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THE USE OF A NON-DIAGONAL WEIGHT MATRIX FOR
MOMENTUM DETERMINATION WITH MAGNETIC SPECTROMETERS

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ABSTRACT

An earlier method of determining five track parameters of a particle by inverting a 5×5 matrix is generalized to the case of known correlations between the expected deviations from a simple model (error matrix).

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We have seen¹⁻³⁾ that the momentum, as well as four other unknown track parameters, of a particle in high-energy physics experiments can be determined by making a least squares fit to observations y_i and z_i , using the model

$$\begin{aligned} y(x) &= a_1 + a_2 x + dY(x) \\ z(x) &= b_1 + b_2 x + dZ(x) . \end{aligned} \quad (1)$$

We thus compare the track model $y(x)$, $z(x)$ with observations y_i , z_i for $i = 1, \dots, N$ taken at $x = x_i$; d is the inverse of the as yet unknown momentum in normalized units, a_1 and b_1 are respectively fitted values of y and z at $x = 0$, a_2 and b_2 are respectively the fitted values of the derivatives y' and z' at $x = 0$.

The definition of the functions $Y(x)$ and $Z(x)$ is

$$\begin{aligned} Y(x) &= \iint_{x_1 x_1}^{x \eta} (1+y'^2+z'^2)^{\frac{1}{2}} \left[B_x z' + B_y y' z' - B_z (1+y'^2) \right] d\xi d\eta \\ Z(x) &= \iint_{x_1 x_1}^{x \eta} (1+y'^2+z'^2)^{\frac{1}{2}} \left[-B_x y' - B_z y' z' + B_y (1+z'^2) \right] d\xi d\eta . \end{aligned}$$

The magnetic field components B_x , B_y , and B_z as well as the derivatives y' and z' are all functions of x . The integrands can be initially estimated with an appropriate interpolation procedure and then iteratively improved. If there is no energy loss or scattering, then $Y(x) = y''(x)/d$ and $Z(x) = z''(x)/d$. We now consider, however, the case that there may be multiple scattering. This we take into account by using a non-diagonal weight matrix $\tilde{W}$, which can be obtained by inverting the, supposedly known, error matrix^{4,5)}. We assume that at $x = x_i$ the expected deviations in $y(x)$ and $z(x)$ are the same. We thus want to minimize

$$S = \tilde{g}_y^T \tilde{W} \tilde{g}_y + \tilde{g}_z^T \tilde{W} \tilde{g}_z ,$$

where

$$\tilde{g}_y \equiv y - e a_1 - x a_2 - Y d$$

with

e a vector with unit elements

x a vector with elements x_i

$\underline{\underline{Y}}$ a vector with elements $Y(x)$ for $x = x_i$
and analogously

$$\underline{\underline{g}}_Z \equiv \underline{\underline{z}} - \underline{\underline{e}}b_1 - \underline{\underline{x}}b_2 - \underline{\underline{Z}}d .$$

We now use abbreviations like

$$(\underline{\underline{x}}, \underline{\underline{y}}) \equiv (\underline{\underline{y}}, \underline{\underline{x}}) \equiv \underline{\underline{x}}^T \underline{\underline{W}} \underline{\underline{y}} \equiv \sum_i \sum_j w_{ij} x_i y_j$$

with $w_{ij} = w_{ji}$ elements of $\underline{\underline{W}}$, so that we write

$$S = (\underline{\underline{g}}_Y, \underline{\underline{g}}_Y) + (\underline{\underline{g}}_Z, \underline{\underline{g}}_Z) . \quad (2)$$

We note that, for example,

$$\begin{aligned} \frac{\partial S}{\partial a_2} &= -2(\underline{\underline{g}}_Y, \frac{\partial \underline{\underline{g}}_Y}{\partial a_2}) - 2(\underline{\underline{g}}_Z, \frac{\partial \underline{\underline{g}}_Z}{\partial a_2}) \\ &= -2\{(\underline{\underline{y}}, \underline{\underline{x}}) - (\underline{\underline{e}}, \underline{\underline{x}})a_1 - (\underline{\underline{x}}, \underline{\underline{x}})a_2 - (\underline{\underline{Y}}, \underline{\underline{x}})d\} . \end{aligned}$$

If we now, in order to minimize S , put

$$\frac{\partial S}{\partial a_1} = \frac{\partial S}{\partial a_2} = \frac{\partial S}{\partial d} = \frac{\partial S}{\partial b_1} = \frac{\partial S}{\partial b_2} = 0$$

we get five linear equations with five unknowns:

$$(\underline{\underline{e}}, \underline{\underline{e}})a_1 + (\underline{\underline{x}}, \underline{\underline{e}})a_2 + (\underline{\underline{Y}}, \underline{\underline{e}})d = (\underline{\underline{y}}, \underline{\underline{e}}) \quad (3)$$

$$(\underline{\underline{e}}, \underline{\underline{x}})a_1 + (\underline{\underline{x}}, \underline{\underline{x}})a_2 + (\underline{\underline{Y}}, \underline{\underline{x}})d = (\underline{\underline{y}}, \underline{\underline{x}}) \quad (4)$$

$$(\underline{\underline{e}}, \underline{\underline{Y}})a_1 + (\underline{\underline{x}}, \underline{\underline{Y}})a_2 + \{(\underline{\underline{Y}}, \underline{\underline{Y}}) + (\underline{\underline{Z}}, \underline{\underline{Z}})\}d + (\underline{\underline{e}}, \underline{\underline{Z}})b_1 + (\underline{\underline{x}}, \underline{\underline{Z}})b_2 = (\underline{\underline{y}}, \underline{\underline{Y}}) + (\underline{\underline{z}}, \underline{\underline{Z}}) \quad (5)$$

$$(\underline{\underline{Z}}, \underline{\underline{e}})d + (\underline{\underline{e}}, \underline{\underline{e}})b_1 + (\underline{\underline{x}}, \underline{\underline{e}})b_2 = (\underline{\underline{z}}, \underline{\underline{e}}) \quad (6)$$

$$(\underline{\underline{Z}}, \underline{\underline{x}})d + (\underline{\underline{e}}, \underline{\underline{x}})b_1 + (\underline{\underline{x}}, \underline{\underline{x}})b_2 = (\underline{\underline{z}}, \underline{\underline{x}}) \quad (7)$$

Eliminating a_2 from Eqs. (3) and (4) we get

$$a_1 = [T_1 - dT_2]/T_3, \quad (8)$$

where

$$T_1 = (\underline{y}, \underline{e})(\underline{x}, \underline{x}) - (\underline{e}, \underline{x})(\underline{x}, \underline{y}) \quad (9)$$

$$T_2 = (\underline{Y}, \underline{e})(\underline{x}, \underline{x}) - (\underline{e}, \underline{x})(\underline{x}, \underline{Y}) \quad (10)$$

and

$$T_3 = (\underline{e}, \underline{e})(\underline{x}, \underline{x}) - (\underline{e}, \underline{x})(\underline{e}, \underline{x}) . \quad (11)$$

Eliminating a_1 from Eqs. (3) and (4), we get

$$a_2 = [T_4 - dT_5]/T_3 , \quad (12)$$

where

$$T_4 = (\underline{e}, \underline{e})(\underline{x}, \underline{y}) - (\underline{e}, \underline{x})(\underline{e}, \underline{y}) \quad (13)$$

$$T_5 = (\underline{e}, \underline{e})(\underline{x}, \underline{Y}) - (\underline{e}, \underline{x})(\underline{e}, \underline{Y}) . \quad (14)$$

The expressions for b_1 and b_2 are analogous:

$$b_1 = [T_6 - dT_7]/T_3 \quad (15)$$

$$b_2 = [T_8 - dT_9]/T_3 , \quad (16)$$

where

T_6 is like T_1 with z instead of y and Z instead of Y ,

T_7 is like T_2 with z instead of y and Z instead of Y ,

T_8 is like T_4 with z instead of y and Z instead of Y ,

and

T_9 is like T_5 with z instead of y and Z instead of Y .

Substitution of Eqs. (8), (12), (15), and (16) into (5) then gives, after multiplication with T_3 and collecting the terms with d ,

$$d = \{(\underline{e}, \underline{Y})T_1 + (\underline{x}, \underline{Y})T_4 + (\underline{e}, \underline{Z})T_6 + (\underline{x}, \underline{Z})T_8 - (\underline{y}, \underline{Y})T_3 - (\underline{z}, \underline{Z})T_3\}/A \quad (17)$$

with the denominator

$$A = (\underline{e}, \underline{Y})T_2 + (\underline{x}, \underline{Y})T_5 + (\underline{e}, \underline{Z})T_7 + (\underline{x}, \underline{Z})T_9 - (\underline{Y}, \underline{Y})T_3 - (\underline{Z}, \underline{Z})T_3 .$$

This also determines the other four track parameters, a_1 , a_2 , b_1 , and b_2 when Eq. (17) is substituted into Eqs. (8), (12), (15), and (16). This is almost the same as for the non-weighted fit¹⁻³⁾, but the definition of T_1 , T_2 , ..., T_9 is more complicated and the evaluations of these now require a number of operations proportional to N^2 rather than N (the number of detectors).

Analogously to Ref. 3 we can now also estimate how much a change in y_k or z_k contributes to a change in the above five track parameters. We can assume no variation in Y , Z and x . This is justified if the variation of $\int B_{\perp} d\ell$ (ℓ being the path) can be neglected for small variations in y_k and z_k .

We see from Eqs. (9) to (11), (13), and (14) that only T_1 and T_4 vary if y_k varies and analogously only T_6 and T_8 for variations in z_k . The denominator, A , does not depend on y_k or on z_k . Differentiation gives

$$\frac{\partial T_1}{\partial y_k} = \frac{\partial T_6}{\partial z_k} = (e, e_k)(x, x) - (e, x)(x, e_k)$$

where e_k is a vector with all elements equal to zero except for the k -th element which is equal to unity; so that, for example

$$(x, e_k) = \sum_i w_{ik} x_i$$

We further find

$$\frac{\partial T_4}{\partial y_k} = \frac{\partial T_8}{\partial z_k} = (e, e)(x, e_k) - (e, x)(e, e_k).$$

So that

$$\frac{\partial d}{\partial y_k} = \{(e, Y)(e, e_k)(x, x) - (e, Y)(e, x)(x, e_k) + (x, Y)(e, e)(x, e_k)$$

$$- (e, Y)(e, x)(e, e_k) - T_3(e_k, Y)\}/A$$

$$= \{(e, e_k)T_7 + (x, e_k)T_5 - (Y, e_k)T_3\}/A$$

and analogously

$$\frac{\partial d}{\partial z_k} = \{(e, e_k)T_7 + (x, e_k)T_9 - (Z, e_k)T_3\}/A$$

For the effect on the other parameters we find

$$\frac{\partial a_1}{\partial y_k} = \{(e, e_k)(x, x) - (e, x)(x, e_k) - T_2 \frac{\partial d}{\partial y_k}\}/T_3$$

$$\frac{\partial a_1}{\partial z_k} = - T_2 \frac{\partial d}{\partial z_k} / T_3$$

$$\frac{\partial a_2}{\partial y_k} = \{ (\tilde{e}, \tilde{e})(\tilde{x}, \tilde{e}_k) - (\tilde{e}, \tilde{x})(\tilde{e}, \tilde{e}_k) - T_5 \frac{\partial d}{\partial y_k} \} / T_3$$

$$\frac{\partial a_2}{\partial z_k} = - T_5 \frac{\partial d}{\partial z_k} / T_3$$

and, once more, analogously for b_1 and b_2 .

We did numerical tests on all of the above formulae.

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