D_1 in the largely 3S_1 ground state is given by

$$\epsilon \simeq \left| \frac{\langle ^3D_1 | V_T(r) S_{12} | ^3S_1 \rangle}{M^{(0)}(1d) - M^{(0)}(1s)} \right|^2 \quad (\text{A.3})$$

Here the contributions from other than the first d-state have been neglected. The mass difference in the denominator can be approximated by the difference between the average, 3730 MeV, of the ψ' and ψ'' masses, and the ψ mass: $M^{(0)}(1d) - M^{(0)}(1s) \simeq 633$ MeV. The estimate for ϵ is therefore

$$\epsilon \simeq \left| \frac{\sqrt{8} \langle 1d | V_T(r) | 1s \rangle}{633 \text{ MeV}} \right|^2 \quad (\text{A.4})$$

To determine approximately the off-diagonal matrix element in (A.4) in terms of the diagonal (A.1) without assumptions about the actual form of the tensor potential, we can exploit the nodeless character of the lowest state of a given angular momentum. For such states the radial wave function can be written as

$$R_l(r) = N_l r^l f_l(r)$$

where $f_l(r)$ is real, finite at $r = 0$, has no zeros, and vanishes at infinity. We choose $f_l(0) = 1$ for definiteness. The matrix elements that we wish to relate can be expressed as

$$\begin{aligned}\langle 1d | V_T | 1s \rangle &= N_0 N_2 \int_0^\infty r^2 dr V_T(r) [r^2 f_0 f_2] \\ \langle 1p | V_T | 1p \rangle &= N_1^2 \int_0^\infty r^2 dr V_T(r) [r^2 f_1 f_1]\end{aligned}$$

To the extent that $f_0 f_2 = f_1^2$, the two integrals can be related via the normalization constants, independent of the shape of $V_T(r)$. For the harmonic oscillator wave functions, the equality is exact. Since we know that the low-lying spectrum and spatial properties of charmonium are described roughly by the oscillator template⁹⁾, we proceed under the assumption that $f_0 f_2 \simeq f_1^2$. For the oscillator, the relevant ratio of normalization constants is $N_0 N_2 / N_1^2 = (3/5)^{1/2}$. We therefore have the estimate, $\langle 1d | V_T | 1s \rangle \simeq (3/5)^{1/2} \langle 1p | V_T | 1p \rangle$, and

$$\epsilon \simeq \frac{24}{5} \left| \frac{10 \text{ MeV}}{633 \text{ MeV}} \right|^2 = 1.2 \times 10^{-3} \quad (\text{A.5})$$

In the text, this number was raised to 2×10^{-3} in a moment of extreme conservatism. Numerical calculations with typical charmonium potentials show that $f_0 f_2 \lesssim f_1^2$, the departure from equality being less than 20% over the region of interest, and $N_0 N_2 / N_1^2$ agrees with the oscillator value to within a few percent.¹⁰⁾

A.2 Model-independent estimate of x_s

Exactly the same trick can be exploited to estimate the parameter x_s of Eq.(3) in terms of the expectation value (A.2). The two quantities are

$$\begin{aligned}x_s &= \mu N_0^2 \int_0^\infty r^2 dr V_{LS}(r) [r^2 f_0^2] \\ \langle 1p | V_{LS} | 1p \rangle &= N_1^2 \int_0^\infty r^2 dr V_{LS}(r) [r^2 f_1^2]\end{aligned}$$

To the extent that $f_0 = f_1$, we have

$$x_s = \mu \langle 1s | r^2 | 1s \rangle \langle 1p | V_{LS} | 1p \rangle \quad (\text{A.6})$$

where the ratio of normalization constants has been expressed in terms of the radius squared of the ground state. For the harmonic oscillator,

$$\mu \langle 1s | r^2 | 1s \rangle = \frac{3}{2\omega_0} \quad (\text{A.7})$$

where ω_0 is the oscillator level spacing. The result (A.7) can be shown to be an upper bound for any central potential when ω_0 is replaced by $E(1p) - E(1s)$.¹¹⁾ It seems, in fact, to be only a slight over-estimate for any plausible charmonium potential.¹²⁾ For the extreme of a purely Coulombic potential, the actual value is 3/4 of (A.7).

With the oscillator value (A.7) and $\omega_0 = 426$ MeV (from the weighted mean of the χ_J masses, $\langle M \rangle = 3523$ MeV, less the ψ mass), the largely model-independent estimate of x_s is $x_s = 0.12$.

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