Ref.TH.2726-CERN

MULTILEPTON CONFIGURATIONS IN b AND t DECAYS

N. Cabibbo
Istituto di Fisica dell'Università, Roma
INFN - Sezione di Roma

and

L. Maiani
CERN -- Geneva
and
Istituto di Fisica dell'Università. Roma

ABSTRACT

The probabilities of emitting one or more prompt charged leptons in b and t decays and in the decay of a $b\bar{b}$ or $t\bar{t}$ pair are computed and found to be large. Multilepton configurations can thus be used to identify the production of new flavours, and to obtain information on the weak mixing angles.

Ref.TH.2726-CERN

22 August 1979

According to the present theoretical picture, the disintegration of heavy flavours into normal, strange or not strange hadrons and leptons, involves many steps in cascade. For instance, t decay may occur through the sequence:

$$\begin{array}{lcl} t & \rightarrow & b + X \\ & \searrow & \downarrow \\ & & c + Y \\ & & \searrow \\ & & s + Z \end{array} \quad (1)$$

where X, Y and Z are normal hadrons or leptons. X and Y may themselves contain charmed hadrons, which would give rise to additional cascade decays. Since at each step there is a sizeable probability ($20 \div 30 \%$) of emitting a charged lepton, the probability for finding one or more leptons among the products of the over-all decay turns out to be rather large. This fact suggests a possible signature for identifying the production of new flavours, for example, in e^+e^- collisions^{*)}. Furthermore, a measurement of the details of the cascade, e.g., the relative probabilities of events with a different number of charged leptons, may give information on the values of the weak mixing angles.

In this note we present a discussion of multi-lepton configurations in b and t decay. More specifically we will:

- i) evaluate the branching ratios for the different b and t decay channels;
- ii) evaluate the probabilities for the emission of one or more leptons in the over-all b and t weak decays and in the decays of $b\bar{b}$ or $t\bar{t}$ pairs.

As a basis of our discussion, we consider the model based on three left-handed doublets, in which the weak mixing is determined by three angles and one CP violating phase^{1),2),3)}.

In evaluating rates for non-leptonic channels of b decay, we take into account the leading QCD corrections^{4),5)}, i.e., corrections involving powers of $\alpha_s \ln(M_W/M_b)$, as well as phase space corrections due to the finite mass of final quarks. Phase space corrections are included also in semi-leptonic decays. Since, at present, there is no computation of non-leading QCD corrections for non-leptonic channels [i.e., corrections of order $\alpha_s(M_b)$] in the evaluation of branching ratios we neglect, for consistency, the $\alpha_s(M_b)$ corrections to semi-leptonic rates computed in Ref. 6).

*) This could be used in conjunction with other proposed signatures, such as a high sphericity.

For $M_t \gtrsim 15$ GeV, the distinction between leading and non-leading QCD corrections is not very meaningful, and the leading corrections are anyway small. For this reason we shall omit QCD corrections to t decay, in the present discussion. To describe the weak mixing we shall adopt the parametrization given in Ref. 7), according to which the transitions generated by the charged weak currents are:

$$u \rightleftharpoons \cos\beta d_c + \sin\beta b \quad (2)$$

$$c \rightleftharpoons \cos\gamma s_c + e^{i\delta} \sin\gamma (-\sin\beta d_c + \cos\beta b) \quad (3)$$

$$t \rightleftharpoons -\sin\gamma e^{-i\delta} s_c + \cos\gamma (-\sin\beta d_c + \cos\beta b) \quad (4)$$

In Eqs (2) to (4) all fields are understood to be left-handed and we have used the shorthand notation:

$$\begin{aligned} d_c &= \cos\theta d + \sin\theta s \\ s_c &= -\sin\theta d + \cos\theta s \end{aligned} \quad (5)$$

This parametrization is more convenient than the KM one¹⁾, in that all non-diagonal transitions have a linear term in β, γ and θ . Furthermore, $\tan\theta$ gives exactly the ratio of $u \rightarrow s$ to $u \rightarrow d$ coupling as in the four quark model, and $\sin^2\beta$ gives directly the size of the violation of the usual weak interaction universality.

The range of the mixing angles can be chosen to be:

$$\begin{aligned} 0 &\leq \theta, \beta \leq \pi/2 \\ -\pi/2 &\leq \gamma, \delta \leq \pi/2 \end{aligned} \quad (6)$$

Further restrictions on β and γ can be derived from^{*)}:

- i) limits on the violation of weak universality;
- ii) $K_L - K_S$ mass difference;
- iii) the rate of $D \rightarrow \pi\pi$ versus $D \rightarrow K\pi$.

^{*)} Bounds on the weak mixing angles in the KM parametrization have been studied in Refs 8),9),10).

The resulting bounds are¹¹⁾:

$$0 \lesssim \beta \lesssim .50 \approx .12 \quad (7)$$

$$-.4 \lesssim \sin \gamma \lesssim 0 \quad (8)$$

The restriction of $\sin \gamma$ to negative values arises from the $D \rightarrow \pi\pi$ branching ratio^{10),11)}.

In Table 1, first column, we give the rates for b quark decay. Rates are in units of:

$$\Gamma_0^b = \frac{G^2}{192 \pi^3} M_b^5 \quad (9)$$

The numerical factors include: i) phase space corrections (we use: $M_b = 5$ GeV, $M_c = 1.75$ GeV, $M_s = 1.80$ GeV); ii) in the case of non-leptonic decays, an enhancement factor^{4),5)}:

$$2 f_+^2 + f_-^2 \approx 3.9 \quad (10)$$

In the absence of gluon corrections, the factor in Eq. (10) reduces to the colour multiplicity, equal to three.

Adding up the entries of the first column of Table 1, one obtains the total b rate as a function of β and γ :

$$\begin{aligned} \Gamma^b = \Gamma_0^b & \left(2.75 \sin^2 \gamma + 7.69 \sin^2 \beta - 5.75 \sin^2 \beta \sin^2 \gamma - \right. \\ & \left. - .25 \sin^4 \gamma \right) + O(\beta^4) \end{aligned} \quad (11)$$

Using the bounds Eqs (7) and (8), we find^{*)}:

$$\begin{aligned} \Gamma^b & \lesssim .53 \Gamma_0^b \\ \tau^b & \gtrsim .32 \left(\frac{5 \text{ GeV}}{M_b} \right)^5 \cdot 10^{-13} \text{ sec} \end{aligned} \quad (12)$$

*) A similar bound has been given in Ref. 12).

This bound does not include small QCD corrections, and puts the b lifetime in a range still suitable to emulsion study. The last three columns of Table 1 give, for each decay channel, the probability for the final state to contain zero, one or two prompt charged leptons (either e or μ). The leptons can be produced either directly, or from the subsequent c and τ decay. We have assumed:

$$\begin{aligned} B(c \rightarrow e, \mu) &= .20 \\ B(\tau \rightarrow e, \mu) &= .34 \end{aligned} \quad (13)$$

From Table 1, one can easily compute the probabilities for the emission of k prompt charged leptons in b decay, B_k ($k = 0, 1, 2$). Neglecting terms higher than second order in $\sin\beta$ and $\sin\gamma$, one obtains:

$$\begin{aligned} B_0 &= (1.48 \sin^2\gamma + 5.28 \sin^2\beta) / R \\ B_1 &= (1.09 \sin^2\gamma + 2.41 \sin^2\beta) / R \\ B_2 &= .18 \sin^2\gamma / R \\ R &= 2.75 \sin^2\gamma + 7.69 \sin^2\beta \end{aligned}$$

The B_k depend only upon the ratio $\sin^2\beta/\sin^2\gamma$. For all values of this ratio, the probability for emitting one lepton is rather high:

$$\begin{aligned} B_1 &\approx .40 \quad (\beta=0) \\ B_1 &\approx .32 \quad (\beta \gg \gamma) \end{aligned} \quad (14)$$

B_2 is much smaller, but not negligible for small β :

$$B_2 \approx 7 \cdot 10^{-2} \quad (\beta=0) \quad (15)$$

Given the B_k , one can further compute the probabilities for the emission of n charged leptons in the weak decay of a $B\bar{B}$ pair, which we denote by $(B\bar{B})_n$. In Fig. 1 we have reported the curves for $(B\bar{B})_0, (B\bar{B})_1$ and for the probability of emitting two or more charged leptons, $(B\bar{B})_{\geq 2}$ (the latter is dominated by the $n = 2$ configuration). As anticipated, lepton emission occurs with high probability: $(B\bar{B})_0$ is smaller than 50%. The probability for two or more charged leptons is the most sensitive to the ratio $\sin^2\beta/\sin^2\gamma$.

We consider now the decay of a t quark. The rate for any given channel can be written as:

$$\begin{aligned}\Gamma(t \rightarrow \alpha) &= \Gamma_0^t X_\alpha g_\alpha \\ \Gamma_0^t &= \frac{G^2}{192\pi^3} M_t^5\end{aligned}\tag{16}$$

We have denoted by X_α the weak coupling factor, including colour multiplicity, while g_α is a phase space correction. Table 2 gives the expressions of X_α and the numerical value of g_α for the three cases: $M_t = 15, 20, 25$ GeV.

Irrespective of the values of the mixing angles, the t lifetime is expected to be extremely short, about 10^{-17} sec for $M_t = 15$ GeV.

Combining Table 2 with Table 1, one may obtain the probabilities for the emission of k prompt charged leptons in t decay, T_k ($k = 0, 1, \dots, 5$). For a given t mass, T_k depends now separately from β and γ . This dependence arises from two different effects. The first is the dependence of b decays upon the ratio $\sin^2\beta/\sin^2\gamma$, the second is due to the appearance, for appreciable $\sin^2\gamma$, of decay channels which do not involve b particles, and which can give rise at most to one charged lepton. Thus $T_k, k \geq 2$, is rather sensitive to the value of γ . To illustrate this, we have reported in Fig. 2 the probabilities T_0, T_1 and $T \geq 2$, as a function of $\sin\gamma$, for the case $\beta = 0$, $M_t = 15$ GeV. Similarly to the $b\bar{b}$ case, one can compute the probabilities for multi-lepton configurations in the weak decays of a $t\bar{t}$ pair, which we denote by $(T\bar{T})_n, n=0, \dots, 10$. $t\bar{t}$ decay turns out to be dominated by configurations with one or more prompt leptons. The average number of prompt leptons per event is of the order of two, in the most favourable case of small γ , and the zero lepton configuration occurs with less than 20% probability, for γ in the range given by Eq. (8). In Fig. 3 we give the values of $(T\bar{T})_0, (T\bar{T})_1, (T\bar{T})_{\geq 2}, (T\bar{T})_{\geq 4}$ as functions of $\sin\gamma$, in the case: $\beta = 0$, $M_t = 15$ GeV.

In conclusion, our analysis indicates that lepton emission in b and t decay occurs with large probability. This may be relevant either as a signature for the opening of the associated thresholds (especially in e^+e^- collisions) or as a tool for determining the values of the mixing angles, β and γ . In the present paper, we have restricted our analysis to the total rates. Further information is contained in the lepton energy distributions¹²⁾. The analysis of this would be quite straightforward, but it can be meaningful only with respect

to a well-determined experimental set-up. Finally we stress that a measurement of the b lifetime should be possible (e.g., by the emulsion technique) and it would give precious information on the value of the mixing angles.

Table 1

First column: decay rates of the b quark in units of Γ_0^b , Eq. (9); ℓ denotes either e or μ , ν_ℓ the corresponding neutrino; $s_\gamma \equiv \sin\gamma$, $c_\gamma \equiv \cos\gamma$ etc. Third to fifth column: probability for each channel to yield 0,1 or 2 prompt charged leptons.

Decay channel	Γ/Γ_0^b	P_0	P_1	P_2
$c \ell^- \bar{\nu}_\ell$	$0.41 s_\gamma^2 c_\beta^2$	0.00	0.80	0.20
$c \tau^- \bar{\nu}_\tau$	$0.08 s_\gamma^2 c_\beta^2$	0.53	0.40	0.07
$u \ell^- \bar{\nu}_\ell$	s_β^2	0.00	1.00	0.00
$u \tau^- \bar{\nu}_\tau$	$0.39 s_\beta^2$	0.66	0.34	0.00
$c \bar{u} d_c$	$1.6 s_\gamma^2 c_\beta^4$	0.80	0.20	0.00
$c \bar{c} s_c$	$0.25 c_\gamma^2 s_\gamma^2 c_\beta^2$	0.64	0.32	0.04
$u \bar{u} d_c$	$3.9 s_\beta^2 c_\beta^2$	1.00	0.00	0.00
$u \bar{c} s_c$	$1.4 s_\beta^2 c_\gamma^2$	0.80	0.20	0.00

Table 2

Weak couplings and phase space factors for t decay, see Eq. (16). ℓ denotes either e or μ ; q denotes either d or s . In evaluating X_α the rates for the latter two cases have been added. The s quark mass has been neglected in phase space factors.

Decay channel	X_α	g_α		
		$M_t = 15$	$M_t = 20$	$M_t = 25$
$b \ell^+ \nu_\ell$	$c_\beta^2 c_\gamma^2$	0.45	0.63	0.74
$b \tau^+ \nu_\tau$	$c_\beta^2 c_\gamma^2$	0.38	0.58	0.71
$q \ell^+ \nu_\ell$	$s_\gamma^2 + c_\gamma^2 s_\beta^2$	1.0	1.0	1.0
$q \tau^+ \nu_\tau$	$s_\gamma^2 + c_\gamma^2 s_\beta^2$	0.89	0.93	0.96
$b u \bar{q}$	$3 c_\gamma^2 c_\beta^4$	0.45	0.63	0.74
$b c \bar{q}$	$3 c_\gamma^2 c_\beta^2 (c_\gamma^2 + s_\gamma^2 s_\beta^2)$	0.37	0.58	0.71
$q u \bar{q}$	$3 c_\beta^2 (s_\gamma^2 + c_\gamma^2 s_\beta^2)$	1.0	1.0	1.0
$q c \bar{q}$	$3 (c_\gamma^2 + s_\gamma^2 s_\beta^2) (s_\gamma^2 + c_\gamma^2 s_\beta^2)$	0.88	0.93	0.96
$q \bar{b} c$	$3 s_\gamma^2 c_\beta^2 (s_\gamma^2 + c_\gamma^2 s_\beta^2)$	0.38	0.59	0.71
$q \bar{b} u$	$3 s_\beta^2 (s_\gamma^2 + c_\gamma^2 s_\beta^2)$	0.45	0.63	0.74
$b \bar{b} c$	$3 c_\gamma^2 s_\gamma^2 c_\beta^4$	0.08	0.30	0.49
$b \bar{b} u$	$3 c_\gamma^2 c_\beta^2 s_\beta^2$	0.12	0.36	0.52

REFERENCES

- 1) M. Kobayashi and K. Maskawa, *Progr. Theor. Phys.* 49 (1973) 652.
- 2) L. Maiani, *Phys. Letters* 62B (1976) 183;
S. Pakvasa and H. Sugawara, *Phys. Rev. D* 14 (1976) 305.
- 3) J. Ellis, M.K. Gaillard and D. Nanopoulos, *Nucl. Phys.* B109 (1976) 213.
- 4) M.K. Gaillard and B.W. Lee, *Phys. Rev. Letters* 33 (1974) 108;
G. Altarelli and L. Maiani, *Phys. Letters* 52B (1974) 351.
- 5) G. Altarelli, N. Cabibbo and L. Maiani, *Phys. Rev. Letters* 35 (1975) 635;
J. Ellis, M.K. Gaillard and D. Nanopoulos *Nucl. Phys.* B100 (1975) 313.
- 6) N. Cabibbo and L. Maiani, *Phys. Letters* 79B (1978) 109;
M. Suzuki, *Nucl. Phys.* B145 (1978) 420.
- 7) L. Maiani, *Proc. of the 1977 Int. Symp. on Lepton and Photon Interactions at High Energies, Hamburg.*
- 8) J. Ellis, M.K. Gaillard, D. Nanopoulos and S. Rudaz, *Nucl. Phys.* B109 (1976) 213.
- 9) R.E. Shrock, S.B. Treiman and L.L. Wang, *Phys. Rev. Letters* (to be published);
V. Barger, W.F. Long and S. Pakvasa, *Phys. Rev. Letters* (to be published);
L. Wolfenstein, *Carnegie-Mellon Univ. preprint*, C00-3066-124, 1979.
- 10) M. Suzuki, *Berkeley LBL preprint*, 1979;
V. Barger, S. Pakvasa, *Univ. of Wisconsin preprint* 1979.
- 11) G. Altarelli, N. Cabibbo and L. Maiani, *in preparation*. See also M.K. Gaillard and L. Maiani in: *Quarks and Leptons, Proc. of the 1979 Cargèse Summer School.*
- 12) V. Barger, W.F. Lang and S. Pakvasa, *Univ. of Hawaii, Univ. of Wisconsin preprint*, 1979.
- 13) N. Cabibbo, G. Corbò and L. Maiani, *Nucl. Phys.* B155 (1979) 93;
A. Ali and E. Pietarinen, *Nucl. Phys.* B154 (1979) 519.

FIGURE CAPTIONS

Fig. 1 : Probabilities for prompt lepton emission in $b\bar{b}$ weak decay versus $\sin^2\beta/\sin^2\gamma$. $a = (B\bar{B})_0$; $b = (B\bar{B})_1$; $c = (B\bar{B})_{\geq 2} \equiv (B\bar{B})_2 + (B\bar{B})_3 + (B\bar{B})_4$.

Fig. 2 : Probabilities for prompt lepton emission in t decay versus $\sin\gamma$.
 $a = T_0$, $b = T_1$, $c = T_{\geq 2} \equiv T_2 + \dots + T_5$.

Fig. 3 : Probabilities for prompt lepton emission in the weak decay of a $t\bar{t}$ pair, versus $\sin\gamma$. $a = (T\bar{T})_0$; $b = (T\bar{T})_1$; $c = (T\bar{T})_{\geq 2} \equiv (T\bar{T})_2 + \dots + (T\bar{T})_{10}$; $d = (T\bar{T})_{\geq 4} \cdot 10 \equiv [(T\bar{T})_4 + \dots + (T\bar{T})_{10}] \cdot 10$.

Fig.1

Fig. 2

Fig. 3