Ref.TH.2746-CERN
LAPP-TH-05

FERMION MASSES AND HIGGS REPRESENTATIONS IN SU(5)

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A B S T R A C T

We argue that the observed pattern of fermion masses, including m_d and m_e , is consistent with the simplest SU(5) grand unified model with just a 24 and a 5 of Higgs fields, if one takes into account possible effective low-energy apparently non-renormalizable interactions scaled by inverse powers of the Planck mass. We give arguments in favour of the simplest possible structure of Higgs representations in SU(5).

Ref.TH.2746-CERN
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5 October 1979

The simplest grand unified theory ¹⁾ of strong, weak and electromagnetic interactions is based on the group $SU(5)$ ²⁾. It is frequently used as a prototype for testing new ideas in the same way that the $SU(2) \times U(1)$ Weinberg-Salam model ³⁾ has long been a standard of comparison for weak electromagnetic unification theories. It now seems as if that "standard model" may actually be correct, at least as a low energy limit of the true theory. May one hope that $SU(5)$ is the "correct" grand unified theory? The model does have some phenomenological problems - for example the calculated ^{4),5)} value of $\sin^2 \theta_w$ is one or two standard deviations below the present experimental value ⁶⁾. More serious problems have arisen with the fermion masses ^{2),5)} in the theory. Although m_b comes out all right, m_s may be somewhat too low, while the most serious problem arises with m_d/m_e . The simplest $SU(5)$ model with only 5's and a 24 of Higgs fields predicts ^{2),5)}

$$\frac{m_d}{m_s} = \frac{m_e}{m_\mu} \quad (1)$$

for the ratio of short-distance current quark masses, which would be $\sim 1/200$ instead of the $0(1/20)$ deduced from current algebra calculations using the pion and kaon masses ⁷⁾. The $SU(5)$ model is qualitatively correct in predicting m_d to be small, but the quantitative prediction (1) is wrong. This failure has been used ⁸⁾ as an argument in favour either of modifying the simple $SU(5)$ model by the inclusion of more Higgs fields in a 45, or of abandoning $SU(5)$ altogether. Arguments to do with generating the baryon number of the universe have also been used ⁹⁾ to motivate more complicated grand unified models.

We argue that the failure of the relation (1) does not necessarily exclude $SU(5)$ with just one (or more) 5 of Higgs fields, and no 45. Our point is that since the grand unification mass is only a few $(4\frac{1}{2}?)$ ¹⁰⁾ orders of magnitude below the Planck mass m_P , it may not always be possible to ignore the Planck mass scale in grand unification considerations, and the relation (1) may be too naïve. In particular, one might expect contributions to the fermion mass matrix due to fermion-antifermion-(Higgs)² interactions which are apparently non-renormalizable but scaled by an inverse power of the Planck mass. One can estimate these contributions to the fermion mass matrix to be a few MeV, sufficient to destroy the unsuccessful relation (1) without affecting other successful predictions of the simplest $SU(5)$ model.

In passing, we also argue that the $SU(5)$ prediction for m_s is not obviously unsuccessful, give a renormalization group argument against the inclusion of a 45 Higgs in the $SU(5)$ model, and recall various arguments against multiple Higgs fields in grand unified models.

Gauge theories are interesting because they are the most general class of renormalizable field theories, and we certainly need a field theory to be renormalizable if it is to function calculably all the way from $m_P \sim 10^{19}$ GeV down to atomic mass scales. On the other hand we know that heavy particles can generate apparently non-renormalizable interactions at low energies. A classical example is the low-energy four-fermion effective interaction mediated by vector boson exchange :

$$\mathcal{L}_{\text{eff}} = \frac{G_F}{\sqrt{2}} (\bar{f} \gamma_\mu (1 - \gamma_5) f') (\bar{f}'' \gamma^\mu (1 - \gamma_5) f''') : \frac{G_F}{\sqrt{2}} = \frac{g^2}{8m_W^2} \quad (2)$$

Similar interactions which are apparently non-renormalizable are generated by the heavy (X, Y : masses $\sim 10^{14}$ to 10^{15} GeV) bosons in grand unified theories. One example ⁵⁾ is a four-fermion interaction analogous to (2) :

$$\frac{G_{Gu}}{\sqrt{2}} = \frac{g^2}{8m_X^2} \quad (3)$$

which mediates baryon decay in such models. The next identifiable mass scale is the Planck mass, and we already know that the apparently non-renormalizable low energy interactions of gravitons are characterized by inverse powers of the Planck mass. We do not know whether this reflects a composite structure of the vertices on a Planck scale, or whether gravitational vertices are intrinsically dimensional, proportional to inverse powers of the Planck mass. The latter possibility is realized ¹¹⁾ in supergravity theories, which also contain a lot of other apparently non-renormalizable interactions. The occurrence of apparently non-renormalizable interactions with a Planck mass scale should not frighten us : when higher order calculations are performed there is presumably a cut-off in the loop momenta of order m_P , so that the scale parameter of such interactions stays $O(m_P)$. Indeed, a purist might argue that the occurrence of such interactions is inevitable given one unrenormalizable interaction (gravity) characterized by this scale. There are in fact calculations ¹²⁾ of apparently non-renormalizable elementary particle interactions of this magnitude generated by non-perturbative gravitational effects.

We therefore argue that the conventional gauge theory Lagrangian should be extended to include arbitrary gauge invariant interactions of mass dimension $d > 4$ scaled by inverse powers of the Planck mass :

$$\mathcal{L} = \mathcal{L}_{d \leq 4}^{\text{Ren.}} + \sum_{d=5}^{\infty} \left(\frac{1}{m_P}\right)^{d-4} \mathcal{L}_d^{\text{unren.}} \quad (4)$$

What are the practical implications of such terms ? Precious few, in view of the enormous ratio between m_P and our conventional interaction scales $\leq m_W = O(10^2)$ GeV. But the grand unification mass of 10^{14} to 10^{15} GeV is not so far below $m_P \sim 10^{19}$ GeV, and the apparently non-renormalizable terms in (4) may have observable implications for grand unified theories. As mentioned earlier, one such implication that we see is for the fermion mass matrix, and to this we now turn.

The lowest dimensional non-renormalizable interactions have dimension $d=5$ - either (Higgs)⁵ or $f\bar{f}$ -(Higgs)². Many terms of the latter type are possible : let us select the SU(5) example

$$\mathcal{L} = O\left(\frac{1}{m_P}\right) \left[\bar{f}_5 H_{24} f_{10} H_5 + \text{hermitian conjugate} \right] \quad (5)$$

This interaction is SU(5) invariant, and could be generated by the exchange of particles with masses $O(m_P)$, but we do not commit ourselves to this mechanism for its existence. Consider an interaction of the form (5) involving the first generation of fermions :

$$\mathcal{L}_1 = O\left(\frac{1}{m_P}\right) (\bar{d}_R, \bar{d}_Y, \bar{d}_B, e^-, \nu_e)_L H_{24} \begin{pmatrix} 0 & \bar{u}_B & -\bar{u}_Y & u_R & d_R \\ -\bar{u}_B & 0 & \bar{u}_R & u_Y & d_Y \\ \bar{u}_Y & -\bar{u}_R & 0 & u_B & d_B \\ -u_R & -u_Y & -u_B & 0 & e^+ \\ -d_R & -d_Y & -d_B & -e^+ & 0 \end{pmatrix}_L H_5 + (\text{hermitian conjugate}) \quad (6)$$

When the first stage of symmetry breaking : $SU(5) \rightarrow SU(3) \times SU(2) \times U(1)$ occurs, H_{24} acquires a vacuum expectation value

$$\langle 0 | H_{24} | 0 \rangle = v \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -\frac{3}{2} \\ 0 & 0 & 0 & -\frac{3}{2} \end{pmatrix} \quad (7)$$

where v is related ⁵⁾ to m_X :

$$v = \frac{2\sqrt{2}}{5g} m_X \quad (8)$$

in terms of the gauge coupling g . Inserting the vacuum expectation value (7) into $\mathcal{L}_1$, Eq. (6), we get

$$\mathcal{L}_1 = O\left(\frac{m_X}{m_P}\right) \left(\frac{2\sqrt{2}}{5g}\right) (\bar{d}_R, \bar{d}_Y, \bar{d}_B, -\frac{3}{2}e^-, -\frac{3}{2}\nu_e)_L \begin{pmatrix} 0 & \bar{u}_B & -\bar{u}_Y & u_R & d_R \\ -\bar{u}_B & 0 & \bar{u}_R & u_Y & d_Y \\ \bar{u}_Y & -\bar{u}_R & 0 & u_B & d_B \\ -u_R & -u_Y & -u_B & 0 & e^+ \\ -d_R & -d_Y & -d_B & -e^+ & 0 \end{pmatrix}_L H_5 + (\text{herm. conj.}), \quad (9)$$

Going down now to the second stage of symmetry breaking : $SU(2) \times U(1) \rightarrow U(1)$, the H_5 field acquires a vacuum expectation value

$$\langle 0 | H_5 | 0 \rangle = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ \frac{v_0}{\sqrt{2}} \end{pmatrix} : \quad v_0 = \frac{2m_W}{g} \quad (10)$$

Inserting the vacuum expectation value (10) into Eq. (9) we get

$$\mathcal{L}_1 = O\left(\frac{m_X m_W}{m_P}\right) \left(\frac{4}{5g^2}\right) (\bar{d}_R d_R + \bar{d}_Y d_Y + \bar{d}_B d_B - \frac{3}{2} \bar{e} e). \quad (11)$$

We therefore have a contribution to the d and e masses which does not have the $SU(4)$ symmetry ^{2),5)} of the renormalizable $\bar{F}_5 f_{10} H_5$ couplings, which was responsible for the successful predictions ⁵⁾ of m_b and m_s (more on this later) and the unsuccessful relation (1). Putting $m_X \sim 10^{14\frac{1}{2}}$ GeV, $m_W \sim 10^2$ GeV and $m_P \sim 10^{19}$ GeV, the numerical factor in front of Eq. (11) is seen to be $O(10)$ MeV. We conclude that this term may not affect the successful

mass predictions of naïve SU(5), but is capable of modifying the unsuccessful naïve SU(5) prediction (1). We will therefore treat it as a small perturbation on the conventional (renormalizable) contributions to the fermion mass matrix. Starting from a fermion basis where this conventional contribution is diagonal, the non-renormalizable piece will generate very little mixing, and its only significant effect is to alter the smallest eigenvalues corresponding to m_d and m_e .

To see what is needed numerically, we take a phenomenological parametrization of the effective light fermion masses at the grand unification mass :

$$\tilde{\mathcal{L}} = a \left[\sum_{i=R,Y,B} \bar{d}_i d_i + \bar{e} e \right] + b \left[\sum_{i=R,Y,B} \bar{d}_i d_i - \frac{3}{2} \bar{e} e \right] \quad (12)$$

and fit the observed mass ratios

$$\frac{m_d}{m_s} = \frac{a+b}{100 \text{ MeV}} = \frac{1}{20} ; \quad \frac{m_e}{m_\mu} = \frac{a - \frac{3}{2}b}{100 \text{ MeV}} = \frac{1}{200} , \quad (13)$$

which determine the parameters

$$a = 3.2 \text{ MeV} , \quad b = 1.8 \text{ MeV}. \quad (14)$$

The required magnitude of the parameter b is quite consistent with the order of magnitude inferred from Eq. (11). So also is the parameter a of Eq. (12). It could also be obtained from a dimension 5 interaction analogous to (5) but with the H_{24} and f_{10} interchanged. It is therefore possible that essentially all the d and e masses come from this non-renormalizable source, and our mechanism gives an order-of-magnitude understanding of their masses.

What are the other implications of non-renormalizable interaction terms like $\mathcal{L}_1(5)$? Clearly they may give Cabibbo mixing between different generations, even if the renormalizable interactions did not. They therefore give an "order of magnitude lower bound" to the amount of Cabibbo mixing to be expected. As an example, an off-diagonal coupling between the first and second generations would give

$$\sin \theta_c \gtrsim \frac{\mathcal{O}(10 \text{ MeV})}{m_\mu} \quad (15)$$

which is certainly consistent with experiment. However, since one vague parameter $O(10)$ MeV earlier (14) turned out to be ~ 1.8 MeV, it seems likely that not all the Cabibbo mixing can be due to this mechanism particularly since it would give very small values of the Kobayashi-Maskawa ¹³⁾ angles s_2 and s_3 inconsistent with their mechanism ¹⁴⁾ for CP violation in the $K^0 - \bar{K}^0$ system ^{*}). There will also be "non-renormalizable" contributions to the charge $2/3$ quark mass matrix which are also $O(10)$ MeV. Thus we get an "order-of-magnitude lower bound" for the masses of all fermions in the $SU(5)$ model - with the exception of the neutrinos which are guaranteed massless because they only have one helicity state. It seems likely that this feature would be common to other grand unified theories. In some cases this may create problems - for example in $SO(10)$ models the neutrinos have two helicity states and can acquire masses. The masses of the ν_e and ν_μ are known ¹⁵⁾ to be small ($< O(1)$ MeV) in which case cosmology ¹⁶⁾ requires that their masses be very small indeed :

$$\sum_{\nu: m_\nu < 1 \text{ MeV}} < 50 \text{ eV},$$

and there is a risk that in some models "non-renormalizable" interactions of the type discussed here may push the neutrino masses above this limit, even if the renormalizable part of the Lagrangian is fixed up so as to give neutrinos sufficiently small masses.

After these remarks about the light fermion masses, we would like to make some comments about other aspects of fermions and Higgs fields in $SU(5)$. The first comment concerns the strange quark for which there is an $SU(5)$ prediction ⁵⁾ that its current (short-distance) mass at a scale $Q = 1$ GeV should be $O(\frac{1}{2})$ GeV. It has been asserted ¹⁷⁾ that $m_s|_{\text{curr.}}$ is more probably ~ 150 MeV. This argument is based on the $\Lambda - N$ mass difference ~ 150 MeV, which is equated with the difference between the constituent masses of the s and d quarks. This in turn identified with the current mass difference :

$$m_\Lambda - m_N \approx 150 \text{ MeV} \approx m_s|_{\text{const.}} - m_d|_{\text{const.}} \approx m_s|_{\text{curr.}} - m_d|_{\text{curr.}}. \quad (16)$$

^{*}) Obtaining all Cabibbo mixing from this source would require breaking discrete symmetries of the renormalizable part of the Lagrangian, and it is not clear whether this can be done.

Since $m_d|_{\text{curr}} \ll m_s|_{\text{curr}}$, it is deduced that $m_s|_{\text{curr}} \sim 150$ MeV. The problem with this derivation is that it assumes additive formulae

$$\left. \begin{aligned} m_s|_{\text{const}} &= m_s|_{\text{curr}} + X \\ m_d|_{\text{const}} &= m_s|_{\text{curr}} + X \end{aligned} \right\} \quad (17)$$

where x is some universal non-perturbative contribution to the masses of order 300 MeV. We see no reason why linear formulae such as (17) should be applicable rather than a quadratic formula

$$m^2|_{\text{const}} = m^2|_{\text{curr}} + X^2 \quad (18)$$

or something more complicated. Indeed quadratic formulae are found in bag models and (with a twist) in QCD in two dimensions ¹⁸⁾. If we assume the Egyptian rule of Pythagoras (18), and take $m_s|_{\text{const}} \approx 500$ MeV, then we find $m_s|_{\text{curr}} \approx 400$ MeV, which is well within the theoretical uncertainties ^{5),19)} in the SU(5) calculation of $m_s|_{\text{curr}}$.

It may be worth pointing out some problems with using more than one Higgs representation to get fermion masses in grand unified theories. First we recall that weak interaction models with more than one isodoublet of Higgs fields have problems ²⁰⁾ with the renormalization of the strong interaction CP violating parameter θ between the scale of weak interaction symmetry ($Q > 100$ GeV) and the scale of the neutron ($Q \sim 1$ GeV) where phenomenologically $\theta(Q)$ should be less than $O(10^{-8})$. Secondly, we recall the problems with natural flavour conservation ²¹⁾ in neutral weak interactions, whose solution is often taken to be pushing some physical weak scalar boson masses up to $O(\sqrt{G_F}) = 300$ GeV. However, it has been pointed out ²²⁾ that when weak Higgs masses are so large, the associated large (Higgs)⁴ couplings tend to go haywire when the renormalization group is used to evolve them up toward the grand unification mass. In the simplest SU(5) model the constraint that the weak interactions stay perturbative up to the grand unification mass requires $m_H < 200$ GeV ²²⁾. This difficulty is avoided by a trick of Georgi and Nanopoulos ²³⁾ which adds arbitrarily large positive mass terms quadratic in all linear combinations of Higgs fields which have vanishing vacuum expectation values. Thirdly, theories with multiple "light" Higgs have to ensure that their vacuum expectation values all occur in the same direction in SU(2) space, in order to avoid problems with

charge conservation. Fourthly we recall that at our present level of understanding it requires ^{5),24),25)} an unnatural condition in the simplest $SU(5)$ model to push the weak interaction symmetry breaking scale down to $O(10^{-12})$ of the grand unification mass, corresponding to arbitrarily miniaturizing a mass parameter. This hierarchy problem is compounded if there are several Higgs representations which must be given vacuum expectation values $O(10^2)$ GeV. There is an $SO(10)$ variant of Georgi and Nanopoulos ²⁶⁾ in which only one linear combination of Higgs is "light". However, if Higgs originally in representations of different dimensionality are to acquire non-vanishing vacuum expectation values, this is equally unnatural ; the previously mentioned trick ²³⁾ cannot be imposed in an $SU(2)$ invariant way. Of course, it may be that the ultimate solution to this hierarchy problem can deal equally easily with one or many "light" Higgs representations : we have to wait and see. Fifthly, we have a remark relevant to the 45 representation H_{45} of $SU(5)$ - or other similarly large Higgs representation in other grand unified models - sometimes invoked to fit the observed fermion mass spectrum. We have seen ²⁵⁾ that in the $SU(5)$ model, an $(H_{24})^4$ coupling which is $O(g^2)$ at $Q=O(m_P)$ evolves rapidly down to zero at $Q=O(10^{14}$ to $10^{15})$ GeV. On the other hand, an $(H_5)^4$ coupling evolves much more slowly, and this + enables us to understand why the H_{24} has a much larger vacuum expectation value than the H_5 in the context of symmetry breaking through radiative corrections with all $(\text{Higgs})^4$ couplings $O(g^2)$ at $Q=O(m_P)$. The relative speed of evolution reflects ²⁵⁾ the larger non-Abelian charge of the H_{24} . The H_{45} has an even larger non-Abelian charge, and if the $(H_{45})^4$ starts at $O(g^2)$ at $Q=O(m_P)$ it is difficult to see how it would avoid going to zero at or above the grand unification scale, and greatly complicating the attractive pattern of symmetry breaking by radiative corrections ^{24),25)}.

None of the above arguments is conclusive, but they create a climate hostile to the survival of many Higgs representations at low energies. We argued earlier that going beyond a single 5 of Higgs at low (weak/electromagnetic unification) energies was not necessary. However, it has been suggested ⁹⁾ that the $SU(5)$ needs multiple Higgs 5 representations in order to give the observed baryon number of the universe. However, these could well be "un-Higgses" - scalar fields with a positive μ^2 which do not participate in spontaneous symmetry breaking, and are not subject to the series of weak criticisms levelled above.

We conclude in the belief that although the simplest $SU(5)$ model has some problems, it is not yet absolutely necessary to complicate it by incorporating a larger than minimal set of Higgs fields. It remains to be seen whether $SU(5)^{2)}$ will be as successful as the Weinberg-Salam model ³⁾.

ACKNOWLEDGEMENTS

We thank M.S. Chanowitz, M. Perry and D.V. Nanopoulos for comments and discussions.

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