Ref.TH.2709-CERN

COLOUR DISTRIBUTION IN HADRON-HADRON SCATTERING

F. Niedermayer ^{*)}
CERN - Geneva

A B S T R A C T

In high p_t quark-quark scattering a simple picture emerges for the colour distribution in the final state: the scattered quark of one of the incident hadrons forms a colour singlet with the spectators of the other hadron, i.e., the four fast coloured objects form two singlets. We assume that the decay of each colour singlet pair - apart from a trivial boost - is similar to that observed in deep inelastic lepton-hadron scattering. This picture leads to measurable anticorrelations for energy fluxes carried by low momentum particles. The predicted asymmetry is significant only in qq scattering, but not in $q\bar{q}$, qg , ... scattering, which leads to specific predictions.

*) Permanent address : Institute for Theoretical Physics, Eötvös University, Budapest

I. INTRODUCTION

There is a general hope that QCD describes both the short distance part of the hadron dynamics, by perturbative methods, and also the long distance behaviour which probably cannot be obtained by usual perturbation theory. The notion of colour plays a quite different rôle in the two regions : at short distances it ensures asymptotic freedom by its non-Abelian nature, while at large distances the assumption that there are only colour singlet states in nature is used to explain confinement, the properties of multihadron production, etc.

The short distance and the large distance colours are, of course, assumed to be the same. This identification is used to explain the nature of Pomeron exchange in hadron-hadron reactions, and in explaining features of multiparticle production on nuclei ¹⁾⁻³⁾.

In this paper we investigate a possible experimental consequence of this picture in hadron-hadron collisions - an elementary coloured gluon exchange and a later non-perturbative evolution of the system - which is sensitive to the particular colour distribution originated at the first step.

In high p_t quark-quark scattering a simple picture emerges for the colour distribution in the final state after the elementary hard scattering via a colour octet gluon exchange : the scattered quark of one of the incoming hadrons forms a colour singlet with the spectators of the other hadron (with a probability of 8/9). That is, in most of the cases the four fast coloured objects form two singlets. We assume that confining forces act only within each of the singlets, and the singlets decay independently into their own hadronic final states. We also make a straightforward assumption that the decay of each colour singlet pair - apart from a trivial boost - is similar to that observed in deep inelastic lepton-hadron scattering. This picture leads to anticorrelation in the fluxes carried by low energy hadrons. The predicted asymmetry is significant only in qq scattering, but not in other elementary processes. This fact implies some specific predictions.

II. COLOUR DISTRIBUTION IN qq SCATTERING

In quark-quark scattering via colour octet exchange the quarks predominantly exchange their colour states, as shown in Fig. 1.

It means that in hadron-hadron collisions the qq scattering contribution leads to a simple situation (neglecting the $1/N$ term) (see Fig.2). Since the quark a has formed a colour singlet with spectators A' in hadron A before the collision, after scattering the quark b' will form a colour singlet with A' (and similarly a' with B'). The result of simple calculation is that this situation will occur with a probability of

$$P = 1 - \frac{1}{N^2} = \frac{8}{9}.$$

(In the remaining cases no two of the four outgoing objects form a colour singlet separately.)

In $q\bar{q}$ scattering the situation is different; the scattered q and $\bar{q}$ form a colour singlet (Fig.3). In the case of quark-gluon or gluon-gluon scattering, no two of four outgoing objects can form a colour singlet.

Now we make a simple assumption that the colour singlet pairs hadronize independently. Imagine, for simplicity, that in a colliding beam experiment both of the quarks have the same momentum ($x_1 = x_2$) and that they are scattered at 90° , as is indicated in Fig. 4.

The "bags" in Fig. 4 denote something which confines the colour singlet state, e.g., the colour electric flux tubes ^{1),2)}.

We assume that the excited colourless ($a'B'$) object decays in its own centre-of-mass frame as the corresponding excited state in deep inelastic lepton-hadron scattering : it leads to hadrons, uniformly distributed in rapidity, with some average transverse momenta. To obtain the whole final state, one should simply boost back both groups of hadrons from their own c.m. frame. This boost will lead to some enhancement of particles in regions I and III with respect to regions II and IV.

III. BOOSTING THE JETS

Now we discuss the simple kinematics for boosting the hadronic final states. Let the two fast coloured objects which form a colour singlet have momenta p_1 and p_2 in the laboratory frame, with an angle Θ between them. We can then boost our system of reference along the bisecting line (angle $\Theta/2$) with an appropriate rapidity to see the two objects flying in opposite directions. Since the ratio of longitudinal momenta (along the bisecting line) to the transverse mass is for both objects

$$\frac{p_t}{m_t} \approx \frac{p \cdot \cos \vartheta/2}{p \cdot \sin \vartheta/2} = \operatorname{ctg} \vartheta/2$$

We have to boost the system of reference with a finite rapidity

$$\operatorname{sh} y_0 = \operatorname{ctg} \vartheta/2.$$

This boost leads to a two-jet picture, as it should be, where the two jets almost do not feel each other. To see it more clearly, imagine that the longitudinally excited object fragments in its c.m. frame with zero transverse momentum. Then (for $\vartheta = 90^\circ$) in the lab. frame we see two hadronic jets with transverse momenta, depending on the rapidity of hadrons as $p_t \approx m_t e^{-|y|}$ (see Fig.5). Consequently, only those hadrons which are slow (not ultrarelativistic) in the lab. frame really feel the effect of colour distribution.

IV. CORRELATION BETWEEN LOW MOMENTUM HADRONS

To have a parameter which measures the asymmetry in the situation indicated in Fig. 4, let us define a normalized correlation function as follows.

Trigger on two high p_t hadrons at two opposite sides around 90° . Measure the energy flux E_1 (E_2) carried by hadrons flying in the direction $+45^\circ$ (-45°). (More precisely, the momentum projection onto the scattering plane should lie, say, between 30° and 60° .) (See Fig.6.)

Take then the averages $\langle E_1 \rangle = \langle E_2 \rangle \equiv \langle E \rangle$, and $\langle E_1 E_2 \rangle$, and calculate the normalized correlation

$$C = \frac{\langle E_1 E_2 \rangle - \langle E \rangle^2}{\langle E \rangle^2} \quad (1)$$

As seen from Fig. 4, we expect in qq scattering an anticorrelation between E_1 and E_2 , i.e., $C < 0$.

We should consider some difficulties at this point :

- a) the hadrons originating from a jet have some random transverse momentum distribution. Will this not screen the effect ?
- b) one cannot detect outgoing quarks and one has to trigger on two high p_t particles at opposite sides with some not too small angular acceptance to have reasonable statistics. Will these uncertainties leave the effect still measurable ?
- c) does the kinematical constraint of momentum conservation affect the value of C ?
- d) the elementary process is not only a qq scattering, it is mixed up with $q\bar{q}$, qg , gg scattering, and even at relatively large p_t values with other possible scattering processes, e.g., in the constituent interchange model ⁴⁾.
- e) we do not know anything about final state interactions between low momentum hadrons, including possible soft colour exchange.

As for the last point we cannot do anything better than to assume that final state interactions do not spoil the asymmetry. (It is enough to assume that they cause independent fluctuations in energy fluxes E_1 and E_2 .)

V. MONTE CARLO CALCULATIONS

To answer question a) of the previous section, we made a simple Monte Carlo calculation, where the two quarks were scattered at 90° , -90° , and we let them fragment via a cascading mechanism ⁵⁾⁻⁷⁾. To be more specific, we considered the colour singlet state (e.g., $a'B'$ of Fig.4) in the rest frame of one of the fast moving particles (say B'), and constructed the hadronic final state by fragmenting the other particle. (The particles which are slow in the lab. frame are in the middle of the plateau of the system $a'B'$, and although the momentum distribution of final hadrons from the cascading mechanism is somewhat different at the two ends of the rapidity region, the slow particles do not feel this asymmetry.)

We took an elementary cascading probability : $3(1-z)^2 dz$ (three hadrons per unit rapidity), and some transverse momentum distribution which has been varied between 0 and 1 GeV. The normalized correlation (1) starts with a value -1 at $\overline{p_t} = 0$, and it is still $C = -0.4$ at ultrarelativistic transverse momenta $m_\pi \ll \overline{p_t} \sim 1$ GeV (Fig.7). At the realistic value $\overline{p_t} \sim 0.3$ GeV :

$$C \approx -0.5$$

Surprisingly enough, the value of C does not depend on different parameters involved in the calculation : on rapidity density of pions, on pion mass, on qq scattering angle (θ and $-\theta$ instead of 90° and -90°), on the angular acceptance $\Delta\theta$ of detected low momentum particles around $\theta/2$ and $-\theta/2$. Obviously, the correlation does not depend also on the energies of the scattered quarks, provided they are big enough to produce jets.

However, if one introduces a lower cut-off for detected particles, whose energies add up to E_1 and E_2 , the value of the anticorrelation increases, since this reduces the number of particles on the "bad" side. The dotted curve in Fig. 7 denotes the correlation for a lower cut-off $p_{\min} = 0.5$ GeV.

Unfortunately, quarks do not scatter only at 90° , -90° , and one may expect that simple momentum conservation will also result in a non-zero value of C . In the situation of Fig. 8a ($x_1 \neq x_2$, $\theta = 90^\circ$, x_1 and x_2 being the momentum fractions carried by initial quarks), we expect a positive correlation, $C > 0$, while for Fig. 8b ($x_1 = x_2$, $\theta_{\text{cm}} \neq 90^\circ$) an anticorrelation, $C < 0$.

To answer this question we have made a more realistic Monte Carlo calculation using a simple quark distribution function inside the incident hadrons and some simple dependence of the elementary cross-section on the scattering angle θ_{cm} . Since we want to trigger on two high p_t hadrons on both sides around 90° , the functional form of these functions is not very important, and we have taken $\sim(1-x)^3$ for the quark distribution function, and a uniform distribution in θ_{cm} .

To see whether the effect of the colour distribution is still observable, one should compare our picture with the commonly used picture of four independent jets. We have made instead a comparison, which is somewhat different, but answers the same question. We compared the following two types of Monte Carlo "experiment". We assumed that :

- A) the scattered objects ($a'b'$) and the spectators ($A'B'$) form colour singlets separately, as in the $q\bar{q}$ scattering (Fig.3);
- B) ($a'B'$) and ($b'A'$) form colour singlets, as in the qq scattering (Fig.2).

If kinematical constraints affect the value of the correlation, it would be seen in both cases A) and B).

In a typical example we generated in both cases $\sim 300'000$ events, and took those of them which have on both sides at least one particle with $|p| > 4$ GeV, with scattering angles between 70° and 110° . In these events we calculated the total energy E_1 within the region $30^\circ < \theta < 60^\circ$ in the production plane, and similarly E_2 within the region $30^\circ < -\theta < 60^\circ$.

We got 3-4 thousand triggered events, with more than 10 thousand particles within $30^\circ < |\theta| < 60^\circ$, hence the statistical error is several per cent.

The result for the normalized correlation (1) was :

$$\text{Case A) : } C = 0.02 \pm 0.03$$

$$\text{Case B) : } C = -0.35 \pm 0.03$$

Consequently, we see that : 1) the effect of colour distribution is still significant even if real smearing in quark distribution, scattering angle and detection angles is introduced; 2) kinematical effects do not lead to large correlations.

We also analyzed the dependence on lower cut-off for momenta of particles in E_1 and E_2 . As expected, the correlation does not change in Case A), and increases in Case B). At the lower cut-off $p_{\min} = 0.5$ GeV we got a value :

$$\text{Case B) : } C = -0.52 \quad (\text{lower cut-off } 0.5 \text{ GeV}).$$

[We would like to stress that only the slow particles are really sensitive to the effect of colour distribution. We have introduced a lower cut-off only because it is natural from an experimental point of view. The lower cut-off should not be taken much more than 0.5 GeV otherwise the statistics becomes worse and other effects, such as three-jet events, kinematical effects, etc., become relatively more important.]

VI. INFLUENCE OF OTHER TYPES OF ELEMENTARY PROCESSES

We have shown that kinematical effects of situations in Figs. 8a and 8b do not lead to significant correlations if the two incident partons have the same x distribution. This is true for valence-valence scattering. For valence-sea scattering, the situation is asymmetric : the average x for the valence parton is larger than that of the sea parton, i.e., the situation in Fig. 8a is enhanced with respect to Fig. 8b, and a larger positive correlation is expected. Indeed, we have got the following results :

- a) $q\bar{q}$ with the same distribution : $C = 0.02$ (for qq : $C = -0.35$)
- b) $q\bar{q}$ with different distributions : $C = 0.07$
- c) $q\bar{q}$ with different distributions, and lower cut-off 0.5 GeV in E_1, E_2 : $C = 0.19$ (for qq : $C = -0.52$)

where the corresponding values for the qq case are quoted for comparison. [In cases b) and c) we used $(1-x)^3$ for the valence, and $(1-x)^8$ for the sea parton distribution, as an example.]

As the Monte Carlo calculations indicate, a significantly non-zero value for C is expected only in qq scattering (because of the colour distribution) and in valence-sea scattering (because of kinematics). Consequently in the real situation :

$$C = \sum_i C_i R_i \approx C_{qq} R_{qq} + C_{vs} R_{vs}$$

where $C_{qq} \approx -0.5$, $C_{vs} \approx +0.2$ (with a lower cut-off 0.5 GeV for slow particles), R_{qq} is the ratio of the number of qq scatterings to the total number of elementary scattering processes in a given reaction, with given triggering conditions. R_{vs} is similarly defined for valence-sea scattering.

The values of R_{qq} , R_{vs} depend strongly on the type of experiment (pp or $p\bar{p}$), on the momentum cut-off for triggered high p_t hadrons and on their type (and on the details of the underlying dynamical model). We do not intend to calculate them but just give a rough estimate for these numbers and mention some qualitative features of their dependence.

Specimen calculations ⁸⁾ indicate that for triggering at $x_t \sim 0.35$ one gets events 2-5 times more often from qq scattering than from quark-gluon scattering, i.e., $R_{qq} \sim 0.7-0.8$, $R_{vs} \sim 0.3-0.4$ while at $x_t \sim 0.2$ they are approximately the same.

These ratios, and hence the predicted correlation, depend on some conditions in a characteristic way.

- 1) The anticorrelation should be larger in pp than in $p\bar{p}$ collisions.
- 2) Increasing the momentum cut-off for triggered particles in the two opposite jets one increases the effective values of x_1 and x_2 and hence the valence-valence contribution increases compared with the valence-sea contribution. [A possible constituent interchange mechanism ⁴⁾ will also be suppressed more effectively.] This will lead to an increased anticorrelation in pp scattering. [With the rough estimates quoted above we have $C \sim -0.15$ at $x_t \approx 0.2$ and $C \sim -0.36$ at $x_t = 0.35$.]

- 3) The types of triggered high p_t particles also influence the values of R 's : in the pp reaction triggering on $\pi^+\pi^+$ or K^+K^+ we expect larger anticorrelations than, e.g., for triggering on K^-K^- , since the constituents of the latter are not present in the valence part of the proton.
- 4) All the above values are given for the case when the two high p_t particles are detected between 70° and 110° . Decreasing the acceptance angle $\Delta\theta$ for triggered particles in the two opposite jets, the kinematical effects contained in C_{vs} become less significant ($C_{vs} \rightarrow 0$) hence the anticorrelation increases. (However, the statistics becomes worse.) This dependence offers a possibility to separate the two origins of the correlation.

VII. CONCLUSIONS

We have shown that under simple assumptions the special colour distribution in an elementary qq scattering process leads to (theoretically) measurable effects. These assumptions include, besides the commonly accepted hard QCD process, the scattering via a colour octet exchange, also some simple assumptions about the confining mechanism.

The proposed test is not a decisive one : if the experiment does not show such an asymmetry, QCD can still be right or wrong. However, if the experimental analysis confirms our expectations it would be a strong argument for our assumptions, since it seems difficult to imagine some other mechanisms which can explain such a specific dependence of the correlation of low momentum hadrons on the type and momentum cut-off of the triggered hadrons.

A final remark on higher QCD corrections:

A further gluon exchange between any two of the four departing objects (which can rearrange colour) is suppressed by $\alpha_s(Q^2)$ and there is no $\log Q^2$ factor ("leading log" which usually comes from collinear singularities) since an exchanged gluon cannot be collinear with any two of them at the same time.

It is assumed that non-perturbative confinement forces act within a colour singlet but not between two colour singlets. However, something similar happens already on the perturbative level. The gluon exchanges between two singlets are suppressed by a factor of N^2 with respect to the corresponding gluon exchanges within one singlet, as illustrated graphically in Fig. 9. It is hoped that this virtue of the $1/N$ expansion is more general; it reflects the dynamics of strong interactions⁹⁾.

ACKNOWLEDGEMENTS

The author acknowledges useful discussions with W. Furmanski, R. Horgan, M. Jacob and C. Sachrajda. He would like to thank G. Veneziano for constant help and suggestions.

The author also would like to thank the CERN Theoretical Physics Division for hospitality.

REFERENCES

- 1) F.E. Low, Phys.Rev. D12, 163 (1975).
- 2) S. Nussinov, Phys.Rev.Letters 34, 1286 (1975); Phys.Rev. D14, 246 (1976).
- 3) Z.J. Rek and G. Wilk, Phys.Letters 78B, 333 (1978).
- 4) D. Sivers, S.J. Brodsky and R. Blankenbecler, Phys.Reports 23C, 1 (1976).
- 5) R.D. Field and R.P. Feynman, Nuclear Phys. B136, 1 (1978).
- 6) A. Krzywicki and B. Petersson, Phys.Rev. D6, 924 (1972).
- 7) F. Niedermayer, Nuclear Phys. B79, 355 (1974).
- 8) R. Horgan, private communication.
- 9) G. Veneziano, Proceedings 19th Int.Conf.High Energy Physics, Tokyo (1978).

FIGURE CAPTIONS

- Figure 1 : Colour rearrangement in qq or in $q\bar{q}$ scattering via colour octet exchange.
- Figure 2 : qq scattering contribution to a high p_t event.
- Figure 3 : $q\bar{q}$ scattering contribution to a high p_t event.
- Figure 4 : Colour distribution in a qq scattering subprocess.
- Figure 5 : Result of the jet boost, neglecting p_t in fragmentation.
- Figure 6 : The type of experiment in which anticorrelations due to colour separation could be measured : two triggered high p_t particles at 90° , -90° ; energy fluxes measured around 45° and -45° .
- Figure 7 : The correlation (1) for qq scattering at 90° , -90° . $\overline{p_t}$ is the average transverse momentum of hadrons with respect to jet axes (the real value ~ 0.3 GeV). The dotted curve is for a lower cut-off 0.5 GeV for slow particles in E_1 and E_2 .
- Figure 8 : Kinematical situations contributing to correlations :
a) $C > 0$, b) $C < 0$.
- Figure 9 : Colour exchange within a singlet (a), between two singlets (b). ($1/\sqrt{N}$ is the normalization factor for initial and final singlet states, and each colour loop gives a factor of N .)

The diagram shows an equation between three Feynman diagrams. The first diagram on the left has four external lines labeled i, j, l, k with arrows pointing towards a central vertex. The internal line is a wavy line. The middle diagram has the same external lines and a solid internal line with an arrow pointing upwards. The third diagram on the right has the same external lines and a dotted internal line. An equals sign is between the first and second diagrams, and a minus sign followed by $\frac{1}{N}$ is between the second and third diagrams.

$$\sum_a t_{ij}^a t_{kl}^a = \delta_{il} \delta_{jk} - \frac{1}{N} \delta_{ij} \delta_{kl}$$

FIG. 1

FIG.2

FIG.3

FIG. 4

FIG. 5

FIG.6

FIG. 7

FIG. 8

a) $\left(\frac{1}{\sqrt{N}}\right)^2 \cdot \text{diagram} \sim \left(\frac{1}{\sqrt{N}}\right)^2 \cdot \text{diagram} \propto (g^2 N)^2$

b) $\left(\frac{1}{\sqrt{N}}\right)^4 \cdot \text{diagram} \sim \left(\frac{1}{\sqrt{N}}\right)^4 \cdot \text{diagram} \propto \frac{1}{N^2} (g^2 N)^2$

FIG. 9