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PIONIC $4f \rightarrow 3d$ TRANSITION IN ^{181}Ta , NATURAL Re , AND ^{209}Bi ,
AND THE STRONG INTERACTION LEVEL SHIFT AND WIDTH OF THE

PIONIC $3d$ STATE

J. Konijn, J.K. Panman^{*)} and J.H. Koch

Institute for Nuclear Physics Research, Amsterdam, The Netherlands

W. van Doesburg

Free University, Amsterdam, The Netherlands

G.T. Ewan

Queens University, Kingston, Canada

T. Johansson and G. Tibell

CERN, Geneva, Switzerland

K. Fransson

Research Institute of Physics, Stockholm, Sweden

L. Tauscher

Institute of Physics, University of Basle, Switzerland

(Submitted to Nuclear Physics A)

^{*)} Present address: CERN, Geneva, Switzerland

ABSTRACT

Owing to a powerful Compton-suppression technique it was possible to observe for the first time the pionic $4f \rightarrow 3d$ X-ray transition in elements heavier than $A = 150$. The strong interaction monopole shifts ϵ_0 and widths Γ_0 as well as the quadrupole splitting of the $3d$ levels have been measured in Ta, Re, and Bi. Thus in addition to the strongly shifted and broadened $5g \rightarrow 4f$ transitions, a second, strongly affected line is available for these elements. For the pionic $4f$ levels, standard optical potentials fit the strong interaction shifts and broadenings quite well. The now observed, deeper-lying $3d$ states in Ta, Re, and Bi have shifts and widths that differ by a factor of 2 or more from the standard optical potential predictions. From the observed relative X-ray intensities of the pionic cascade we also extract the strong interaction widths of the $5g$ and $4f$ levels.

1. INTRODUCTION

Pionic atoms provide a powerful tool for investigating the strong pion-nucleus interaction at low energies. As the captured negative pion cascades down through atomic states, it finally reaches circular orbits close enough to the nucleus to feel the short-range strong pion-nucleus interaction. Radiative transitions between two mesic levels during this latter part of the cascade therefore show deviations from the purely electromagnetic case. The energy levels involved are shifted by the strong interaction and, since pions are being absorbed, there is a broadening of each level in addition to the radiative width.

These effects grow very rapidly from one level to the next lower one, and the increase in absorption leads to a fast decrease in intensity of X-rays for subsequent transitions. Therefore, once the strong shift and width are large enough to be directly observable for a given transition, the X-ray for the next one is even much broader and weaker. This has limited the measurement of both the strong shift and width to usually only one level for a given target: the 1s state in the lightest nuclei ($Z \leq 11$), the 2p state in nuclei with $8 \leq Z \leq 33$, the 3d state for $39 \leq Z \leq 60$, and the 4f state in the heaviest nuclei ($Z \geq 67$). From this information, a semi-phenomenological optical potential has been constructed, containing terms linear and quadratic in the nuclear density. Besides fitting the available data on π -mesic atoms reasonably well, this potential has also been used successfully to predict pion photoproduction cross-sections near threshold. It has also been shown that this potential is consistent with the optical potentials used for low-energy π -scattering.

To learn more about the pion nucleus interaction and to get a better understanding of the higher-order terms in the optical potential, one would like to measure also strong interaction effects for the lower-lying levels, e.g. for the 3d levels in heavy nuclei. These states have a much larger nuclear overlap than the previously studied 4f states. Together with the observed effects for the 4f states in the same nuclei this provides a test for the density dependence of the pion-nucleus optical potential.

One way to observe such low-lying states has been proposed by Leon¹⁾. It makes use of the dynamical polarization which occurs when a nuclear excitation of the target is degenerate with the transition energy between two pionic atom states. However, this method does not determine the strong interaction shift and width separately, and can be used for only a few targets, where this accidental degeneracy exists.

In this paper we report a more direct measurement of the strong interaction shifts and widths for the low-lying 3d level in Ta, Re, and Bi. This was done by using a background suppression technique²⁾ which enabled us to resolve the broad, very weak, $4f \rightarrow 3d$ X-ray. In contrast to the resonance method of Leon, this approach can be applied, in principle, to any target. The nuclei studied here have spins $I > 1$ and each pionic level is split into a hyperfine (hf) complex. Therefore, in addition to the monopole shift ϵ_0 and width Γ_0 we also measure the quadrupole splitting for the 3d level, and discuss the quadrupole shift ϵ_2 .

2. EXPERIMENT AND ANALYSIS

Recently, an experiment was performed at the muon channel of the CERN Synchrocyclotron (SC) stopping pions in ^{181}Ta and ^{209}Bi targets. Results on the observed neutron multiplicities after pion capture and their implication for the capture mechanism were presented by Beetz et al.³⁾. In another paper⁴⁾ the determination of the effective quadrupole moment from pionic X-rays was reported. In the latter analysis the hyperfine splittings of the transitions to the 4f level were studied; this is traditionally considered to be the last observable level of the cascade in these nuclei.

However, the experimental set-up was especially designed to detect weak transitions by sacrificing some of the resolution in energy in order to obtain a higher photo-peak efficiency. This was achieved by replacing the usual LEPS small-volume detectors by a highly efficient large-volume diode with a high peak-to-Compton ratio. A further reduction of the background was accomplished by incorporating this detector into a Compton suppression system^{2,5)}.

The spectra were recorded in a time differential way³⁾, allowing for a clean separation between the prompt X-rays and the delayed γ -ray background.

In this paper we report on the results of an analysis of these data which takes full advantage of the high background suppression of the experimental set-up. We were able to separate from the background the broad line corresponding to the pionic $4f \rightarrow 3d$ transition. Also included in our analysis are new data from pions stopped in a natural Re target⁶⁾.

The relevant part of the prompt and delayed γ -ray spectra from pionic tantalum and bismuth are shown in Figs. 3 and 6 of Ref. 3. Figure 1 shows an enlarged part of the prompt Ta spectrum. A broad bump is observed between 965 keV and 1040 keV, identified below as the hf complex of the $4f \rightarrow 3d$ transition in ^{181}Ta . In the midst of this complex can be seen the pronounced 1014.46 keV γ -ray produced by inelastic neutron scattering on aluminium in the Ge(Li) detector cap. This line was used for calibration purposes. Since the broad bump is not present in the delayed spectra, it is clearly due to a prompt process. We can rule out the possibility of any Compton or pair processes being the source of this bump since there are no associated photo peaks.

Furthermore, the complex is observed in both our Ge(Li) detectors. Since the energy of the pionic $4f \rightarrow 3d$ X-ray is expected at around 1000 keV, we can safely assign the complex to the $4f \rightarrow 3d$ pionic X-ray transition in ^{181}Ta . The same criteria were used to identify the corresponding hf complexes in Re and Bi.

The pionic Re data⁶⁾ (Fig. 2) have been analysed under the assumption that the strong interaction shifts of ^{185}Re and ^{187}Re are equal within experimental errors. The centre of the $4f \rightarrow 3d$ complex in Re occurs at 1059 keV, and therefore correction has to be made for the influence from the nearby 1040 keV γ -ray transition due to inelastic neutron scattering from ^{70}Ge in the Ge(Li) detector itself. The position and line shape of this background peak are well known and are taken from our experiments with the other targets.

In the spectra from pionic ^{209}Bi (Fig. 3) a prompt γ -ray of 1304.1 keV falls in the middle of the $4f \rightarrow 3d$ complex. It originates from the γ -ray cascade in ^{204}Pb , produced by the $(\pi^-, 5n)$ reaction³⁾. The intensity of this transition was calculated from that of the 776.9 keV γ -ray from the same cascade⁷⁾ and was then kept constant throughout the analysis. The intensities of the ^{127}I and ^{63}Cu lines were obtained from a comparison with the spectra recorded with other targets.

The nuclei considered here have spins $I > 1$ and each pionic level (n, ℓ) is split into an hf multiplet. Our analysis of the X-ray transitions for such nuclei -- determination of hyperfine splittings, line widths, energies and intensities -- follows closely the method in Ref. 4, and we refer to this article for our notation and further details. The dominant hf splitting is due to nuclear quadrupole effects, but we also include in our discussion the small magnetic dipole splitting. The shift of each member away from the central value of the hf multiplet can then be written as

$$E_{\text{hf}} = A_1^{(n, \ell)} \gamma(I, \ell, F) + A_2^{\text{eff}} C(I, \ell, F) , \quad (1)$$

where A_1 is the magnetic hf constant, which we take to be the point nucleus value; $A_2^{\text{eff}}(n, \ell)$ is the quadrupole hf constant for the level (n, ℓ) . The decomposition of A_2^{eff} into an electric part A_2 and a strong interaction piece ϵ_2 , according to

$$A_2^{\text{eff}} = A_2 - \epsilon_2 , \quad (2)$$

will be discussed in Section 4.

In ^{181}Ta we measure a difference of 18.3 ± 1.2 keV between the $^{13/2}(4f) \rightarrow ^{11/2}(3d)$ and $^{11/2}(4f) \rightarrow ^{9/2}(3d)$ transitions. Using the value $A_2^{\text{eff}}(4f) = 1.75 \pm 0.016$ keV from Ref. 4 we obtain for the 3d level $A_2^{\text{eff}} = 9.11 \pm 0.51$ keV. This corresponds to an "effective quadrupole moment" (see Ref. 4) of $Q_{\text{eff}} = 3.10 \pm 0.17b$. In Re the splitting between the $^{11/2}(4f) \rightarrow ^{9/2}(3d)$ and $^{9/2}(4f) \rightarrow ^{7/2}(3d)$ lines is 12.0 ± 1.7 keV. For our natural Re target, we have to take into account that it contains the isotopes ^{185}Re and ^{187}Re with abundances of 37.4% and 62.6%, respectively. The ratio of quadrupole moments of these isotopes has been determined by Segel⁸⁾ by a reso-

nance method to be $Q_{185}/Q_{187} = 1.056709(17)$. Assuming that our quadrupole effects scale in this way and with $A_2^{\text{eff}}(4f) = 1.163 \pm 0.010$ keV (see Ref. 6), we obtain $A_2^{\text{eff}}(3d) = 5.39 \pm 0.63$ keV for ^{187}Re ($Q_{\text{eff}} = 1.68 \pm 0.20b$). In Bi the expected quadrupole splitting of hf transitions is only about 2 keV. Since the Lorentzian width of each transition is about 25 keV, it is impossible to determine the quadrupole effects accurately. Therefore in our fits for the Bi data we used the calculated value of 2.1 keV for the splitting between the dominant $^{15/2}(4f) \rightarrow ^{13/2}(3d)$ and $^{13/2}(4f) \rightarrow ^{11/2}(3d)$ transitions. This value was kept constant throughout the analysis.

From the analysis described above, the centroid positions of the hf complex are determined, yielding the $4f \rightarrow 3d$ X-ray transition energies. Our results are shown in Table 1. The quoted electromagnetic values are the point Coulomb values with corrections for vacuum polarization, Lamb shift, and electron screening. The remaining difference between the calculated electromagnetic and experimental value is due to finite-size Coulomb and strong interaction effects. Correcting this difference for the previously determined shift of the 4f level, $\epsilon_0(4f)$, we obtain the monopole shift $\epsilon_0(3d)$. Table 2 shows our results, along with a compilation of other recent measurements in neighbouring nuclei^{3,4,6,9-13}, for $\epsilon_0(3d)$ and $\epsilon_0(4f)$.

The strong interaction broadening of the 3d levels is extracted from the data by fitting the hf components with a Lorentzian line shape. For this fit, the calculated value for Γ_2/Γ_0 was used (see Table 3). The results for the monopole width Γ_0 are rather insensitive to this choice. Subtracting from the fitted width the radiative widths of initial and final levels, we obtain the results for $\Gamma_0(3d)$ shown in Table 2.

All available data for $\epsilon_0(3d)$ and $\Gamma_0(3d)$ together with our new results are plotted as a function of Z in Figs. 4 and 5.

3. PIONIC X-RAY CASCADE

From the recorded spectra, we obtain the X-ray intensities of the transitions of the pionic cascade. The relative X-ray intensities have been determined from

the measured spectra by correcting the efficiency of the Ge(Li) detector in the Compton-suppression system for self-absorption in the target. The angular dependence of the target thickness is negligible, since the solid angle extended by the target with respect to the Ge(Li) detector is only 0.6 msr. In Tables 4 and 5 the observed transition energies and intensities are presented for ^{181}Ta and ^{209}Bi , respectively. The data for Re can be found in Ref. 6. The calculated energies include the full electromagnetic interaction, including finite size, electron screening, higher-order vacuum polarization, and Lamb shift corrections, but without strong interaction effects. The observed relative intensities are in good agreement with the predictions of the cascade program of Tauscher¹⁴).

Given the relative intensities of the pionic X-rays, we can determine the strong interaction width Γ_0 of the (n, ℓ) level according to

$$\Gamma_0(n, \ell) = \Gamma_{\text{rad}}(n, \ell) \{ [1/Y(n, \ell)] - 1 \} ,$$

where $\Gamma_{\text{rad}}(n, \ell)$ is the electromagnetic width given in Table 3. In our cases the Auger rates can be neglected as compared to the radiative width of the 5g and 4f levels. The ratio $Y(n, \ell)$ is that of the intensity of the transition depopulating the level (n, ℓ) to the next lower circular orbit over the sum of intensities of transitions feeding into (n, ℓ) :

$$Y(n, \ell) = \frac{I[(n, \ell) \rightarrow (n-1, \ell-1)]}{\sum_{n_x \ell_x} I[(n_x \ell_x) \rightarrow (n, \ell)]} .$$

The widths obtained in this fashion for the 5g and 4f levels are shown in Table 6. They are in good agreement with the widths determined directly from line shapes and with the theoretical predictions.

4. DISCUSSION AND CONCLUSIONS

In calculating the strong interaction monopole shifts and level widths, the standard Kisslinger optical potential was adopted:

$$V_{\pi}(\vec{r}) = - \frac{4\pi}{2\mu} \left[q(\vec{r}) - \bar{v} \frac{\alpha(\vec{r})}{1 + \frac{4\pi}{3} \xi \alpha(\vec{r})} \bar{v} \right] \quad (3a)$$

where

$$q(\vec{r}) = \left(1 + \frac{m_\pi}{M_p}\right) \{b_0 \rho(\vec{r}) + b_1 [\rho_n(\vec{r}) - \rho_p(\vec{r})]\} \\ + \left(1 + \frac{m_\pi}{2M_p}\right) B_0 \rho^2(\vec{r}) \quad (3b)$$

and

$$\alpha(\vec{r}) = \left(1 + \frac{m_\pi}{M_p}\right)^{-1} \left\{ c_0 \rho(\vec{r}) + c_1 [\rho_n(\vec{r}) - \rho_p(\vec{r})] \right\} \\ + \left(1 + \frac{m_\pi}{2M_p}\right)^{-1} C_0 \rho^2(\vec{r}), \quad (3c)$$

where μ is the reduced pion-nucleus mass, b_0 and b_1 are the isoscalar and isovector s-wave scattering lengths, c_0 and c_1 are the corresponding scattering volumes for the p-wave. The complex parameters B_0 and C_0 represent the pion absorption on two nucleons, and the term proportional to ξ has been introduced to account for nuclear correlations. There have been a variety of fits¹⁵⁻¹⁷⁾ of these parameters to the available data¹⁸⁾ from pionic atoms. Table 7 shows three representative sets, which we will use in this discussion. Also shown are the nuclear density parameters c and t , and the deformation β . For simplicity, we assume that the matter density ρ and the neutron and proton densities ρ_n and ρ_p are proportional:

$$\rho : \rho_n : \rho_p = A : N : Z .$$

To the strong π -nucleus potential V_π we add the Coulomb potential for a finite nucleus. Expanding to order β^2 in the nuclear deformation parameter, we decompose the total potential into a monopole and quadrupole part. We then solve a modified Klein-Gordon equation with the $\beta = 0$ monopole part only. The β^2 monopole terms are treated in perturbation theory. After subtracting out the point Coulomb value, we obtain the monopole shift ϵ_0 and width Γ_0 ¹⁹⁾. The results for the three optical potentials are shown in Table 2. The calculations reproduce the 4f shifts in Ta, Re, and Bi reasonably well. However, for the 3d shift, due to finite size and strong interaction effects, the potentials predict a shift ϵ_0 much larger than observed. Turning off the strong interaction, one obtains for

$\epsilon_0(3d)$ a repulsive finite size shift of -3.6, -4.3 and -10.1 keV for Ta, Re and Bi, respectively.

A similar discrepancy exists for the widths Γ_0 . There is some variation among the different predictions of the models, but all potentials overestimate ϵ_0 and Γ_0 by at least a factor of two for the 3d level.

With the pion wave functions obtained from solving the monopole part of the modified Klein-Gordon equation, the quadrupole part of the potential is evaluated in perturbation theory. The electromagnetic part yields A_2 , the electric quadrupole constant, which differs from the point value because of finite size and pion distortion effects. Evaluating the strong interaction quadrupole potential, the strong interaction quadrupole shift and width ϵ_2 and Γ_2 are obtained. These values are shown in Table 8. Together, A_2 and ϵ_2 yield the total "effective" quadrupole constant

$$A_2^{\text{eff}} = A_2 - \epsilon_2 . \quad (4)$$

All theoretical predictions for the full $A_2^{\text{eff}}(3d)$ fall considerably below the experimental values. Subtracting the calculated A_2 from the observed A_2^{eff} to obtain the "experimental" ϵ_2 is of course not model-independent. However, in the case of the 4f level, A_2 is practically unchanged from its point nuclear value, and the resulting "experimental" $\epsilon_2(4f)$ therefore does not depend on the particular optical potential model. For the 3d level on the other hand, the calculated A_2 differs from the point nuclear value (Ta: 9.70 keV, Re: 6.57 keV) and there is a 5% variation among the three models. Using arbitrarily Set I for an estimate, $\epsilon_2(3d) = 0.8 \pm 0.5$ keV is obtained for Ta and 1.3 ± 0.6 keV for Re. The error, of course, only accounts for the experimental one in A_2^{eff} and not for the model uncertainty.

It is interesting to note that while the monopole shift ϵ_0 remains attractive in the 4f and 3d levels (see Table 2), ϵ_2 changes sign. Theory and experiment agree on a negative ϵ_2 for the 4f level^{4,6,9}, but we obtain a positive $\epsilon_2(3d)$ for the theoretical as well as the estimated experimental value. This is because

in the 4f level the attractive p-wave part still dominates $\epsilon_2(4f)$, while in the 3d state the repulsive s-wave part takes over. This sensitivity should be explored further.

Some improvements can be made in the strong interaction potential without going much beyond the form assumed in Eqs. (3). For example, one can introduce a different radius for the neutron density or adjust the absorption term such that neutron pairs are not included. While these changes may lead to some improvement for the 4f level, they are not large enough to remedy the disagreement for the 3d state.

The standard optical potentials have been fitted to the strong interaction effects in the last observed X-ray transitions in various nuclei. This involved only relatively small overlap between the pion wave function and the tail of the nuclear density. The measurements for the 3d level in heavy nuclei reported here involve a considerably larger overlap than, for example, the previously observed 4f state in the same targets. The disagreement between theory and experiment for the lower level indicates that the standard optical potentials are inadequate for these large overlaps. This trend also appears when one follows, for a fixed state, the shifts and widths towards heavier targets (Figs. 6 and 7). As Z increases and the overlap with the nuclear density grows, the disagreement between theory and experiment rises to a factor of about four. Finally, Fig. 8 shows that the theory predicts the right relative magnitudes for the shifts and widths. The predicted ratio ϵ_0/Γ_0 agrees quite well with experiment. As shown above, the negative finite size effects in the shifts $\epsilon_0(3d)$ are very large in this region. The trend towards smaller (and finally negative) values is, therefore, due to the fact that the repulsive s-wave interaction as well as the finite size effects become more dominant²⁰⁾ over the attractive p-wave potential.

As mentioned in the Introduction, the combined shift and broadening of low-lying "hidden" levels can also be studied by making use of the dynamical nuclear polarization effect in some pionic atoms¹⁾. A detailed discussion of this phenomenon can be found in the paper by Dubach et al.²¹⁾. These authors also used the

optical potentials shown in Table 7. Our finding that the standard optical potential might overestimate the strong interaction effects for low-lying levels in heavy nuclei is not inconsistent with the conclusions by Dubach et al.²¹⁾. For example, simply cutting the strong interaction effects in half where unobserved "hidden" levels are involved still yields reasonable agreement^{*)} with the attenuation measurements.

Clearly, both approaches -- the direct measurement reported here and the indirect resonance method -- will be extremely powerful in yielding a more detailed understanding of the pion-nucleus interaction for low-lying pionic states in heavy nuclei.

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*) For ^{150}Sm the attenuation from this simple estimate is somewhat too high. For ^{110}Pd , where still some additional nuclear structure information is needed²¹⁾, the agreement with experiment is greatly improved.

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Table captions

- Table 1. Experimental 4f-3d X-ray transition energies. The calculated values are those for the point nucleus with corrections for vacuum polarization to higher orders, Lamb shift, and electron screening for realistic populations of the atomic K- and L-shells at the moment of the pionic X-ray transition. From these data the full finite size and strong interaction shift $\epsilon_0(3d)$ is obtained after correcting for the observed shift $\epsilon_0(4f)$ of the upper level.
- Table 2. Experimental and theoretical monopole shifts ϵ_0 and widths Γ_0 for pionic 4f and 3d levels with respect to the point nucleus. The theoretical values are obtained from optical potentials using sets I-III in Table 7.
- Table 3. Radiative width Γ_{rad} of the pionic levels and the ratio Γ_2/Γ_0 obtained from theory (Set I, Table 7).
- Table 4. Pionic transitions in ^{181}Ta . Calculated results without strong interaction effects.
- Table 5. Pionic transitions in ^{209}Bi . Calculated results without strong interaction effects.
- Table 6. Relative yields and strong interaction widths for pionic 5g and 4f levels obtained from experimental transition intensities. The direct line-shape measurements and the theoretical values are shown for comparison.
- Table 7. Optical potential parameter sets, used to calculate the strong interaction level shifts and widths. Also shown are parameters for a deformed Fermi distribution.
- Table 8. Calculated strong interaction quadrupole shift ϵ_2 and width Γ_2 for the 4f and 3d levels with optical potential parameter Sets I-III. A_2 is the calculated electromagnetic quadrupole constant. Also shown is the theoretical and experimental effective quadrupole constant,

$$A_2^{\text{eff}} = A_2 - \epsilon_2 .$$

Table 1

Nucleus	Energy of the pionic $4f \rightarrow 3d$ transition		The strong interaction monopole shift	
	Experimental values (keV)	Calculated values (keV)	$\epsilon_0(4f)$ (keV)	$\epsilon_0(3d)$ (keV)
^{181}Ta	1002.0 ± 1.0	992.26	0.549 ± 0.10 a)	10.2 ± 1.0
Re	1059.1 ± 1.0	1048.83	0.685 ± 0.050 b)	11.1 ± 1.0
^{209}Bi	1301.3 ± 1.3	1292.51	1.83 ± 0.15 c)	10.6 ± 1.0

a) Data from Beetz et al.⁴⁾, relative to the point nucleus value.

b) Data from Konijn et al.⁶⁾.

c) Data from Beetz et al.³⁾, relative to the point nucleus value.

Table 2

Nucleus	$\varepsilon_0(4f)$ in eV				References
	Experiment	I	II	III	
^{165}Ho	295 ± 80	231	240	246	Ebersold et al. ⁹⁾
^{175}Lu	597 ± 70	401	415	430	Ebersold et al. ⁹⁾
^{181}Ta	549 ± 100	524	539	568	Beetz et al. ⁴⁾
Re	685 ± 50	680	700	743	Konijn et al. ⁶⁾
^{209}Bi	1830 ± 150	1794	1850	1959	Beetz et al. ³⁾
^{209}Bi	1870 ± 60	1794	1850	1959	Batty et al. ¹⁰⁾
^{238}U	5320 ± 250	4291	4049	5062	Jenkins et al. ¹¹⁾
^{238}U	5190 ± 200	4291	4049	5062	Batty et al. ¹⁰⁾
$\Gamma_0(4f)$ in eV					
^{165}Ho	190 ± 40	128	136	144	Ebersold et al. ⁹⁾
^{175}Lu	200 ± 70	237	250	272	Ebersold et al. ⁹⁾
^{181}Ta	225 ± 57	322	335	378	Beetz et al. ¹⁴⁾
Re	547 ± 50	428	442	517	Konijn et al. ⁶⁾
^{209}Bi	1157 ± 70	1216	1228	1484	Beetz et al. ³⁾
^{209}Bi	1240 ± 140	1216	1228	1484	Batty et al. ¹⁰⁾
^{238}U	4050 ± 860	3905	3709	4964	Jenkins et al. ¹¹⁾
^{238}U	3650 ± 650	3905	3709	4964	Batty et al. ¹⁰⁾

Table 2 (cont'd)

Nucleus	$\epsilon_0(3d)$ in keV				References
	Experiment	I	II	III	
^{139}Ba	5.10 ± 0.27	5.92	5.92	6.79	Jenkins et al. ¹²⁾
^{140}Ce	6.57 ± 0.29	7.30	7.46	8.13	Jenkins et al. ¹²⁾
^{142}Ce	6.75 ± 0.33	7.34	7.33	8.37	Jenkins et al. ¹²⁾
^{148}Nd	7.18 ± 0.32	8.48	8.29	9.75	Kunselman et al. ¹³⁾
^{150}Nd	7.07 ± 0.22	8.13	7.60	9.61	Kunselman et al. ¹³⁾
^{181}Ta	10.2 ± 1.0	21.22	17.86	26.93	Present result
Re	11.1 ± 1.0	24.31	20.36	31.59	Present result
^{209}Bi	10.6 ± 1.3	34.51	28.19	47.55	Present result
$\Gamma_0(3d)$ in keV					
^{138}Ba	4.3 ± 0.9	5.84	5.82	6.76	Jenkins et al. ¹²⁾
^{140}Ce	5.6 ± 1.0	7.51	7.54	8.50	Jenkins et al. ¹²⁾
^{142}Ce	6.5 ± 0.9	7.63	7.58	8.64	Jenkins et al. ¹²⁾
^{148}Nd	8.8 ± 1.2	9.76	9.55	11.38	Kunselman et al. ¹³⁾
^{150}Nd	9.2 ± 1.1	9.98	9.60	11.80	Kunselman et al. ¹³⁾
^{181}Ta	17.7 ± 3.3	41.89	38.15	49.85	Present result
Re	16.8 ± 2.0	50.16	45.18	60.87	Present result
^{209}Bi	25.4 ± 5.3	90.50	79.37	109.8	Present result

Table 3

Nucleus	$\Gamma_{\text{rad}}(3d)$ (eV)	$\Gamma_{\text{rad}}(4f)$ (eV)	$\Gamma_{\text{rad}}(5g)$ (eV)	$\Gamma_2/\Gamma_0(3d)$	$\Gamma_2/\Gamma_0(4f)$
^{181}Ta	369.9	79.0	24.4	-0.0046	-0.130
^{187}Re	412.0	88.2	27.1	+0.0014	-0.081
^{209}Bi	617.4	131.6	40.6	-0.0024	+0.010

Table 4

$n_i \ell_i \rightarrow n_f \ell_f$	Transition energies in keV		Rel. X-ray intensity, experimental	Remarks
	Calculated	Experimental		
4f $\rightarrow$ 3d	989.01	1002.0 $\pm$ 1.0	31.6 $\pm$ 2.5	Contains ^{172}Yb
5g $\rightarrow$ 4f	453.020	453.55 $\pm$ 0.11	100 $\pm$ 7	
6h $\rightarrow$ 5g	244.55	244.07 $\pm$ 0.13	73.5 $\pm$ 5	
6g $\rightarrow$ 5f	246.52	246.88 $\pm$ 0.13	6.5 $\pm$ 0.7	Contains ^{171}Hf
6g $\rightarrow$ 4f	696.330	696.89 $\pm$ 0.23	15.2 $\pm$ 1.5	
7h $\rightarrow$ 5g	390.89	390.53 $\pm$ 0.11	14.7 $\pm$ 1.5	Contains ^{27}Al
7g $\rightarrow$ 5f	393.29	393.37 $\pm$ 0.12	2.0 $\pm$ 0.4	
7g $\rightarrow$ 4f	843.11	843.43 $\pm$ 0.42	6.9 $\pm$ 1.1	Contains ^{174}Hf
8i $\rightarrow$ 6h	241.68	241.64 $\pm$ 0.13	5.8 $\pm$ 0.6	
8h $\rightarrow$ 5g	485.83	485.52 $\pm$ 0.90	3.0 $\pm$ 0.5	
8g $\rightarrow$ 5f	488.49	489.44 $\pm$ 0.50	1.0 $\pm$ 0.5	Contains ^{173}Hf
9i $\rightarrow$ 6h	306.61	306.39 $\pm$ 0.16	11.1 $\pm$ 1.1	
9h $\rightarrow$ 5g	550.87	550.35 $\pm$ 0.24	2.3 $\pm$ 0.4	Contains ^{173}Hf
10i $\rightarrow$ 6h	352.96	352.79 $\pm$ 0.32	4.2 $\pm$ 0.6	

Table 5

$n_i \ell_i \rightarrow n_f \ell_f$	Transition energies in keV		Rel. X-ray intensity, experimental	Remarks
	Calculated	Experimental		
4f $\rightarrow$ 3d	1282.47	1301.3 $\pm$ 1.3	10.4 $\pm$ 0.9	Contains also ^{208}Pb and ^{127}I
5g $\rightarrow$ 4f	588.013	589.68 $\pm$ 0.10	100 $\pm$ 5	
6h $\rightarrow$ 5g	316.96	317.10 $\pm$ 0.90	92.7 $\pm$ 5	
6g $\rightarrow$ 5f	320.21	320.1 $\pm$ 1.0	1.3 $\pm$ 0.6	
6g $\rightarrow$ 4f	902.965	904.92 $\pm$ 0.11	8.7 $\pm$ 1.2	
7i $\rightarrow$ 6h	190.26	190.40 $\pm$ 0.15	50 $\pm$ 8	
7h $\rightarrow$ 6g	191.35			
7h $\rightarrow$ 5g	506.27	506.25 $\pm$ 0.53	14.3 $\pm$ 1.6	
7g $\rightarrow$ 6f	193.12	194.55 $\pm$ 0.69	0.6 $\pm$ 0.3	
7g $\rightarrow$ 4f	1092.99	1094.52 $\pm$ 0.30	7.1 $\pm$ 0.8	
8i $\rightarrow$ 6h	312.83	312.96 $\pm$ 0.18	9.9 $\pm$ 1.0	
8h $\rightarrow$ 6g	314.22	314.46 $\pm$ 0.42	3.2 $\pm$ 0.6	
9j $\rightarrow$ 8i	206.94	207.15 $\pm$ 0.16	9.5 $\pm$ 1.0	
9i $\rightarrow$ 7h	207.52			
9h $\rightarrow$ 7g	208.34	396.88 $\pm$ 0.25	4.1 $\pm$ 0.8	
9i $\rightarrow$ 6h	396.84			
9h $\rightarrow$ 5g	713.33	712.94 $\pm$ 0.32	1.9 $\pm$ 0.6	
10k $\rightarrow$ 8j	144.01	144.05 $\pm$ 0.14	8.1 $\pm$ 1.6	
10j $\rightarrow$ 8i	144.26			
10i $\rightarrow$ 8h	144.63	267.49 $\pm$ 0.59	3.8 $\pm$ 0.5	
10j $\rightarrow$ 7i	266.85			
10i $\rightarrow$ 7h	267.50	456.47 $\pm$ 0.37	2.5 $\pm$ 1.1	
10i $\rightarrow$ 6h	456.82			

Table 6

Nucleus	Y(5g)	Y(4f)	$\Gamma_0(5g)$		$\Gamma_0(4f)$		
			Rel. intens. (eV)	Theory Set I (eV)	Rel. intens. (eV)	Line shape (eV)	Theory Set I (eV)
^{181}Ta	1.05 ± 0.11	0.255 ± 0.028	1.3 ± 2.7	0.7	231 ± 34	225 ± 57	322
Re	0.96 ± 0.08	0.159 ± 0.012	1.1 ± 2.3	1.0	466 ± 41	547 ± 50	428
^{209}Bi	0.87 ± 0.07	0.089 ± 0.009	6.0 ± 3.7	3.7	1343 ± 155	1157 ± 70	1216

Table 7

	ξ	b_0 m_π^{-1}	b_1 m_π^{-1}	$\text{Re } B_0$ m_π^{-4}	$\text{Im } B_0$ m_π^{-4}	c_0 m_π^{-3}	c_1 m_π^{-3}	$\text{Re } C_0$ m_π^{-6}	$\text{Im } C_0$ m_π^{-6}
Set I a)	1	-0.0293	-0.078	0	0.0428	0.227	0.18	0	0.076
Set II b)	1	-0.017	-0.13	-0.0475	0.0475	0.255	0.17	0	0.090
Set III c)	0	-0.0293	-0.078	0	0.0428	0.172	0.22	0	0.036
Nucleus									
		c (fm)	t (fm)	β					
	^{181}Ta	6.347	2.15	0.28					
	Re	6.40	2.15	0.22					
	^{209}Bi	6.687	2.30	-0.024					

- a) Tauscher¹⁵⁾
b) Batty et al.¹⁶⁾
c) Tauscher¹⁵⁾

Table 8

Nucleus	Set	4f level				3d level			
		Γ_2 (keV)	ϵ_2 (keV)	A_2 (keV)	A_2^{eff} (keV)	Γ_2 (keV)	ϵ_2 (keV)	A_2 (keV)	A_2^{eff} (keV)
^{181}Ta	I	-0.042	-0.034	1.652	1.686	-0.19	3.81	9.93	6.12
	II	-0.038	-0.029	1.654	1.683	+0.51	4.47	9.73	5.26
	III	-0.044	-0.039	1.654	1.693	+0.50	4.09	10.21	6.12
	Experiment				1.750 ± 0.016 a)				9.11 ± 0.51
Re	I	-0.032	-0.026	1.117	1.143	0.09	2.77	6.68	3.91
	II	-0.028	-0.022	1.117	1.139	0.50	3.20	6.54	3.34
	III	-0.033	-0.029	1.118	1.147	0.76	3.00	6.91	3.91
	Experiment				1.163 ± 0.010 b)				5.39 ± 0.63 b)

a) Beetz et al.⁴⁾

b) Konijn et al.⁶⁾

Figure captions

- Fig. 1 : Part of the prompt gamma-ray spectrum from pions stopped in a ^{181}Ta target, showing the 4f-3d hf complex (complete spectrum is shown in Fig. 3 of Ref. 3).
- Fig. 2 : Part of the prompt gamma-ray spectrum from pions stopped in a natural Re target, showing the 4f-3d hf complex (complete spectrum is shown in Ref. 6).
- Fig. 3 : Part of the prompt gamma-ray spectrum from pions stopped in a ^{209}Bi target, showing the 4f-3d transition (full spectrum is shown in Fig. 6 of Ref. 3).
- Fig. 4 : Experimental shift ϵ_0 of the 3d level with respect to the point nuclear value, corrected for vacuum polarization, electron screening, and Lamb shift as a function of Z.
- Fig. 5 : Experimental strong interaction width Γ_0 of the 3d level as a function of Z.
- Fig. 6 : Ratio of theoretical and experimental level shifts of the 3d level ϵ_0 as a function of Z (closed dots: Set I; open dots: Set III).
- Fig. 7 : Ratio of theoretical and experimental widths of the 3d level Γ_0 as a function of Z. Labelling as in Fig. 6.
- Fig. 8 : Ratio of ϵ_0/Γ_0 obtained from experiment (closed dots) and theory (open dots: Set I; crosses: Set III).

Fig. 1

Fig. 2

Fig. 3

Fig. 4

Fig. 5

Fig. 6

Fig. 7

Fig. 8