ANALYSIS OF SINGLE AND COHERENT NUCLEON DIFFRACTION DISSOCIATION
AT ISR ENERGIES

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ABSTRACT

We present an analysis of a combined measurement of neutron and proton diffraction dissociation into $(N\pi)$ and $(N\pi\pi)$ final states, both on proton and deuteron targets.

The experiment was performed at the CERN Intersecting Storage Rings with the Split Field Magnet detector using proton and deuteron colliding beams.

Mass spectra and decay angular distributions are compared with the predictions of a dual-Reggeized Deck model. Strong decay angular correlations are observed in all cases. Pronounced structures observed in the differential cross-sections of the coherent diffractive processes for small produced masses are interpreted in terms of constructive interference between the single- and double-scattering terms in a Glauber analysis. These interference effects cannot be reproduced by a central dissociation mechanism and require a peripheral description of the elementary nucleon-nucleon diffractive production amplitudes.

(Submitted to Nuclear Physics B)

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1. INTRODUCTION

Hadronic diffraction dissociation has long been the subject of extensive experimental and theoretical investigation [1]. Only recently has it been possible to obtain data on nucleon diffraction with high statistical accuracy at very high energies. For the case of single diffraction, neutron dissociation into proton and negative pion has been measured in great detail at Fermilab [2], and a similar experiment was performed at the CERN Intersecting Storage Rings (ISR) to measure proton dissociation into neutron and positive pion [3].

The analysis of the data in terms of refined versions [4] of the Deck model has in general been successful in describing the main features of this class of reactions, although several details still remain unexplained.

For a further study of inelastic single nucleon diffraction at very high energies, we performed a systematic investigation of neutron and proton dissociation in two- and three-body final states both on nucleon and on deuteron targets. The experiment was performed at the CERN ISR with proton-proton, proton-deuteron, and deuteron-deuteron colliding beams. It consisted of a combined measurement of the four processes:

$$pp \rightarrow (p\pi^+\pi^-)p \quad \text{at} \quad \sqrt{s} = 53 \text{ GeV} , \quad (1)$$

$$np \rightarrow (p\pi^-)p \quad \text{at} \quad \sqrt{s} = 37 \text{ GeV} , \quad (2)$$

$$pd \rightarrow (p\pi^+\pi^-)d \quad \text{at} \quad \sqrt{s} = 53 \text{ GeV} , \quad (3)$$

$$nd \rightarrow (p\pi^-)d \quad \text{at} \quad \sqrt{s} = 37 \text{ GeV} . \quad (4)$$

The aim of the experiment was to compare, under identical experimental conditions, the coherent diffraction dissociation processes (3) and (4) with the corresponding reactions on free nucleons (1) and (2). To this end the simple and well-known deuteron structure provides, through strong interference effects, a unique analyser of the detailed properties of the production dynamics. The simultaneous analysis of our nucleon-nucleon and coherent diffraction data is, in fact, able to produce new information on elementary nucleon diffraction dissociation in terms of the helicity structure of production amplitudes.

Preliminary results on reactions (2) and (4) have already been reported in previous publications [5-7].

The present paper is organized as follows: Section 2 describes the experimental apparatus and the analysis procedure used for data reduction. In Section 3 the features of the invariant mass distributions are described and compared with the theoretical predictions of a Deck-type model. Section 4 shows the differential t -distributions and the effects of slope-mass correlation. In Section 5 the decay angular correlations and decay moments are presented and compared to the Deck model predictions. In Section 6 we first present the theoretical basis of our Glauber analysis and then investigate the differential cross-sections of the coherent processes (3) and (4) using as input the single-diffractive data. Our conclusions are given in Section 7.

2. EXPERIMENTAL SET-UP AND PROCEDURE

In this section we describe the experimental apparatus used to detect diffraction dissociation events from reactions (1) to (4) and the trigger conditions defining the selection criteria. We then discuss the properties of the off-line events reconstruction and the characteristics of the final event samples.

Details on the aspects of the experimental procedure have been described in recent publications [8-10].

2.1 Apparatus and trigger

The data for this experiment were obtained at the CERN ISR with 26.6 GeV/c proton-proton [reaction (1)], proton-deuteron [reactions (2) and (3)] and deuteron-deuteron [reaction (4)] colliding beams. Events from the four reactions were detected in the forward telescopes of the Split Field Magnet (SFM) detector.

The SFM has been described in detail elsewhere [11]; a plan view of the detector is shown in fig. 1. It consists of two symmetric magnetic spectrometers with a gap of 1.1 m and a field strength of up to 1 T. The detector is fully equipped with multiwire proportional chambers, which provide momentum measurement of the charged secondaries over nearly the full solid angle.

Additional chambers in the compensator magnets allow detection of particles down to about 8 mrad to the beams with a momentum resolution $\Delta p/p$ of about 10.07 at 26.6 GeV/c.

In order to select events corresponding to the topology of reactions (1) to (4), a two-step trigger, operating at a fast and at a slower level, was adopted. The fast trigger required the signature of a beam-beam interaction. The slow trigger [12], operating on the memory-OR signals of groups of 32 wires, was complemented with a special hardware processor for reactions involving deuterons. It required an elastic-like track in one arm pointing to the vertex within a narrow angular aperture and, for reactions (2) and (4), a track in the opposite arm corresponding to the spectator proton from deuteron break-up within a specific angular interval. An approximate topological condition on the number of charged tracks in the telescope of the dissociating nucleon was also applied in a wider acceptance region.

Events with charged particles in the central region were rejected so as to enhance the diffractive nature of the data sample. The limits of the kinematical acceptance of the trigger conditions were set at values of invariant mass and four-momentum transfer largely in excess of the upper limits given by the sensitivity of the experiment.

The trigger for reaction (4) was symmetric in the two spectrometer arms.

The steepness of the deuteron form factor required, as an additional feature, different scaling factors in different intervals of four-momentum transfer to enrich our high- t sample of events for the coherent diffractive reactions (3) and (4).

2.2 Data reduction, normalization, and acceptance

After going through a preliminary off-line filter, refining the requirements of the electronic triggers and performing topological tests at the space-point level, the surviving events were processed by the standard SFM program chain [13] performing pattern recognition and geometrical reconstruction. For the case of coherent reactions (3) and (4), a simple momentum cut prior to the kinematical fit was very effective in isolating the elastic peak in the deuteron arm, since

the deuteron break-up products peak at 13.3 GeV/c, around half the beam momentum, with a limited spread due to the Fermi motion. The remaining events were processed by a four-constraint kinematical fit. Events corresponding to a confidence level greater than 1% were retained for the final analysis. In these final event samples the background contamination was found to be smaller than 1% for all reactions.

The absolute cross-section normalization was obtained by recording monitor counts simultaneously with data-taking, using two scintillation counter hodoscopes placed in front of the compensator magnets. During data-taking the beam-beam interaction rate, the beam-gas background level, and time-of-flight information between the two hodoscopes systems were continuously monitored and recorded. The calibration of the monitor was obtained for each run with the Van der Meer method [14] by determining the overlap profile of the beam densities at the interaction point.

The acceptance of the apparatus, of the trigger, and of the off-line programs was calculated by Monte Carlo methods, i.e. by processing Monte Carlo generated events with full simulation of the detector geometry, backgrounds, trigger, and chamber efficiency, in the same way as the real events. In particular, the effects of absorption and scattering in the beam tubes and in the chamber frames, together with losses in the filtering, pattern recognition, and reconstruction chain, were considered in detail.

In all cases the over-all acceptance is almost uniform up to t -values in excess of 2 GeV² and up to masses of the dissociated systems in excess of 3 GeV. However, owing to the presence of the beam vacuum tubes in the forward telescopes, elastic-like particles with four-momentum transfer $|t| < 0.04$ GeV² remained undetected. This results in an experimental cut in the differential cross-section at $|t| = 0.05$ GeV² for all reactions.

All the quantities relevant to the final data samples of the four reactions are given in table 1 *).

*) The calibration constant for reaction (1) was not reliable because of losses in the recording process. The corresponding quantities are given in arbitrary units.

3. INVARIANT MASS DISTRIBUTIONS

3.1 Experimental data

The experimental invariant mass distributions for reactions (1) to (4) are shown in fig. 2. All four distributions display very similar features, namely a sharp rise above threshold, a rather limited range of mass values, and a clear presence of resonant structures. The three-body final states display broader distributions as compared with two-body final states, for which the low-mass enhancement, on the other hand, appears distinctively more pronounced. If we compare the distributions of the coherent diffraction processes with the corresponding ones on free nucleons, a striking similarity appears for both two-body and three-body final states. This is strictly related to the diffractive nature of the processes, for which nucleon dissociation factorizes with respect to the target vertex and is dominated by isoscalar exchange in the very high energy limit.

An increased weighting of the low-mass part of the distributions is apparent for the coherent reactions and can be interpreted as the combined effect of the very steep deuteron form factor and the slope-mass correlation. This can also be seen in fig. 3, where the same mass distributions are shown for $|t|$ -values greater than 0.25 GeV^2 . After this cut, which to a large extent selects the double-scattering region for coherent processes, it can be observed that the main features of two- and three-body nucleon diffraction excitation are still present when rescattering occurs.

In both fig. 2 and fig. 3, clear resonant signals appear for all four reactions at the position of the $N(1688)$ isobar, lower-lying structures being masked to a large extent by the dominant low-mass enhancement. The structure of the mass spectra appears somewhat more evident in the backward hemisphere of the Gottfried-Jackson frame of the decaying systems, as shown in fig. 4.

3.2 Deck-model calculations

Low-mass exclusive diffraction dissociation has usually been described in terms of Deck models. The original formulation [15], containing essentially the pion exchange term (fig. 5a), has later been refined by the introduction of Reggeization [16] and absorptive corrections [17].

To compare our experimental results with the prediction of the model, we used the three-component dual Deck approach of Cohen-Tannoudji et al. [18] in which all the diagrams of fig. 5 are considered simultaneously, together with their interference terms. This model has been successful in describing the mass, t and decay angle correlations without the need of absorptive corrections for both single [18] and double diffraction dissociation [19].

The total amplitude, including pion exchange, baryon exchange, and direct pole graphs (figs. 5a,b,c), reads:

$$D = i g_{a1\pi} \left[\frac{s_3 \Sigma_{1b}}{u_1 - m_1^2} + \frac{s_2 \Sigma_{\pi b}}{t_1 - m_\pi^2} + \frac{(s_2 + s_3) \Sigma_{ab}}{s_1 - m_a^2} \right] =$$

$$= i g_{a1\pi} \left[\frac{s_3 Z_{su}}{(s_1 - m_a^2)(u_1 - m_1^2)} + \frac{s_2 Z_{st}}{(s_1 - m_a^2)(t_1 - m_\pi^2)} \right], \quad (5)$$

where

$$\Sigma_{ib} = \sigma_{ib}^{\text{off}} e^{B_{ib}^{\text{off}}(t_2/2)} \quad (i = 1, \pi, a) \quad (\text{off} = \text{off-mass-shell values})$$

$$Z_{st} = \Sigma_{\pi b}(s_1 - m_a^2) + \Sigma_{ab}(t_1 - m_\pi^2)$$

$$Z_{su} = \Sigma_{1b}(s_1 - m_a^2) + \Sigma_{ab}(u_1 - m_1^2)$$

and the kinematical variables are defined in fig. 5.

After dualization the amplitude (5) becomes:

$$D_d = i g_{a1\pi} \left\{ s_3 Z_{su} \frac{\Gamma(-\alpha_s)\Gamma(-\alpha_u)}{\Gamma(1 - \alpha_s - \alpha_u)} + s_2 Z_{st} \frac{\Gamma(-\alpha_s)\Gamma(-\alpha_t)}{\Gamma(1 - \alpha_s - \alpha_t)} \right\}. \quad (6)$$

The trajectories are assumed to be:

$$\alpha_t = \alpha_\pi(t_1) = t_1 - m_\pi^2$$

$$\alpha_u = \alpha_N(u_1) = u_1 - m_1^2$$

$$\alpha_s = \alpha_N(s_1) = s_1 - m_a^2 + i\lambda \sqrt{s_1 - (m_1 + m_\pi)^2}. \quad (7)$$

This model contains a rough parametrization for low-mass resonances in the s -channel, which is controlled by the parameter λ ; this parametrization, though, is too rigid to reproduce the whole structure of the mass spectra in detail. In our calculation we therefore adopted the asymptotic dual-Reggeized Deck parametrization [18], in which the Veneziano functions are replaced by their Regge limits:

$$D_d^R = i g_{a1\pi} \left\{ s_3 Z_{su} (s_1 - m_a^2)^{\alpha_N(u_1)-1} \Gamma[-\alpha_N(u_1)] e^{-i\pi\alpha_N(u_1)} + \right. \\ \left. + s_2 Z_{st} (s_1 - m_a^2)^{\alpha_\pi(t_1)-1} \Gamma[-\alpha_\pi(t_1)] e^{-i\pi\alpha_\pi(t_1)} \right\}. \quad (8)$$

In this way we expect to account for at least the low-mass enhancement originally explained by the simple pion exchange amplitude. In fact, as already shown in ref. 6, the predictions of the latter two amplitudes are not significantly different, as far as the mass spectra are concerned.

In comparing the model with the experimental data, we assumed for the parameters the following values:

$$\sigma_{1N} = 30 \text{ mb}, \quad \sigma_{\pi N} = 25 \text{ mb}, \quad \sigma_{aN} = 40 \text{ mb}$$

with slopes $B_{1N} = B_{aN} = 9 \text{ GeV}^{-2}$, $B_{\pi N} = 10 \text{ GeV}^{-2}$.

In the case of the coherent reactions (3) and (4), the total cross-sections on deuterons have been taken to be twice the corresponding ones on free nucleons and, in order to account for the deuteron form factor, the following slopes have been assumed [8]:

$$B_{\pi D} = 35 \text{ GeV}^{-2}, \quad B_{1D} = B_{aD} = 34 \text{ GeV}^{-2}.$$

For neutron diffraction dissociation into $(p\pi^-)$ [reactions (2) and (4)], proper care was taken about the spin complications, which however do not alter essentially the features of the model.

In the case of proton diffraction into $(p\pi^+\pi^-)$ [reactions (1) and (3)], we assumed that the process proceeds mainly via Δ^{++} production, as can be argued from the $(p\pi^+)$ mass spectra, which appear entirely dominated by the Δ^{++} peak. In this case spin effects have not been taken into account and the same cross-sections have been assumed as for the nucleon.

The predictions of the model are shown by the continuous curves in fig. 2 and are normalized to the integrated spectra after subtraction of the N(1688) peak. The model seems generally capable of reproducing reasonably well the threshold behaviour of exclusive diffraction, together with the bulk of the low-mass

enhancement for two-body dissociation. The predictions of the model will also be compared in Section 5 with the experimental decay angular distributions.

4. DIFFERENTIAL CROSS-SECTIONS

The differential cross-sections for the four reactions are shown in fig. 6 as a function of the four-momentum transfer squared. In all cases the data cover the interval between $|t| = 0.05 \text{ GeV}^2$ and $|t| = 1 \text{ GeV}^2$, the lower limit corresponding to the experimental cut associated with the ISR vacuum chamber. The differential cross-sections for the diffractive reactions on nucleons appear very similar, as is also the case for the two coherent processes on deuterons. For the latter case the strong damping due to the very steep deuteron form factor is evident.

The continuous curves in fig. 6 represent a phenomenological parametrization of the data which were fitted to the assumed form $d\sigma/dt = A_1 e^{b_1 t} + A_2 e^{b_2 t}$. The numerical results of the fits are given in table 2.

For the case of single neutron diffraction dissociation [reaction (2)], extrapolating to $t = 0$ and taking into account shadow corrections, we obtain a total cross-section of $206 \pm 24 \text{ } \mu\text{b}$, in very good agreement with the previous measurements of two-body nucleon diffraction at the ISR [3,5]. We consider a similar extrapolation for the coherent processes (3) and (4) to be not reliable because of the limited validity of the assumed parametrization in reproducing the low- t behaviour of the deuteron form factor.

The differential cross-sections for the four reactions in different mass bins display the usual correlation between the forward slope and the mass of the diffractively produced systems. This is shown in figs. 7 and 8, where the exponential slope in the region $0.05 < |t| < 0.25 \text{ GeV}^2$ appears to be a monotonically decreasing function of the mass. Data on coherent proton diffraction are not shown because of the limited statistical accuracy. The curves in figs. 7 and 8 show an empirical parametrization of the dependence of the forward slope on the produced mass, by fitting the data to the form [20]

$$b(M) = A + BM^{-\alpha} . \quad (9)$$

Owing to the limited energy interval covered by the experiment, a logarithmic term in the centre-of-mass energy squared has been neglected. The numerical results of the fits are given in table 3.

Full information on the differential cross-sections of the coherent reactions (3) and (4) in different mass bins is given in Section 6, where it is used for the detailed Glauber analysis.

5. DECAY ANGULAR DISTRIBUTIONS

5.1 Decay angular correlations

It is well known that the Deck model for diffraction dissociation predicts specific correlations in the decay angular variables of the produced systems. The azimuthal and polar decay angular distributions are expected to show accumulations in specific regions of the phase space where different exchange mechanisms (see fig. 5) dominate separately.

The relevant information on the decay angular distributions of the four reactions is summarized in figs. 9 to 12; we shall comment on them jointly, since they display a very similar behaviour.

In the first frame (a) of each figure we show a scatter plot in the $(\cos \theta_J - \phi_J)$ plane for a sample of events of each reaction.

The three-vectors chosen as analysers are the nucleon momentum for two-body final states and the Δ momentum for the three-body final states.

The second frame (b) of each figure shows the same information as a three-dimensional surface after local smoothing of the event density. In both cases a remarkable symmetry around the $\phi_J = 0^\circ$ plane is evident and strong peaks appear in the double differential cross-sections. The two most prominent forward peaks can be associated, for all reactions, to the dominant pion-exchange term, whereas the backward structure at opposite azimuthal angles is essentially due to the baryon exchange term.

In frames c and d (figs. 9 to 12) the projections on the $\cos \theta_J$ and ϕ_J distributions are compared to the predictions of the model of subsection 3.2. A cut for $|\cos \theta_J| > 0.9$ has been applied to the data samples in order to avoid decay angular

regions in which the acceptance is more critically affected by the presence of the beam vacuum chamber. The azimuthal distributions are folded around the $\phi_J = 0^\circ$ plane, owing to the expected and observed symmetry.

The polar decay angle distributions appear to be generally well described by the model, apart from the very forward bins. The azimuthal distributions, on the other hand, show an excess of events around $\phi_J = 0^\circ$. In a previous publication [6] we have shown that this excess is an increasing function of the produced mass above threshold in the case of coherent neutron dissociation. The same behaviour (not shown) is also found for single neutron dissociation [reaction (2)].

5.2 Decay moments

The decay angular distributions have been analysed in terms of spherical harmonic moments, by evaluating the integrals

$$\langle Y_\ell^0(\cos \theta_J) \rangle = \frac{\int_{-1}^1 Y_\ell^{m=0}(\cos \theta_J) W(\cos \theta_J) d(\cos \theta_J)}{\int_{-1}^1 W(\cos \theta_J) d(\cos \theta_J)}, \quad (\ell = 1, \dots, 6), \quad (10)$$

where $Y_\ell^{m=0}(\cos \theta_J)$ is the ℓ^{th} spherical harmonic function with $m = 0$ and $W(\cos \theta_J)$ is the measured differential cross-section in the polar angle of the proper analyser for each reaction in the t-channel helicity frame. The moments were evaluated separately in 17 mass bins for each of the 6 values of ℓ , and the results are shown in figs. 13 to 16.

In these plots the moments up to $\ell = 3$ are significantly different from 0 for most of the mass range. The other moments show some fluctuations, although with limited significance, and their values are generally consistent with zero over the whole mass interval. The curves are the theoretical predictions for the mass dependence of the moments from the model of subsection 3.2. The fact that the moments $\langle Y_{20} \rangle$ and $\langle Y_{30} \rangle$ are different from zero means that diffractive states with spin higher than $1/2$ are present in the final state.

For the three-body final states of reactions (1) and (3) as well as for the two-body neutron diffraction (2), the qualitative behaviour of the moments appears

to be satisfactorily reproduced by the model. This is not the case for coherent neutron diffraction (4), at least for the lowest moments, which display a rapid variation at low masses for $\ell = 1, 2, 3$.

The real and imaginary parts of the moments $\langle Y_\ell^m \rangle$ for $m \neq 0$ are consistent with zero throughout.

6. GLAUBER ANALYSIS OF THE DIFFERENTIAL CROSS-SECTIONS FOR COHERENT REACTIONS

6.1 Basic theory

At high energies the theory of coherent production is usually derived from the so-called "ordered" form of Glauber theory [21], formulated by Glauber and Franco [22] for the deuteron case. The production amplitude is

$$\langle \beta | F | \alpha \rangle = \frac{i p_\ell}{2\pi} \int d^2b e^{i \vec{q}_T \cdot \vec{b}} \times \left\langle \psi, \beta \left| \left[\gamma \left(\vec{b} - \frac{\vec{s}}{2}, \frac{z}{2} \right) + \gamma \left(\vec{b} + \frac{\vec{s}}{2}, \frac{-z}{2} \right) - \right. \right. \right. \\ \left. \left. \left. - \gamma \left(\vec{b} - \frac{\vec{s}}{2}, \frac{z}{2} \right) \gamma \left(\vec{b} + \frac{\vec{s}}{2}, \frac{-z}{2} \right) \theta(z) - \gamma \left(\vec{b} + \frac{\vec{s}}{2}, \frac{-z}{2} \right) \gamma \left(\vec{b} - \frac{\vec{s}}{2}, \frac{z}{2} \right) \theta(-z) \right] \right| \psi, \alpha \right\rangle, \quad (11)$$

where p_ℓ is the laboratory momentum of the projectile α ; $\vec{s}, z$ are the transverse and longitudinal parts of the relative distance $\vec{r}$ of the two nucleons in the deuteron; $\vec{q}_T, q_L$ are the transverse and longitudinal part of the momentum transfer; and $|\psi, \alpha\rangle$ and $|\psi, \beta\rangle$ are the initial and final states of the deuteron and hadron. The step function $\theta(z)$ determines the z -ordering of the interactions in the double-scattering terms, which becomes relevant when the scattering functions γ do not commute. In the case of production, that is $\beta \neq \alpha$, one usually includes as intermediate states only β and α [23]. With this assumption, and if the scattering functions γ are independent from isospin, the amplitude becomes

$$\langle \beta | F | \alpha \rangle = \frac{i p_\ell}{2\pi} \int d^2b e^{i \vec{q}_T \cdot \vec{b}} \left\langle \psi \left| \gamma_{\beta\alpha} \left(\vec{b} - \frac{\vec{s}}{2} \right) e^{i(z/2)q_L} + \right. \right. \\ \left. \left. + \gamma_{\beta\alpha} \left(\vec{b} + \frac{\vec{s}}{2} \right) e^{-i(z/2)q_L} - \left[\gamma_{\beta\alpha} \left(\vec{b} - \frac{\vec{s}}{2} \right) e^{i(z/2)q_L} \times \right. \right. \right. \\ \left. \left. \times \gamma_{\alpha\alpha} \left(\vec{b} + \frac{\vec{s}}{2} \right) \theta(z) + \gamma_{\beta\beta} \left(\vec{b} - \frac{\vec{s}}{2} \right) \gamma_{\beta\alpha} \left(\vec{b} + \frac{\vec{s}}{2} \right) e^{-i(z/2)q_L} \times \right. \right. \\ \left. \left. \times \theta(z) + \left(\begin{smallmatrix} \vec{s} & \rightarrow & -\vec{s} \\ z & \rightarrow & -z \end{smallmatrix} \right) \right] \right| \psi \right\rangle, \quad (12)$$

where the t-matrix elements are related to the operators of (11) in the following way:

$$\gamma_{\beta\alpha}\left(\vec{b} - \frac{\vec{s}}{2}\right) e^{i(z/2)q_L} = \langle\beta|\gamma\left(\vec{b} - \frac{\vec{s}}{2}, \frac{z}{2}\right)|\alpha\rangle \quad (13)$$

and analogously for the diagonal terms, but for the absence of the longitudinal phase; actually, $q_L = (M_\beta^2 - M_\alpha^2)/2p_\ell$. Expressing the amplitudes in (12) as Fourier-Bessel transforms, we get

$$\begin{aligned} \langle\beta|F|\alpha\rangle = & \left\langle\psi\left|f_{\beta\alpha}(\vec{q}_T) e^{iq_L(z/2)} e^{i\vec{s}\cdot(\vec{q}_T/2)} + f_{\beta\alpha}(\vec{q}_T) e^{-iq_L(z/2)} \times \right.\right. \\ & \times e^{-i\vec{s}\cdot(\vec{q}_T/2)} + \frac{i}{2\pi p_\ell} \left[\theta(z) e^{iq_L(z/2)} \int d^2q'_T f_{\beta\alpha}\left(\vec{q}'_T + \frac{\vec{q}_T}{2}\right) \times \right. \\ & \times f_{\alpha\alpha}\left(-\vec{q}'_T + \frac{\vec{q}_T}{2}\right) e^{i\vec{s}\cdot\vec{q}'_T} + \theta(z) e^{-iq_L(z/2)} \int d^2q'_T f_{\beta\beta}\left(\vec{q}'_T + \frac{\vec{q}_T}{2}\right) \times \\ & \left. \left. \times f_{\beta\alpha}\left(-\vec{q}'_T + \frac{\vec{q}_T}{2}\right) e^{i\vec{s}\cdot\vec{q}'_T} + \left(\begin{smallmatrix} \vec{s} \rightarrow -\vec{s} \\ z \rightarrow -z \end{smallmatrix}\right) \right] \right| \psi \rangle, \quad (14) \end{aligned}$$

where the f amplitudes are normalized, so that

$$\frac{d\sigma}{d\Omega_{lab}} = |f|^2.$$

At ISR energies, the corresponding laboratory momenta are of the order of several hundred GeV/c and q_L is very small: therefore it is reasonable to neglect it in formula (14), which therefore becomes

$$\begin{aligned} \langle\beta|F|\alpha\rangle = & 2\hat{S}(\vec{q}_T/2) f_{\beta\alpha}(\vec{q}_T) + \frac{i}{2\pi p_\ell} \int d^2q'_T \hat{S}\left(\frac{\vec{q}_T}{2} - \vec{q}'_T\right) f_{\beta\alpha}(\vec{q}'_T) \times \\ & \times \left[f_{\alpha\alpha}(\vec{q}_T - \vec{q}'_T) + f_{\beta\beta}(\vec{q}_T - \vec{q}'_T) \right]. \quad (15) \end{aligned}$$

We have used here the relation $\theta(z) + \theta(-z) = 1$, and the expression of the deuteron form factor operator

$$\hat{S}(\vec{q}_T) = \langle\psi|e^{i\vec{q}_T\cdot\vec{s}}|\psi\rangle = S_0(q_T) - \left[3(\vec{J}\cdot\vec{q}_T)^2 - 2\right]S_2(q_T), \quad (16)$$

where

$$S_0(q) = \int_0^\infty j_0(qr)[u^2(r) + w^2(r)] dr, \quad S_2(q) = \sqrt{2} \int_0^\infty j_2(qr)\left[u(r)w(r) - \frac{w^2(r)}{\sqrt{8}}\right] dr,$$

u and w being the S and D radial wave functions of the deuteron and $\vec{J}$ its total angular momentum.

In the application of the above formula to our case, α represents the incident nucleon and β a diffractive state, produced coherently on the deuteron. As is well known, β is a composite system of the nucleon and one or two pions, which might be the decay products of a resonance. Since the spin of β is not known, we distinguish different states β by their mass range.

In formula (15), the amplitude $f_{\alpha\alpha}$ is known from the measurement of the elastic proton-proton cross-section:

$$f_{\alpha\alpha}(q_T) = \frac{ip_\ell}{4\pi} \sigma_T (1 - i\rho) e^{-Bq_T^2}, \quad (17)$$

where σ_T is the proton-proton total cross-section, ρ is the ratio of the real to the imaginary part, and B is half the slope of the differential cross-section [24].

In a similar fashion $f_{\beta\beta}$ is connected to the parameters of the β -nucleon interaction,

$$f_{\beta\beta}(q_T) = \frac{ip_\ell}{4\pi} \sigma_T^* (1 - i\rho^*) e^{-B^*q_T^2}. \quad (18)$$

These parameters are not known and are usually kept free in the analysis of coherent production.

The production amplitude $f_{\alpha\beta}$ is determined from nucleon-nucleon experiments; it is usually assumed to be spin independent, purely imaginary and its slope is determined from an exponential fit of the differential cross-section. To take into account the structure of the cross-section visible in the latest experimental data [2,3,5], one has to parametrize it in terms of two exponentials

$$T(q_T) = \frac{\sqrt{\pi}}{p_\ell} f_{\alpha\beta}(q_T) = i \left[a_1 e^{-b_1 q_T^2} + e^{i\phi} a_2 e^{-b_2 q_T^2} \right]; \quad (19)$$

T is normalized as

$$d\sigma/dt = |T(q_T)|^2,$$

where $q_T^2 = -t$. In this case the overlap function has a maximum at $b = 0$, exactly like in the elastic case [1].

While the elastic amplitude is constrained to be central by unitarity, this is not the case for diffraction dissociation, which is unlikely to occur at small impact parameters: in this region of space the open production channels are mainly non-diffractive, by analogy with classical collisions. Therefore a more appropriate form for the profile function comes from allowing the possibility of helicity flips and parametrizing the helicity amplitudes in terms of Bessel functions:

$$T_{\Delta\lambda}(q_T) = iC_{\Delta\lambda} e^{-Aq_T^2} J_{\Delta\lambda}(q_T b_0) e^{i\Delta\lambda\phi}, \quad (20)$$

where $\Delta\lambda$ is the amount of helicity flip; $J_{\Delta\lambda}$ is the Bessel function of order $\Delta\lambda$; ϕ is the azimuthal angle of q_T ; and $C_{\Delta\lambda}$, A , b_0 are parameters which depend on the mass of the diffractive systems. The need of non-negligible terms for $\Delta\lambda \neq 0$ is dictated by the absence of very deep minima in the differential cross-sections; these minima are predicted by the amplitude (20) for $\Delta\lambda = 0$. This form was suggested by Humble [25] and reformulated by Fäldt and Osland [26] to compute the production cross-section on heavy nuclei. The original model contains an arbitrary form for the mass dependence of the coefficients $C_{\Delta\lambda}$, in order to satisfy an approximate spin-mass correlation. In our case these coefficients were kept as free parameters, obtaining in a natural way the spin-mass correlation from the data.

As an example we show in fig. 17 the overlap function in impact parameter space of neutron single diffraction [reaction (2)] for both central [eq. (19)] and peripheral [eq. (20)] amplitudes.

Inserting eqs. (17), (18), and (20) into formula (15) we get^{*)}

$$\begin{aligned} M_{\Delta\lambda} = \frac{p_\ell}{\sqrt{\pi}} \langle \beta | F_{\Delta\lambda} | \alpha \rangle = iC_{\Delta\lambda}(M) & \left\{ 2J_{\Delta\lambda}(q_T b_0) e^{-Aq_T^2} \hat{S}\left(\frac{q_T}{2}\right) + \frac{i}{2p_\ell} \left[f_{\alpha\alpha}(0) + \right. \right. \\ & + f_{\beta\beta}(0) \left. \right] \sum_i \frac{A_i e^{-(\alpha_i + 4B)q_T^2/4}}{\alpha_i + B + A} \times \exp \left[\frac{(\alpha_i + 2B)^2 q_T^2 - b_0^2}{4(\alpha_i + B + A)} \right] \times \\ & \times J_{\Delta\lambda} \left[\frac{(\alpha_i + 2B)q_T b_0}{4(\alpha_i + B + A)} \right] \left. \right\}. \end{aligned} \quad (21)$$

^{*)} More details on the derivation are given in Appendix A.

In the first term of the above expression, the single-scattering term, the azimuthal angle ϕ was taken equal to 0; $\hat{S}$ is defined in eq. (16). In the second term we have neglected the contribution of the deuteron quadrupole form factor, which gives a small correction ($\leq 10\%$), and we have assumed $B^* = B$; A_i, α_i are the parameters of a Gaussian representation of the charge form factor

$$S_0(q) = \sum_i A_i e^{-\alpha_i q^2} \quad *).$$

It is interesting to note that the double-scattering amplitude in eq. (21) is expressed in terms of Bessel functions of the same order as for single scattering but with a different argument; we therefore expect very peculiar interference effects due to the particular structure of these functions.

We have calculated the Glauber formula (15) analytically also for the central case. The result is**))

$$\begin{aligned} \hat{M} = \langle \beta | F | \alpha \rangle \frac{p_\ell}{\sqrt{\pi}} = & 2T(q_T) \hat{S}(\vec{q}_T/2) + \frac{i}{2p_\ell} \left[f_{\alpha\alpha}(0) + f_{\beta\beta}(0) \right] \times \\ & \times \left[ia_1 \sum_i \exp \left[-\frac{4Bb_1 + \alpha_i(B + b_1)}{B + b_1 + \alpha_i} \frac{q_T^2}{4} \right] \frac{A_i}{B + b_1 + \alpha_i} - \right. \\ & - ia_1 \sum_i \exp \left[-\frac{4Bb_1 + \beta_i(B + b_1)}{B + b_1 + \beta_i} \frac{q_T^2}{4} \right] \frac{B_i}{(B + b_1 + \beta_i)^2} \times \\ & \times \left\{ \frac{3}{2}(\vec{J} \cdot \hat{q}_T)^2 + \frac{3}{2}(\vec{J} \cdot \hat{n})^2 - 2 + \frac{(B - b_1)^2 q_T^2}{4(B + b_1 + \beta_i)} [3(\vec{J} \cdot \hat{q}_T)^2 - 2] \right\} + \\ & \left. + e^{i\phi} \begin{pmatrix} a_1 \rightarrow a_2 \\ b_1 \rightarrow b_2 \end{pmatrix} \right], \end{aligned} \quad (22)$$

where $\hat{n}$ defines the unit vector orthogonal to the scattering plane. Here we have taken into account the D-wave contribution in the double-scattering term, using a Gaussian parametrization of the quadrupole form factor

$$S_2(q) = q^2 \sum_i B_i e^{-Bq^2} \quad *).$$

*) The parameters are given in Alberi et al. [27].

**) More details on the derivation are given in Appendix A.

In this case the double-scattering term has the usual exponential form in q_T^2 , and the results for the coherent differential cross-section will have the same pattern as in the elastic case.

6.2 Analysis of experimental data

In fig. 18, we show our experimental data for the differential cross-section for neutron coherent diffraction [reaction (4)] for five different mass bins. The data are compared with the prediction of Glauber formulas given in Section 6.1. The dashed lines are the results obtained with a central profile function in the production amplitude, and the continuous lines refer to the peripheral case. The parameters for the input amplitudes were determined by fitting the data for the reaction $pp \rightarrow p(n\pi^+)$ [3]. These data are completely consistent with our data for reaction (2), but have higher statistical accuracy.

The total cross-section for the composite system σ_T^* was kept fixed and taken equal to 20 mb and $\rho^* = 0$; these values are suggested by a recent Glauber analysis of coherent production on heavy nuclei [28].

While the Glauber predictions for the central case have the typical features of the elastic scattering, with two slopes and the break at $t = -0.3 \text{ GeV}^2$, the data distinctly show a deviation from this behaviour at low masses, $M_{p\pi^-} < 1.44 \text{ GeV}$ (fig. 18a,b).

In this mass region for $-t < 0.5 \text{ GeV}^2$, the data points lie clearly above the dashed line, displaying a pronounced shoulder instead of the usual break; for a larger value of $|t|$, the data points seem to decrease faster than the dashed line. This deviation becomes more evident if one plots the ratio of data and peripheral theory to the central theory (fig. 19a). For peripheral production amplitudes the peculiar behaviour of the experimental data is beautifully reproduced, both below and above $|t| = 0.5 \text{ GeV}^2$.

The analytic structure of the peripheral amplitude (20) with zeros and maxima is essential in order to get the complex behaviour of fig. 19a. For a better understanding of this behaviour it is useful to plot separately the imaginary part of the single- and double-scattering amplitudes for $\Delta\lambda = 0$, which represents

the dominant contribution to the total production cross-section at low masses (fig. 19b). Since the amplitude (21) is spin-dependent, we plot the matrix element $\langle J_z = 1 | \text{Im } M_{\Delta\lambda} | J_z = 1 \rangle$, where the polarization axis is taken to be the normal to the scattering plane. Figure 19b clearly shows the first zero of the Bessel function in the single scattering at $|t| \approx 0.17 \text{ GeV}^2$, and the first zero of the double scattering at $|t| \approx 0.6 \text{ GeV}^2$; this happens because the Bessel functions have different arguments in the single- and double-scattering term (21). Between the two zeros the two amplitudes interfere *constructively*, while at lower and higher t -values there is the usual destructive interference. This explains why the continuous line is below the dashed line for $|t| > 0.6$ in fig. 19a. Even if $\Delta\lambda = 0$ is the most important contribution to the integrated cross-section, there are regions of the angular distribution where the $\Delta\lambda \neq 0$ part plays an important role, filling the minima of the $\Delta\lambda = 0$ part. This is shown in fig. 20, where we show separately the $\Delta\lambda = 0$ (dashed line) and the $\Delta\lambda \neq 0$ contribution (continuous line) for the second-lowest mass bin. Around the first zero and in the high-momentum region, the contribution of helicity flip is essential for producing a smooth behaviour of the differential cross-section. For higher masses the same effect seems to be present although in less dramatic terms, as shown in figs. 18c,d,e.

The experimental data for coherent proton diffraction [reaction (3)] are shown in fig. 21 for four different mass intervals. The features of the data are very similar to those of coherent neutron diffraction, in spite of lower statistics. This can be seen from the comparison of the results with the model predictions for central and peripheral production amplitudes. These amplitudes have been determined from a best fit to the data of reaction (1). The value of σ_T^* is taken independent of the mass and equal to 20 mb, with $\rho^* = 0$, as in the previous case.

The effects of constructive interference are visible at low masses (fig. 21a,b), while at higher masses (fig. 21c,d) a large effect of destructive interference is visible in the comparison between data and theory for the central case.

For low masses we can make the same analysis as for the $(p\pi^-)$ case: this is shown in fig. 22a,b. As for neutron coherent diffraction, there is a pronounced maximum in the ratio R for $1.44 < M_{p\pi^+\pi^-} < 1.6$ GeV, corresponding to the constructive interference between the two zeros of the single- and double-scattering amplitudes. Figure 23 shows the contribution of the term with $\Delta\lambda \neq 0$ for the same range of masses which are, as in the previous case, important around the minimum of the $\Delta\lambda = 0$ contribution. The amplitudes for $(p\pi^-)$ production by neutrons and for $(p\pi^+\pi^-)$ production by protons appear to have the same peripheral features, probably shared by all diffractive processes.

In the above analysis we have used for the parameters σ_T^* and ρ^* a fixed equal value for the two cases, for the purpose of studying the properties of the diffractive amplitudes; we can now relax this constraint and try to determine the values of σ_T^* and of ρ^* from a best fit to the data. This is in the old philosophy of determining the scattering properties of unstable systems on nucleons from coherent production data on nuclei [23,29]. Using an exponential amplitude we have obtained large values of ρ^* that cannot be accepted as reasonable values for the elastic amplitude f_{pp} : this is one more reason to discard the exponential parametrization of the production amplitude. One could still keep the value of ρ^* fixed and equal to 0, and determine σ_T^* by a best fit of the data. It is clear that, in this case, one should get large values for σ_T^* at low masses and small values at large masses. This is clearly shown by the gap between the data points and the dashed curves in figs. 21a-d, which is negative at low masses and positive at large masses. This could be the reason why Edelstein et al. [30] found a behaviour of this type for coherent production on heavy nuclei, as also suggested by the "theoretical experiment" of Fäldt and Osland [26].

Taking on the other hand as production amplitudes the peripheral forms (20), the best fit to the neutron coherent diffraction data yields much better results, with ρ^* values close to zero. The corresponding values for the interaction cross-section σ_T^* are shown in fig. 24 and are given in table 4. In spite of the large errors, a rise of σ_T^* as a function of the mass of the produced system can be seen above 1.4 GeV. A similar feature is present in the coherent production of (3π) systems on nuclei [31], for states of definite spin parity.

7. SUMMARY AND CONCLUSIONS

Using a combined measurement of neutron and proton dissociation into $(N\pi)$ and $(N\pi\pi)$ final states, we performed a systematic investigation of nucleon inelastic diffraction at very high energy, both on proton and deuteron targets at the CERN ISR. The experiment allowed a direct comparison of the basic nucleon dissociation mechanisms in nucleon-nucleon and coherent interactions under identical experimental conditions.

From the analysis of both types of processes a homogeneous and consistent picture of all four reactions emerges which confirms a universal high-energy behaviour of nucleon diffraction dissociation.

The general aspects of the production and decay of low-mass systems are reproduced by a Deck calculation; in particular, the strong decay angular correlations are observed in all four reactions.

The analysis of the coherent diffraction processes, on the grounds of the corresponding nucleon-nucleon diffraction data, makes it possible to obtain new fundamental information on the nature of elementary production amplitudes. This information is derived from the peculiar features of the coherent differential cross-sections, which show a pronounced structure at intermediate values of four-momentum transfer for low produced masses.

We interpret these structures in terms of a strong constructive interference between the single- and double-scattering terms of the Glauber amplitude. This interference is only possible for a peripheral nature of the diffractive production dynamics, and can be observed thanks to the peculiar structure of the deuteron. Exponential amplitudes with a central profile function are unable to reproduce the data.

All these features hold for both two- and three-body final states, thus suggesting a universal peripheral description of nucleon diffraction dissociation.

APPENDIX

We derive here formulas (21) and (22), using peripheral and central production amplitudes [eqs. (19) and (20), respectively]. The other ingredients are Gaussian elastic amplitudes [(17), (18)] and Gaussian deuteron form factors.

We discuss in particular the integral in the double-scattering term of eq. (15):

$$\langle \beta | M^{\text{double}} | \alpha \rangle = \frac{i}{2\pi p_\ell} \int d^2 \vec{q}_T' f_{e1}(\vec{q}_T - \vec{q}_T') T_{\beta\alpha}(\vec{q}_T') S\left(\frac{\vec{q}_T}{2} - \vec{q}_T'\right), \quad (\text{A.1})$$

where $f_{e1} = f_{\alpha\alpha}$ or $f_{\beta\beta}$.

In the peripheral case, we take into account the charge form factor only.

For a definite helicity flip $\Delta\lambda$, the result is the following:

$$\begin{aligned} \langle \beta | M_{\Delta\lambda}^{\text{double}} | \alpha \rangle &= \frac{i}{2\pi p_\ell} \int d^2 \vec{q}_T' \left[f(0) e^{-B(\vec{q}_T - \vec{q}_T')^2} \right] \left[i C_{\Delta\lambda} e^{i\Delta\lambda\phi'} e^{-A\vec{q}_T'^2} J_{\Delta\lambda}(b_0 q_T') \right] \times \\ &\times \left[\sum_i A_i e^{-\alpha_i \left(\frac{\vec{q}_T}{2} - \vec{q}_T' \right)^2} \right] = \frac{i}{2\pi p_\ell} f(0) i C_{\Delta\lambda} \sum_i A_i e^{-(\vec{q}_T^2/4)(\alpha_i + 4B)} \times \\ &\times \int d q_T' q_T' e^{-q_T'^2(\alpha_i + B + A)} J_{\Delta\lambda}(b_0 q_T') \int_0^{2\pi} d\phi' e^{i\Delta\lambda\phi'} e^{q_T q_T' \cos\phi' (2B + \alpha_i)} = \\ &= \frac{i}{2\pi p_\ell} f(0) i C_{\Delta\lambda} \sum_i A_i e^{-(q_T^2/4)(\alpha_i + 4B)} \int_0^\infty d q_T' q_T' e^{-q_T'^2(\alpha_i + B + A)} \times \\ &\times 2\pi J_{\Delta\lambda}(b_0 q_T') I_{\Delta\lambda}[(2B + \alpha_i) q_T q_T'] = \frac{i}{2\pi p_\ell} f(0) i C_{\Delta\lambda} \sum_i A_i \times \\ &\times \exp \left[-\frac{q_T^2}{4} (\alpha_i + 4B) + \frac{(2B + \alpha_i)^2 q_T^2 - b_0^2}{4(B + \alpha_i + A)} \right] \frac{1}{B + \alpha_i + A} \times \\ &\times J_{\Delta\lambda} \left[\frac{b_0 (2B + \alpha_i) q_T}{2(B + \alpha_i + A)} \right]. \end{aligned} \quad (\text{A.2})$$

Here we have used some simple formulas (see, for example, refs. 32 and 33); $J_{\Delta\lambda}$ and $I_{\Delta\lambda}$ are the Bessel and modified Bessel functions of the first kind, respectively

$$I_{\Delta\lambda}(z) = \exp \left[-i \frac{\pi}{2} \Delta\lambda \right] J_{\Delta\lambda}(e^{i\pi/2} z) .$$

In the calculation for the central case it is possible to take into account both the charge and the quadrupole form factors. We use here the expression $T_{\beta\alpha}(q) = i a_1 e^{-b_1 q^2}$; it is easy to extend the result to the amplitude in formula (19):

$$\begin{aligned}
 \langle \beta | M^{\text{double}} | \alpha \rangle &= \frac{i}{2\pi p_\ell} \int d^2 q_T' f_{el} \left(\vec{q}_T' + \frac{\vec{q}_T}{2} \right) T_{\beta\alpha} \left(-\vec{q}_T' + \frac{\vec{q}_T}{2} \right) S(\vec{q}_T') = \\
 &= \frac{i}{2\pi p_\ell} \int d^2 q_T' \left\{ f(0) \exp \left[-B \left(\vec{q}_T' + \frac{\vec{q}_T}{2} \right)^2 \right] \right\} \left\{ i a_1 \exp \left[-b_1 \left(-\vec{q}_T' + \frac{\vec{q}_T}{2} \right)^2 \right] \right\} \times \\
 &\times \left\{ \sum_i A_i e^{-\alpha_i \vec{q}_T'^2} - [3(\vec{J} \cdot \vec{q}_T') - 2] \sum_i B_i e^{-\beta_i \vec{q}_T'^2} \vec{q}_T'^2 \right\} = \\
 &= \left[\frac{i}{2\pi p_\ell} f(0) i a_1 e^{-(q_T^2/4)(B+b_1)} \right] \left\{ \sum_i A_i \int d^2 q_T' e^{-\vec{q}_T'(\alpha_i+B+b_1)} \times \right. \\
 &\times e^{-(B-b_1)\vec{q}_T' \cdot \vec{q}_T} - \sum_i B_i \int d^2 q_T' e^{-\vec{q}_T'(B_i+B+b_1)} e^{-(B-b_1)\vec{q}_T' \cdot \vec{q}_T} \times \\
 &\times [3(\vec{J} \cdot \vec{q}_T')^2 - 2\vec{q}_T'^2] \Big\} = [\dots] \left\{ \sum_i A_i \int d^2 q_T'' e^{-(\alpha_i+B+b_1)q_T''^2} \times \right. \\
 &\times \exp \left[\frac{\vec{q}_T^2}{4} \frac{(B-b_1)^2}{\alpha_i+B+b_1} \right] - \sum_i B_i \int d^2 q_T'' e^{-(\beta_i+B+b_1)q_T''^2} \times \\
 &\times \exp \left[\frac{\vec{q}_T^2}{4} \frac{(B-b_1)^2}{\beta_i+B+b_1} \right] \left[3 \left(\vec{J} \cdot \vec{q}_T'' - \frac{\vec{J} \cdot \vec{q}_T}{2} \frac{(B-b_1)^2}{\beta_i+B+b_1} \right)^2 - \right. \\
 &\left. \left. - 2 \left(\vec{q}_T'' - \frac{\vec{q}_T}{2} \frac{B-b_1}{B+b_1+\beta_i} \right)^2 \right] \right\}. \tag{A.3}
 \end{aligned}$$

It is very easy to perform the integration in $d\phi''$. In fact, $\vec{q}_T'' = |q_T''| \hat{q}_T \cos \phi'' - |q_T''| \hat{n} \sin \phi''$ in our notation, and

$$\begin{aligned}
 \int_0^{2\pi} d\phi'' \cos \phi'' &= \int_0^{2\pi} d\phi'' \cos \phi'' \sin \phi'' = 0 \\
 \int_0^{2\pi} d\phi'' \cos^2 \phi'' &= \int_0^{2\pi} d\phi'' \sin^2 \phi'' = \pi = \int_0^{2\pi} d\phi'' \frac{1}{2}.
 \end{aligned}$$

Inserting these relations in (A.3), we finally get:

$$\begin{aligned}
 \langle \beta | M^{\text{double}} | \alpha \rangle &= [\dots] \left(\sum_i A_i \exp \left[\frac{\vec{q}_T^2}{4} \frac{(B - b_1)^2}{\alpha_i + B + b_1} \right] \frac{\pi}{\alpha_i + B + b_1} - \right. \\
 &\quad - \sum_i B_i \exp \left[\frac{\vec{q}_T^2}{4} \frac{(B - b_1)^2}{\beta_i + B + b_1} \right] \left\{ \int d^2 \vec{q}_T'' e^{-q_T''^2 (\beta_i + B + b_1)} q_T''^2 \times \right. \\
 &\quad \times \left[\frac{3}{2} (\vec{J} \cdot \hat{q}_T)^2 + \frac{3}{2} (\vec{J} \cdot \hat{n})^2 - 2 \right] + \int d^2 \vec{q}_T'' e^{-q_T''^2 (\beta_i + B + b_1)} \times \\
 &\quad \times \left[3 \left(\frac{\vec{J} \cdot \vec{q}_T}{2} \frac{B - b_1}{B + b_1 + \beta_i} \right)^2 - 2 \left(\frac{\vec{q}_T}{2} \frac{B - b_1}{B + b_1 + \beta_i} \right)^2 \right] \Big\} \Bigg) = \\
 &= \frac{i}{2p_\ell} f(0) i a_1 \left(\sum_i \frac{A_i}{\alpha_i + B + \beta_i} \exp \left[-\frac{q^2}{4} \frac{4Bb_1 + \alpha_i (B + b_1)}{\alpha_i + B + b_1} \right] - \right. \\
 &\quad - \sum_i \frac{B_i}{(\beta_i + B + b_1)^2} \exp \left[-\frac{q^2}{4} \frac{4Bb_1 + \beta_i (B + b_1)}{\beta_i + B + b_1} \right] \times \\
 &\quad \times \left\{ \frac{3}{2} (\vec{J} \cdot \hat{q}_T)^2 + \frac{3}{2} (\vec{J} \cdot \hat{n})^2 - 2 + \frac{(B - b_1)^2}{B + b_1 + \beta_i} \frac{q_T^2}{4} [3(\vec{J} \cdot \hat{q}_T)^2 - 2] \right\} \Bigg) .
 \end{aligned}
 \tag{A.4}$$

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Table 1
Relevant parameters for reactions (1) to (4)

Reaction Parameters	$pp \rightarrow (p\pi^+\pi^-)p$	$np \rightarrow (p\pi^-)p$	$pd \rightarrow (p\pi^+\pi^-)d$	$nd \rightarrow (p\pi^-)d$
$\sqrt{s}$ (GeV)	53.2	37.2	53.2	37.2
Equivalent p_{lab}	1474	740	735	368
Selected events	4807	6646	2320	14053
$\int \mathcal{L} \text{ (}\mu\text{b}^{-1}\text{)}$	-	8525	8794	12455
Statistical + systematic error	-	11.8%	11.4%	11.2%
σ (μb) for $ t > 0.05 \text{ GeV}^2$	-	122.8 ± 14.5	24.3 ± 2.8	25.6 ± 2.9
Scale uncertainty	-	8%	10%	9%

Table 2

Numerical results of the fits to the differential cross-sections of reactions (1) to (4) to the assumed form $d\sigma/dt = A_1 e^{b_1 t} + A_2 e^{b_2 t}$

Reaction	A_1 ($\mu\text{b GeV}^{-2}$)	b_1 (GeV^{-2})	A_2 ($\mu\text{b GeV}^{-2}$)	b_2 (GeV^{-2})
$pp \rightarrow (p\pi^+\pi^-)p$	5916 ± 4350 a)	-21.0 ± 1.0	1804 ± 49 a)	-5.7 ± 0.5
$np \rightarrow (p\pi^-)p$	1848 ± 97	-16.5 ± 1.5	422 ± 43	-5.0 ± 0.5
$pd \rightarrow (p\pi^+\pi^-)d$	4188 ± 576	-34.2 ± 2.0	41 ± 9	-6.5 ± 1.0
$nd \rightarrow (p\pi^-)d$	9566 ± 813	-35.9 ± 1.5	146 ± 18	-7.9 ± 0.5

a) Arbitrary units.

Table 3

Numerical results of the fits to the dependence of the forward slope on the produced mass according to eq. (9)

Reaction	A (GeV^{-2})	B ($\text{GeV}^{\alpha-2}$)	α
$pp \rightarrow (p\pi^+\pi^-)p$	4.3 ± 0.5	57.0 ± 4	4.3 ± 0.5
$np \rightarrow (p\pi^-)p$	4.0 ± 0.5	60.0 ± 9	6.5 ± 0.5
$nd \rightarrow (p\pi^-)d$	19.0 ± 1.0	39.5 ± 3	3.8 ± 0.5

Table 4

Values of the interaction cross-section σ_T^* (in mb) for proton and neutron coherent diffraction

M (GeV)	< 1.3	1.3-1.44	1.44-1.6	1.6-1.8	> 1.8
$nd \rightarrow (p\pi^-)d$		7.8 ± 3.5	20.6 ± 4.0	21.8 ± 5.1	41.3 ± 5.8
$pd \rightarrow (p\pi^+\pi^-)d$	33 ± 15.5		2.1 ± 14.0	23.8 ± 8.6	27.4 ± 7.4

Figure captions

- Fig. 1 : Plan view of the Split Field Magnet detector.
- Fig. 2 : Invariant mass distributions for reactions (1) to (4). The curves are the predictions of a Deck model calculation described in the text.
- Fig. 3 : Invariant mass distributions for reactions (1) to (4) for four-momentum transfer values greater than 0.25 GeV^2 .
- Fig. 4 : Invariant mass distributions for reactions (1) to (4) for backward decay angles in the Jackson frame.
- Fig. 5 : The three components of the generalized Deck model:
a) pion exchange; b) baryon exchange; c) direct pole.
- Fig. 6 : Differential cross-sections for reactions (1) to (4). The curves are a double-exponential fit to the data described in the text.
- Fig. 7 : Slope-mass correlation for single nucleon diffraction:
a) three-body final state [reaction (1)];
b) two-body final state [reaction (2)].
The curves are a power-law parametrization described in the text.
- Fig. 8 : Slope-mass correlation for coherent neutron diffraction [reaction (4)].
- Fig. 9 : Decay angular distributions for single proton diffraction [reaction (1)]:
a) scatter plot of events in the Jackson frame;
b) double differential cross-section (after local smoothing) in the Jackson plane;
c) and d) polar and azimuthal angle distributions; the curves are model calculations described in the text.

- Fig. 10 : Decay angular distributions for single neutron diffraction [reaction (2)]:
- a) scatter plot of events in the Jackson frame;
 - b) double differential cross-section (after local smoothing) in the Jackson plane;
 - c) and d) polar and azimuthal angle distributions; the curves are model calculations described in the text.
- Fig. 11 : Decay angular distributions for coherent proton diffraction [reaction (3)]:
- a) scatter plot of events in the Jackson frame;
 - b) double differential cross-section (after local smoothing) in the Jackson plane;
 - c) and d) polar and azimuthal angle distributions; the curves are model calculations described in the text.
- Fig. 12 : Decay angular distributions for coherent neutron diffraction [reaction (4)]:
- a) scatter plot of events in the Jackson frame;
 - b) double differential cross-section (after local smoothing) in the Jackson plane;
 - c) and d) polar and azimuthal angle distributions; the curves are model calculations described in the text.
- Fig. 13 : Normalized spherical harmonic moments $\langle Y_{\ell}^0 \rangle$ of the Δ^{++} direction of motion for reaction (1) in the t-channel helicity frame as a function of mass for $\ell = 1$ to 6. The curves are a model calculation described in the text.
- Fig. 14 : Normalized harmonic moments $\langle Y_{\ell}^0 \rangle$ of the nucleon direction of motion for reaction (2) in the t-channel helicity frame as a function of mass for $\ell = 1$ to 6. The curves are a model calculation described in the text.

- Fig. 15 : Normalized harmonic moments $\langle Y_{\ell}^0 \rangle$ of the Δ^{++} direction of motion for reaction (3) in the t-channel helicity frame as a function of mass for $\ell = 1$ to 6. The curves are a model calculation described in the text.
- Fig. 16 : Normalized harmonic moments $\langle Y_{\ell}^0 \rangle$ of the nucleon direction of motion for reaction (4) in the t-channel helicity frame as a function of mass for $\ell = 1$ to 6. The curves are a model calculation described in the text.
- Fig. 17 : Overlap function in impact parameter space of neutron single diffraction assuming central (a) and peripheral (b) amplitudes (see text). The arrows indicate the value of $b_{\text{r.m.s.}}$ for both cases.
- Fig. 18 : Differential cross-sections for neutron coherent diffraction [reaction (4)] in five different mass bins. The continuous curves correspond to the peripheral model, the dashed curves to the central model (see text).
- Fig. 19 : a) Data points and results of the peripheral model normalized to the predictions of the Glauber model with central amplitudes as a function of $|t|$ in neutron coherent diffraction [reaction (4)] for the mass interval $M(p\pi^-) < 1.3$ GeV.
b) t-dependence of the imaginary part of the coherent production amplitudes, for $\Delta\lambda = 0$, in reaction (4). The continuous curve is the single-scattering term, the dashed curve the double-scattering one. The sign of each part of the amplitude is also shown on the curves.
- Fig. 20 : Differential cross-section for coherent neutron diffraction [reaction (4)] in the second-lowest mass bin. The curves represent two separate contributions to the peripheral model calculation (continuous line in fig. 17b); the dashed line is the contribution of the $\Delta\lambda = 0$ amplitude, the continuous line the contribution of the helicity-flip terms.

- Fig. 21 : Differential cross-sections for proton coherent diffraction [reaction (3)] in four different mass bins. The continuous curves correspond to the peripheral model, the dashed curves to the central model (see text).
- Fig. 22 : a) Data points and results of the peripheral model normalized to the predictions of the Glauber model with central amplitudes as a function of $|t|$ in proton coherent diffraction [reaction (3)] for the mass interval $1.44 < M(p\pi^+\pi^-) < 1.6$ GeV.
b) t -dependence of the imaginary part of the coherent production amplitudes, for $\Delta\lambda = 0$, in reaction (3). The continuous curve is the single-scattering term, the dashed curve the double-scattering one. The sign of each part of the amplitude is also shown on the curves.
- Fig. 23 : Differential cross-section for coherent proton diffraction [reaction (3)] in the second-lowest mass bin. The curves represent two separate contributions to the peripheral model calculation (continuous line in fig. 20b); the dashed line is the contribution of the $\Delta\lambda = 0$ amplitude, the continuous line the contribution of the helicity-flip terms.
- Fig. 24 : Dependence of the total interaction cross-section σ_T^* on produced mass from coherent proton and neutron diffraction. The dashed line indicates the value used in the previous analyses on the production amplitudes. The solid line is a linear fit to guide the eye through the points above 1.4 GeV.

Fig. 1

Fig. 2

Fig. 3

Fig. 4

Fig. 5

Fig. 6

Fig. 7

Fig. 8

Fig. 9

Fig. 10

Fig. 11

Fig. 12

PP- $P(\pi^+\pi^-)$

Fig. 13

PN→P(PT⁻)

Fig. 14

$PD \rightarrow (P\pi^+\pi^-)D$

Fig. 15

ND $\rightarrow$ (PT $^+$)D

Fig. 16

Fig. 17

Fig. 18

Fig. 19

Fig. 20

Fig. 21

Fig. 22

Fig. 23

Fig. 24