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QUANTUM CORRECTIONS TO A MASS FORMULA IN BROKEN SUPERSYMMETRY

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A B S T R A C T

We investigate the quantum corrections to a mass formula which holds for a general class of models with broken supersymmetry. Three models (supersymmetric QED, the O'Raifeartaigh model and the Wess-Zumino model with explicit breaking) are discussed in detail. We find that, in general, quantum corrections modify the mass formula. However, in the case of spontaneous symmetry breaking, the mass formula remains exact to first order in the symmetry breaking parameter but to all orders in perturbation theory. Finally, we include some remarks about the possible origin of the mass relation at the tree level.

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It has been shown recently¹⁾ that a very large class of globally or locally supersymmetric models satisfy, at the tree level, a remarkable mass formula when supersymmetry is broken spontaneously or, under certain conditions, explicitly. Let m_J be the mass of the particle of spin J , then the formula reads:

$$\sum_J (-)^{2J} (2J+1) m_J^2 = 0 \quad (1)$$

The purpose of this paper is to investigate the quantum corrections to Eq. (1) which, for the case of a renormalizable theory, are expected by the standard arguments to be finite and calculable. At the end we shall also comment on the possible origin of this mass formula at the tree level.

Let us first start with the quantum corrections in models with spontaneous symmetry breaking. The renormalizable ones fall into two classes: the first is essentially the supersymmetric extension of QED²⁾ in which a term proportional to the D component of the vector multiplet has been added³⁾ and the second represents a set of interacting scalar multiplets with a linear F term⁴⁾. They both share a common feature, namely the auxiliary field which triggers spontaneous symmetry breaking belongs to a massless supermultiplet. Our treatment is similar for both classes.

The simplest example of a model of the second class is the one of Ref. 4). The Lagrangian can be written, in terms of the component fields as:

$$\begin{aligned} \mathcal{L} = & -\frac{1}{2} \left[(\partial A_n)^2 + (\partial B_n)^2 + i \bar{\Psi}_n \not{\partial} \Psi_n - F_n^2 - G_n^2 \right] \\ & + m \left[F_1 A_2 + F_2 A_1 + G_1 B_2 + G_2 B_1 - i \bar{\Psi}_2 \Psi_1 \right] \\ & + g \left[F_0 (A_1^2 - B_1^2) + A_0 (2 F_1 A_1 + 2 G_1 B_1 - i \bar{\Psi}_1 \Psi_1) \right. \\ & \quad + 2 G_0 A_1 B_1 + B_0 (2 G_1 A_1 - 2 F_1 B_1 + i \bar{\Psi}_1 \gamma_5 \Psi_1) \\ & \quad \left. - 2 i \bar{\Psi}_0 (A_1 - \gamma_5 B_1) \Psi_1 \right] \\ & + \xi F_0 \end{aligned} \quad (2)$$

where the index n in the first line runs from 0 to 2. This Lagrangian describes three scalar supermultiplets ϕ_0, ϕ_1 and ϕ_2 , the first massless, with a $\phi_0 \phi_1^2$ interaction. Internal symmetries⁴⁾ guarantee that this simple form is in fact stable under renormalization.

Spontaneous breaking of supersymmetry is achieved by shifting the F_0 field in order to absorb the last term. Notice that for this to happen, both ξ and m must be different from zero. In fact, if $m = 0$, one can eliminate the linear term by shifting $A_1 \rightarrow A_1 + a_1$. One then creates a non-diagonal mass term of the form $m_{01}\phi_0\phi_1$ with $m_{01} = 2ga_1$ and $\xi + ga_1^2 = 0$. Therefore the symmetry breaking parameter can be viewed to be either ξ or m . Let us first consider the case in which $|2g\xi| < m^2$. Eliminating the F_0 term we generate masses for A_1 and B_1 $m_A^2 = m^2 + 2g\xi$ and $m_B^2 = m^2 - 2g\xi$. All other particles are unaffected in the tree approximation.

The renormalization of Eq. (3) is performed in the standard way. All counter terms are computed for the symmetric theory ($\xi = 0$) with symmetric renormalization conditions. All particles belonging to the same supermultiplet have the same renormalized mass and the full Lagrangian, including the counterterms, is supersymmetric invariant. We now switch on the term ξF_0 . The quantum corrections at first order in ξ are given by the three point functions containing one zero momentum F_0 insertion (the tadpole diagrams of Fig. 1). Notice that, since we are only interested in the $O(\xi)$ contributions, all the masses in the internal lines are the symmetric ones (zero or m whichever the case may be) and the external momenta are taken on the symmetric mass-shell. Individual diagrams may develop infra-red divergences and a cut-off may be necessary at intermediate stages of the calculation. However, as we shall argue presently, these contributions in fact vanish as a result of the Ward identities. It is straightforward to verify this statement at the one loop level. Indeed due to the $F_0(A_1^2 - B_1^2)$ term in the Lagrangian which gives the only coupling of F_0 , all such insertions vanish because the diagrams in which the F_0 is attached to an internal B_1 or A_1 line cancel. This result can be extended to any number of loops, provided one still neglects $O(\xi^2)$ contributions. The simplest way to see it, is to analyze diagrammatically each term and realize that, again, the A_1 and B_1 contributions cancel (Fig. 2). A more formal proof can be constructed by noticing that the Lagrangian (2), without the F_0 term, is invariant under two parameter group of axial transformations of the form:

$$\begin{aligned} \delta A_n &= \beta_n^A B_n & \delta F_n &= \beta_n^F G_n & \delta \Psi_n &= \beta_n^\Psi \gamma_5 \Psi_n \\ \delta B_n &= -\beta_n^A A_n & \delta G_n &= -\beta_n^F F_n \end{aligned} \quad (3)$$

where n runs from 0 to 2 and

$$\begin{aligned}
 \beta_1^A &= \beta & \beta_1^F &= \beta' & \beta_1^\Psi &= \frac{\beta - \beta'}{2} \\
 \beta_0^A &= \beta' - \beta & \beta_0^F &= 2\beta & \beta_0^\Psi &= \frac{\beta' - 3\beta}{2} \\
 \beta_2^A &= \beta' & \beta_2^F &= \beta & \beta_2^\Psi &= \frac{\beta' - \beta}{2}
 \end{aligned} \tag{4}$$

The physical meaning of the transformations (3) and (4) is easy to understand in superspace. For a chiral superfield $\phi_n(x, \theta)$ they correspond to the transformation

$$\phi_n(x, \theta) \rightarrow e^{i\alpha_n} \phi_n(x, e^{i\delta} \theta) \tag{5}$$

The parameters β and β' are functions of α and δ . Notice that this is a global symmetry (β and β' are constants) and it is preserved in a straightforward way by renormalization. For example, the regularization scheme of higher derivatives used in Ref. 5) respects both supersymmetry and the axial symmetry. We now write the corresponding Ward identities. We introduce a set of external sources $J(x)$, one for every field of (2). Let $W[J]$ be the generating functional of the connected Green functions. In the notations of Ref. 5) the Ward identities read:

$$\begin{aligned}
 \sum_{n=0}^2 \left\{ -i \frac{\delta W}{\delta J_{A_n}} (\partial_\mu \bar{\eta}_n) \gamma^\mu + i \frac{\delta W}{\delta J_{B_n}} (\partial_\mu \bar{\eta}_n) \gamma_5 \gamma^\mu + i \frac{\delta W}{\delta J_{F_n}} \bar{\eta}_n \right. \\
 \left. + i \frac{\delta W}{\delta J_{G_n}} \bar{\eta}_n \gamma_5 - \frac{\delta W}{\delta \eta_n} \left[\partial_\mu (J_{F_n} - \gamma_5 J_{G_n}) \gamma^\mu + J_{A_n} + \gamma_5 J_{B_n} \right] \right\} = 0 \tag{6} \\
 \sum_{n=0}^2 \left\{ \beta_n^A \left(J_{A_n} \frac{\delta W}{\delta J_{B_n}} - J_{B_n} \frac{\delta W}{\delta J_{A_n}} \right) + \beta_n^F \left(J_{F_n} \frac{\delta W}{\delta J_{G_n}} - J_{G_n} \frac{\delta W}{\delta J_{F_n}} \right) \right. \\
 \left. + \beta_n^\Psi \bar{\eta}_n \gamma_5 \frac{\delta W}{\delta \bar{\eta}_n} \right\} = 0 \tag{7}
 \end{aligned}$$

It is now straightforward to verify that, as a consequence of Eqs (6) and (7), (i) the form of the Lagrangian (2) is stable under renormalization (for example a ϕ_0^2 mass term vanishes since it is not invariant under (3)) and (ii) the higher order mass corrections due to an F_0 tadpole insertion vanish. Therefore the formula (1) holds at first order in ξ but all orders in g . This argument fails if one considers higher order ξ contributions because, in this case, the insertions on A_1 and B_1 lines do not cancel.

Finally, we would like to point out that the case $|2g\xi| > m^2$ can be treated along similar lines. Let us choose, without loss of generality, $g\xi < 0$. (Otherwise we can change the sign of ξ by a chiral transformation.) We shift the fields $A_1 \rightarrow A_1 + a_1$, $F_0 \rightarrow F_0 + f_0$ and $F_2 \rightarrow F_2 + f_2$. Eliminating the linear terms gives, at the tree level, the conditions:

$$2g f_0 = m^2 ; \quad 2g^2 a_1^2 = -m^2 - 2g\xi ; \quad f_2 = -m a_1, \quad (8)$$

The Lagrangian, after the shifts reads:

$$\mathcal{L} = \mathcal{L}_{\text{KIN.}} + m\phi_1\phi_2 + \frac{1}{2}m^2(A_1^2 - B_1^2) + 2ga_1\phi_1\phi_0 + g\phi_1^2\phi_0 \quad (9)$$

The supersymmetric invariant mass terms $\phi_1\phi_0$ and $\phi_1\phi_2$ leave one massless supermultiplet. Let us write:

$$\tilde{\phi}_0 = \frac{m\phi_2 + 2ga_1\phi_0}{\sqrt{m^2 + 4g^2a_1^2}} \quad (10a)$$

$$\tilde{\phi}_2 = \frac{2ga_1\phi_2 - m\phi_0}{\sqrt{m^2 + 4g^2a_1^2}} \quad (10b)$$

In terms of the new fields (9) gives:

$$\mathcal{L} = \mathcal{L}_{\text{KIN.}} + \sqrt{m^2 + 4g^2a_1^2} \phi_1 \tilde{\phi}_0 + \frac{1}{2}m^2(A_1^2 - B_1^2) + g\phi_1^2 \left[\frac{2ga_1\tilde{\phi}_0 - m\tilde{\phi}_2}{\sqrt{m^2 + 4g^2a_1^2}} \right] \quad (11)$$

In this case, the symmetry breaking parameter is m^2 . We shall again renormalize the symmetric theory, namely the one with $m = 0$.

At first order in m the Lagrangian (11) is equivalent to

$$\mathcal{L} = \mathcal{L}_{\text{KIN}} + 2ga_1 \phi_1 \tilde{\phi}_0 + g \phi_1^2 \tilde{\phi}_0 - \frac{m}{2a_1} \phi_1^2 \tilde{\phi}_2 + m a_1 \tilde{F}_2 \quad (12)$$

and the masses are calculated by the diagrams with one F_2 , zero momentum insertion. Notice, however, that, unlike the previous case of Fig. 1, the fact that $\tilde{\phi}_0$ is now a massive multiplet, prevents the complete cancellation between the insertions on A_1 and B_1 lines and one obtains non-zero mass-shifts even at first order in m^2 . This can be traced to the fact that the symmetric ($m = 0$) theory is now invariant only under a subgroup of the chiral transformations (3) and (4), the one which corresponds to $\beta = 0$. The rest is broken by the ξ term. However, even in this case, the mass formula (1) remains exact at first order in m^2 , a fact that we have also verified by explicit calculation at the one loop level.

We now turn our attention to the model of Ref. 3). After eliminating the auxiliary fields F and G we write the Lagrangian as:

$$\begin{aligned} \mathcal{L} = & -\frac{1}{2} \left[(\partial_\mu A_1)^2 + (\partial_\mu B_1)^2 + i \bar{\Psi}_1 \not{\partial} \Psi_1 + 1 \rightarrow 2 \right] \\ & - \frac{1}{2} m^2 (A_1^2 + B_1^2 + 1 \rightarrow 2) - \frac{i}{2} m (\bar{\Psi}_1 \Psi_1 + \bar{\Psi}_2 \Psi_2) \\ & - \frac{1}{4} V_{\mu\nu}^2 - \frac{i}{2} \bar{\lambda} \not{\partial} \lambda + \frac{1}{2} D^2 \\ & + g \left\{ D (A_1 B_2 - A_2 B_1) - V_\mu (A_1 \partial^\mu A_2 - A_2 \partial^\mu A_1 + \right. \\ & \quad \left. + B_1 \partial^\mu B_2 - B_2 \partial^\mu B_1 - i \bar{\Psi}_1 \gamma^\mu \Psi_2) \right. \\ & \quad \left. - i \bar{\lambda} [(A_1 + \gamma_5 B_1) \Psi_2 - (A_2 + \gamma_5 B_2) \Psi_1] \right\} \\ & - \frac{1}{2} g^2 V_\mu^2 (A_1^2 + A_2^2 + B_1^2 + B_2^2) \\ & + \xi D \end{aligned} \quad (13)$$

The last term induces a shift $D \rightarrow D - \xi$ and is responsible for the spontaneous breaking of supersymmetry. The scalars A_i and B_i acquire off-diagonal mass terms which, after diagonalization, split the boson masses to $m^2 \pm \xi g$ leaving the ψ mass equal to m . The relation (1) holds.

We now compute the quantum corrections at first order in ξ . All renormalizations are again performed in the symmetric theory ($\xi = 0$). Notice that the Lagrangian (13) has this simple form only in the Wess-Zumino gauge²⁾ in which supersymmetry is not manifest. Therefore the counterterms will not be supersymmetric²⁾. This is not a problem, however, since the theory is determined by the renormalization conditions and they can be chosen supersymmetric. The important problem is to show that the Ward identities can be preserved⁶⁾. In particular all masses will remain equal.

The one-loop diagrams are shown in Fig. 3. Since D is pseudoscalar, these tadpole diagrams only contribute to off-diagonal terms of the form $A_2 B_1$ and $A_1 B_2$. A crucial observation is that, to this order, the fermion masses remain unchanged. In fact, the diagram of Fig. 3c, due to the fact that λ is massless, gives a contribution of the form $\sim g^3 \xi \bar{\psi}_1 \gamma_5 \psi_2$. It is easy to see that such a term gives only $O(\xi^2)$ corrections to the fermion masses. On the other hand, the boson masses are again split symmetrically, as in the tree approximation and the relation (1) holds.

This result can again be extended to any number of loops provided we neglect $O(\xi^2)$ contributions. If, however, we include higher order terms in ξ the cancellation argument does not apply. We have checked by direct calculations at the one loop level that, in this case, the mass relation (1) is indeed violated.

The situation changes when we look at models where supersymmetry is explicitly broken. The mass relation (1) may still be valid in the tree approximation but it is violated by the quantum corrections even at first order in the symmetry breaking parameter. The simplest such model is the self-interacting massive scalar multiplet with a symmetry breaking term proportional to the A field⁵⁾:

$$\begin{aligned} \mathcal{L} = & -\frac{1}{2} \left[(\partial_\mu A)^2 + (\partial_\mu B)^2 + i \bar{\Psi} \not{\partial} \Psi - F^2 - G^2 \right] \\ & + m \left[FA + GB - \frac{i}{2} \bar{\Psi} \Psi \right] \\ & + g \left[FA^2 - FB^2 + 2 GAB - i \bar{\Psi} \Psi A + i \bar{\Psi} \gamma_5 \Psi B \right] \\ & + c A \end{aligned} \tag{14}$$

It has been shown⁵⁾ that no spontaneous breaking can occur in this model but the addition of the last term breaks supersymmetry explicitly but softly. A linear shift of the fields $A \rightarrow A + a$ and $F \rightarrow F + f$ can eliminate the last term and gives, in the tree approximation, a mass spectrum of the form

$$m_\psi = m + 2ga ; \quad m_A^2 = m_\psi^2 - \frac{2gc}{m_\psi} ; \quad m_B^2 = m_\psi^2 + \frac{2gc}{m_\psi} \quad (15)$$

This mass spectrum was in fact the first example of the relation (1). It was also shown in Ref. 5) that this relation receives only finite quantum corrections. We have checked, by explicit calculation, that these corrections, even at first order in the symmetry breaking parameter c , do not respect Eq. (1). It is instructive to trace the origin of this failure: in order to calculate the physical masses at first order in c one needs the connected three-point functions of the general form $W_{\phi_1 \phi_2 A}(p, -p, 0)|_{p^2 = -m^2}$ and $W_{\phi_1 \phi_2 F}(p, -p, 0)|_{p^2 = -m^2}$ where ϕ_1 and ϕ_2 represent any of the fields A, B, G, F or ψ . Notice that the A or F insertions are of zero momentum but the ϕ fields are taken on the symmetric mass-shell. One can now write the Ward identities connecting these Green functions. They are contained in Eq. (17) of Ref. 5). It is easy to see that the F insertions satisfy simple Ward identities because, as it was shown in Ref. 5), we have

$$W_{\phi_1 \phi_2 F}(p, -p, 0) \sim \frac{\partial}{\partial m} W_{\phi_1 \phi_2}(p, -p) \quad (16)$$

i.e., they are given by the derivative with respect to the bare mass of the corresponding two-point functions. On the other hand, the A insertions satisfy simple Ward identities only when $p = 0$. For $p \neq 0$ we obtain relations containing derivatives of the Green functions. These extra terms are responsible for the breaking of Eq. (1). In turn they are due to the transformation properties of the fields under supersymmetry which often contain derivatives of the fields. In other words, supersymmetry gives simple relations only for zero momentum Green functions. We also understand now why Eq. (1) is simple at the tree level since, in this approximation, the three-point functions are independent of the external momenta.

Before closing we would like to add some remarks on the quite general validity of Eq. (1) at the classical level. We would like to know if it is a consequence of the supersymmetry algebra alone, or if it is due to the particular couplings one has so far considered. We have no general answer to this question and we shall restrict ourselves into providing general classes of examples which do not satisfy Eq. (1).

In fact, essentially every example we have considered, which contains an F or D field directly coupled to the fermions and/or terms in the interaction Lagrangian with more than one power of F or D fields, violates the mass relation. Since all these terms have dimensions higher than four, all these examples will be non-renormalizable. Furthermore, they all share a common feature, namely at least one of the fields appears in the interaction Lagrangian through a term proportional to its kinetic energy. This feature was recognized already in supergravity^{7),1)} in association with the non-validity of Eq. (1).

In particular, consider a scalar multiplet ϕ . By using Eqs (3) and (5) of Ref. 2) we construct the vector multiplets $\phi \times \phi$ and $(\phi \times \phi)^2$. We then consider the Lagrangian

$$\mathcal{L} \sim (\phi \times \phi)_D + m (\phi^2)_F + \frac{g}{m^2} [(\phi \times \phi)^2]_D \quad (17)$$

The interaction contains terms of the form $F^2 A^2$ and $\bar{\psi} \psi A F$. A linear shift of the A and F fields produces a mass spectrum which does not satisfy Eq. (1).

Another example, with an interaction Lagrangian of dimension four, is provided by a massive vector multiplet V interacting with a massive or massless scalar multiplet ϕ through a $[V \cdot (\phi \times \phi)]_D$ coupling. It contains terms of the form $F^2 C$ and $\bar{\lambda} \psi F$. The symmetry can be broken either explicitly, by linear terms proportional to F , A , C , D and N , or spontaneously by adding only F and D terms (in the last case the ϕ multiplet must be chosen massless). In both cases the mass relation (1) is not satisfied.

Further examples are provided by self-coupled massive vector multiplets with couplings of the form $(V^n)_D$. There is little merit in writing any of these complicated Lagrangians in detail but the moral of the story seems to be that Eq. (1) is not an algebraic consequence of supersymmetry alone. However, if one considers only renormalizable interactions, F or D cannot appear with a power higher than first and cannot couple directly to the fermions. Since their shift breaks supersymmetry (a shift of an A field gives only a common mass to all members of the supermultiplet) it follows that, in the renormalizable models, the fermion masses remain unchanged and the boson masses are split symmetrically around

the fermion mass. On the other hand it is clear that renormalizability is not a necessary condition for the validity of the mass formula as it can be seen by the example of the ϕ^n , $n > 3$, model¹⁾ and, in fact, in extended supergravity even formulae with higher powers of m^2 are valid⁸⁾, but we still think that the origin may be traced to the particular form of couplings like $F(A^2 - B^2)$ or $D(A_1 B_2 - A_2 B_1)$.

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FIG. 1

FIG. 2

FIG.3