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COVARIANT QUANTIZATION OF ANTISYMMETRIC TENSOR GAUGE FIELDS

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ABSTRACT

Covariant quantization of the antisymmetric tensor field $B_{\mu\nu}$ with gauge transformation $\delta B_{\mu\nu} = \delta_\mu \xi_\nu - \delta_\nu \xi_\mu$ reveals two unusual features. Firstly, one mode in the covariant $B_{\mu\nu}$ propagator is not cancelled by a Faddeev-Popov ghost but instead decouples by itself as a consequence of Ward identities. This is analogous to the spin 3/2 field ψ_μ . Secondly, the Faddeev-Popov ghost Lagrangian is itself gauge invariant and requires further commuting ghosts for its covariant quantization. These features are exhibited by a diagrammatic analysis of unitarity.

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1. INTRODUCTION: MODES IN THE PROPAGATOR

The Lagrangian

$$\mathcal{L} = \frac{1}{2} B_{\mu\nu} \square B_{\mu\nu} + (\partial_\mu B_{\mu\nu})(\partial_\nu B_{\mu\nu}) \quad (1.1)$$

is invariant under the gauge transformation

$$\delta B_{\mu\nu} = \partial_\mu \xi_\nu - \partial_\nu \xi_\mu \quad (1.2)$$

for the antisymmetric tensor field $B_{\mu\nu}$. This is a gauge theory describing a spinless particle¹⁾. Considering that a spinless particle can be described very simply by means of a single scalar field, there might seem to be no point in considering more complicated fields. However, antisymmetric tensor fields appear naturally as auxiliary fields in theories of extended supergravity^{2),3)} and in locally superconformal theories they become gauge fields subject to the transformation (1.2)³⁾. Moreover, antisymmetric tensor gauge fields have found their way into the recent literature for a number of other reasons⁴⁾⁻⁶⁾ and Freedman has provided a non-Abelian generalization of the Lagrangian (1.1)⁷⁾.

The motivation for this paper, however, is simply that the Lagrangian (1.1) provides a model gauge theory with some interesting and unusual features connected with the counting of degrees of freedom in the quantum theory. Before discussing these it is instructive to make a space/time decomposition of the Lagrangian and to construct the Hamiltonian. The result is

$$H = \frac{1}{2} \pi_{ij}^2 + \frac{1}{4} [\epsilon^{ijk} \partial_i B_{jk}]^2 + 2 B_{0k} (\partial_j \pi_{jk}) \quad (1.3)$$

π_{0k} , the momentum conjugate to B_{0k} is identically zero and B_{0k} itself is a Lagrange multiplier enforcing the constraint $\partial_j \pi_{jk} = 0$. This can be solved for two of the three components of π_{jk} leaving one independent canonical momentum field. Since the constraint $\partial_j \pi_{jk}$ also generates the gauge transformation of B_{ij} under which H is invariant, two of the three components of B_{ij} can also be eliminated. This leaves one pair of conjugate fields which can only describe a single spinless particle.

But suppose that we want to quantize the theory covariantly with the apparatus of Faddeev-Popov ghosts etc. Then it is not so easy to see how it is that there is still only one physical degree of freedom. Since the gauge transformation of

(1.2) involves only the transverse part of ξ_ν we can introduce transverse ghost and anti-ghost fields, C_μ^T, C_μ^{T*} . Then, choosing the gauge fixing term to be $-(\partial_\mu B_{\mu\nu})^2$, we obtain the effective quantum Lagrangian

$$\mathcal{L}_{\text{eff.}} = \frac{1}{2} B_{\mu\nu} \square B_{\mu\nu} + C_\mu^{T*} \square C_\mu^T \quad (1.4)$$

Now the propagator for $B_{\mu\nu}$ is

$$p^2 \overline{B_{\mu\nu} B_{\rho\sigma}} = \frac{1}{2} (\delta_{\mu\rho} \delta_{\nu\sigma} - \delta_{\nu\rho} \delta_{\mu\sigma}) \quad (1.5)$$

which propagates six modes (i.e., all components of $B_{\mu\nu}$). The ghost field C_μ^T has three components so that at first sight C_μ^T and C_μ^{T*} would appear to subtract all six modes leaving no physical degrees of freedom. However, the ghost Lagrangian is equivalent to a complex Maxwell Lagrangian in the Lorentz gauge, and we know that a photon has only two physical degrees of freedom. So we would expect that C_μ^T and C_μ^{T*} actually cancel only four of the six modes of $B_{\mu\nu}$. This still leaves two instead of one physical degree of freedom.

To see what happens to these two modes we can decompose the propagator of (1.5) into its various channels by means of the transverse δ function $\bar{\delta}_{\mu\nu}$ defined by

$$\begin{aligned} \bar{\delta}_{\mu\nu} &= \delta_{\mu\nu} - (p_\mu \bar{p}_\nu + p_\nu \bar{p}_\mu) / (p \cdot \bar{p}) \\ &= \epsilon_\mu^+ \epsilon_\nu^{+*} + \epsilon_\mu^- \epsilon_\nu^{-*} \end{aligned} \quad (1.6)$$

where

$$p_\mu = (\vec{p}, p_4) ; \quad \bar{p}_\mu = (\vec{p}, -p_4) \quad (1.7)$$

and ϵ_μ^+ and ϵ_μ^- are the usual spin 1 circular polarization vectors. The physical propagator for $B_{\mu\nu}$ is obtained from (1.5) by the replacement $\delta_{\mu\nu} \rightarrow \bar{\delta}_{\mu\nu}$

$$\begin{aligned} p^2 \overline{B_{\mu\nu} B_{\rho\sigma}}(\text{phys.}) &= \frac{1}{2} (\bar{\delta}_{\mu\rho} \bar{\delta}_{\nu\sigma} - \bar{\delta}_{\nu\rho} \bar{\delta}_{\mu\sigma}) \\ &= \frac{1}{2} (\epsilon_\mu^+ \epsilon_\nu^- - \epsilon_\nu^+ \epsilon_\mu^-) (\epsilon_\rho^{+*} \epsilon_\sigma^{-*} - \epsilon_\sigma^{+*} \epsilon_\rho^{-*}) \end{aligned} \quad (1.8)$$

which evidently describes the propagation of a spin zero particle. This is exactly analogous to the treatment of the usual spin 1,2⁸⁾ and 3/2⁹⁾ fields. Now we can write (1.8) as follows

$$\begin{aligned} \frac{1}{2}(\bar{\delta}_{\mu\rho}\bar{\delta}_{\nu\sigma} - \bar{\delta}_{\nu\rho}\bar{\delta}_{\mu\sigma}) &= \frac{1}{2}(\delta_{\mu\rho}\delta_{\nu\sigma} - \delta_{\nu\rho}\delta_{\mu\sigma}) \\ - \left[\frac{1}{2} \frac{(\bar{\delta}_{\mu\rho}\bar{p}_\sigma - \bar{\delta}_{\mu\sigma}\bar{p}_\rho)}{(q \cdot \bar{p})} p_\nu + 3 \text{ terms} \right] \\ - \left[\frac{1}{2} (p_\mu p_\sigma \bar{p}_\rho \bar{p}_\nu) / (q \cdot \bar{p})^2 + 3 \text{ terms} \right] \end{aligned} \quad (1.9)$$

This shows how the various modes in the physical propagator can be decomposed into the usual covariant propagator plus other modes which must either be cancelled by the ghosts C_μ^T, C_μ^{T*} or decouple separately. Taking the trace of Eq. (1.9) one finds the equation

$$1 = 6 - 4 - 1 \quad (1.10)$$

one can already see, from this, that four of the six modes will be cancelled by the ghosts while one further mode must decouple separately. In the following section we will demonstrate that this is indeed what happens.

2. UNITARITY

We will need various Ward identities of the theory which can be derived most easily by considering the BRS transformations¹⁰⁾ of the effective Lagrangian

$$\begin{aligned} \delta B_{\mu\nu} &= (\partial_\mu C_\nu^T - \partial_\nu C_\mu^T) \Lambda \\ \delta C_\nu^{*T} &= -2(\partial_\mu B_{\mu\nu}) \Lambda \\ \delta C_\nu^T &= 0 \end{aligned} \quad (2.1)$$

Λ is the anticommuting BRS constant. These transformations leave the effective action invariant; hence by taking the BRS variation of the equation $\langle C_\nu^{*T}, B_{\alpha\beta} \rangle = 0$ one finds the identity

$$2\langle \partial_\mu B_{\mu\nu}, B_{\alpha\beta} \rangle = \langle C_\nu^{*T}, \partial_\alpha C_\beta^T - \partial_\beta C_\alpha^T \rangle \quad (2.2)$$

On the left-hand-side of (2.2) we take the incoming $B_{\mu\nu}$ to attach to a vertex $\Gamma_{\rho\sigma}$ and on the right-hand-side we take the incoming C_V^{T*} to attach to a ghost vertex Γ_σ^T . We get, therefore, for the momentum space version of (2.2), the equation

$$2 p_\mu \left(\frac{\delta_{\mu\sigma} \delta_{\nu\sigma}}{p^2} \right) \langle \Gamma_{\rho\sigma}, B_{\alpha\beta} \rangle = \frac{\delta_{\nu\sigma}}{p^2} \langle \Gamma_\sigma^T, -p_\alpha C_\beta^T + p_\beta C_\alpha^T \rangle \quad (2.3)$$

To get the right-hand-side, we have used $\delta_{\nu\sigma}^T/p^2$ as the ghost propagator since this is the inverse of $\delta_{\nu\sigma}^T p^2$ in the space of transverse functions, and ∂_μ is replaced by $-p_\mu$ for an outgoing ghost. But since Γ_σ^T is transverse (C_μ^T can only couple to a transverse quantity) we may replace $\delta_{\nu\sigma}^T$ by $\delta_{\nu\sigma}$. Thus

$$\frac{\delta_{\nu\sigma}}{p^2} \langle (p \cdot \Gamma)_\sigma, B_{\alpha\beta} \rangle = \frac{\delta_{\nu\sigma}}{p^2} \langle \Gamma_\sigma^T, \frac{-p_\alpha C_\beta^T + p_\beta C_\alpha^T}{2} \rangle \quad (2.4)$$

Effectively, $\partial_\mu B_{\mu\nu}$ represents a longitudinal gauge line whose propagator has been replaced by a ghost propagator. Amputating this propagator on the incoming lines gives the identity shown in Fig. 1. Similarly, by considering an equation of the form $\langle C_\rho^{T*} \partial_\mu B_{\mu\nu} \rangle = 0$ and taking its BRS variation we can derive

$$2 \langle \partial_\sigma B_{\sigma\rho}, \partial_\mu B_{\mu\nu} \rangle = \langle C_\rho^{T*} \square C_\nu^T \rangle = 0 \text{ (on shell)} \quad (2.5)$$

since $\square C_\nu^T = 0$ on shell. This represents the decoupling of longitudinally polarized gauge lines and is shown in Fig. 2. The identities obtained in this way, or slight generalizations of them, can now be used to prove unitarity by the method of 't Hooft⁸⁾. We will consider only two particle unitarity and this only briefly, since more details of this method can be found elsewhere^{8),9)}.

Now consider the diagram of Fig. 3(a) with two cut gauge lines with physical residues. The decomposition of the physical residue given in (1.9) is represented diagrammatically in Fig. 4. Making this decomposition on the upper cut line of Fig. 3(a) one derives immediately Fig. 3(b) by using Fig. 1. Now make the same decomposition on the lower cut line to get Fig. 3(c). Applying Fig. 1 again we get Fig. 3(d). Now applying Fig. 1 to the second and third diagrams of Fig. 3(d) we get a factor of the form

$$-\frac{(\bar{\delta}_{\mu\sigma} \bar{p}_\sigma - \bar{\delta}_{\mu\sigma} \bar{p}_\sigma) \times p_\sigma}{(p \cdot \bar{p})} = -\bar{\delta}_{\mu\sigma} \quad (2.6)$$

where μ and ρ are the indices associated with the points to which the lower cut ghost line attaches. But on shell $\bar{\delta}_{\mu\nu} = \delta_{\mu\nu}^T$, which is the residue of the ghost propagator, so that we get the closed ghost loops of Fig. 3(e) with the relative minus as shown. Finally, we apply Fig. 2 to the last diagram of Fig. 3(d) which shows that it vanishes. Thus we derive Fig. 3(e) which establishes two-particle unitarity. Actually, one needs a generalization of the Ward identities of Figs. 1 and 2 to include cut diagrams etc., for a proper proof of unitarity. It is not our purpose here to give such a proof because it would not differ essentially from that given for Yang-mills theories originally by 't Hooft⁸⁾. It is our purpose to show what happens to the six degrees of freedom in the $B_{\mu\nu}$ propagator. The above analysis shows that four are cancelled by ghosts while one decouples by itself leaving one physical degree of freedom. This is very similar to the spin 3/2 case and we will comment on this point in the conclusions.

Of course the Lagrangian (1.1) describes a free field theory so that all modes decouple from each other and the S matrix is trivial. But interactions can be added to (1.1) easily, either interactions with other fields¹⁾, or in the non-Abelian case, self-interactions⁷⁾. These interactions have been omitted because they are inessential for the above analysis. Now we turn to a second unusual feature of antisymmetric tensor gauge fields.

3. GHOSTS FOR GHOSTS

The analysis of the previous section is not completely in the spirit of covariant quantization because the ghost field is restricted to be transverse. Also it is not so easy to see why the ghost Lagrangian only cancels four rather than six of the degrees of freedom of $B_{\mu\nu}$. This problem is easily overcome by writing (1.4) as;

$$\mathcal{L}_{\text{eff.}} = \frac{1}{2} B_{\mu\nu} \square B_{\mu\nu} + C_{\mu}^* (\square \delta_{\mu\nu} - \partial_{\mu} \partial_{\nu}) C_{\nu} \quad (3.1)$$

Only the transverse part of C_{μ} and C_{μ}^* actually appear in this Lagrangian because of the gauge invariances

$$\delta C_{\mu} = \partial_{\mu} \alpha, \quad \delta C_{\mu}^* = \partial_{\mu} \beta \quad (3.2)$$

where α, β are anticommuting scalar functions. According to the usual procedure we fix these gauges by adding

$$-(\partial \cdot C^*)(\partial \cdot C) = \mathcal{L}_{\text{fix}}(\text{ghost}) \quad (3.3)$$

to $\mathcal{L}_{\text{eff}}$. We must then add a corresponding ghost Lagrangian

$$\mathcal{L}_{\text{ghost}}(\text{ghost}) = \xi^* \square \xi + \phi^* \square \phi \quad (3.4)$$

where ξ and ϕ are two complex commuting scalar fields. The total effective Lagrangian is therefore

$$\mathcal{L}_{\text{eff. (tot.)}} = \frac{1}{2} B_{\mu\nu} \square B_{\mu\nu} + C_\mu^* \square C_\mu + \xi^* \square \xi + \phi^* \square \phi \quad (3.5)$$

and this Lagrangian is invariant under the BRS transformations

$$\begin{aligned} \delta B_{\mu\nu} &= (\partial_\mu C_\nu - \partial_\nu C_\mu) \Lambda \\ \delta C_\nu^* &= (-2 \partial_\mu B_{\mu\nu} + \partial_\nu \xi) \Lambda \\ \delta C_\nu &= (\partial_\nu \phi) \Lambda \\ \delta \phi^* &= (\partial_\nu C^*) \Lambda \quad ; \quad \delta \xi^* = (-\partial_\nu C) \Lambda \\ \delta \phi &= \delta \xi = 0 \end{aligned} \quad (3.6)$$

It is now easy to see that ϕ, ϕ^*, ξ, ξ^* will cancel four of the eight degrees of freedom in C_μ, C_μ^* leaving only four. But it is no longer so easy to see which mode decouples altogether because the analogue of Fig. 2 no longer holds. In fact, by considering the BRS variation of $\langle C_\sigma^*, \partial_\mu B_{\mu\nu} - \partial_\nu \xi^* \rangle = 0$ one obtains

$$-2 \langle \partial_\sigma B_{\sigma\sigma}, \partial_\mu B_{\mu\nu} \rangle + \langle C_\sigma^* \square C_\nu \rangle - \langle \partial_\sigma \xi, \partial_\nu \xi^* \rangle = 0 \quad (3.7)$$

Now $\square C_\nu = 0$ is on shell so we find the on shell Ward identity

$$2 \langle \partial_\sigma B_{\sigma\sigma}, \partial_\mu B_{\mu\nu} \rangle + \langle \partial_\sigma \xi, \partial_\nu \xi^* \rangle = 0 \text{ (on shell)} \quad (3.8)$$

so that longitudinal gauge lines no longer decouple. Nevertheless, it is still possible to demonstrate that the theory is unitary. The analysis is straightforward and we will not reproduce it here as it differs little from the analysis of the previous section.

4. COMMENTS

The decoupling of modes in the covariant $B_{\mu\nu}$ propagator is analogous to the decoupling of a Majorana spin $\frac{1}{2}$ mode in the covariant spin 3/2 propagator. The spin 3/2 field ψ_μ has 16 components, the complex spin $\frac{1}{2}$ supersymmetry ghost has eight components. This leaves $(16 - 8)/2 = 4$ degrees of freedom (we divide by

two because half the components of spinor fields are the conjugate momenta). We need two degrees of freedom for a massless spin 3/2 field so that two degrees of freedom are superfluous. Decomposition of the spin 3/2 propagator reveals that there are two additional spin $\frac{1}{2}$ modes that cannot be cancelled by the Faddeev-Popov ghosts, but fortunately they decouple by themselves as a consequence of Ward identities of supersymmetry⁹⁾. There is, however, one important difference between the spin 3/2 field and the antisymmetric tensor field. The gauge fixing term for supersymmetry is usually taken to be

$$\frac{1}{4} \bar{\psi} \cdot \gamma \phi \gamma \cdot \psi \quad (4.1)$$

This can be obtained from the path integral expression

$$Z = \int [d\psi_\mu] \delta(\gamma \cdot \psi - f) e^{\mathcal{L}} (\det \phi)^{-1} \quad (4.2)$$

by means of 't Hooft's exponentiation trick. One finds

$$Z = \int [d\psi_\mu] e^{\left\{ \mathcal{L} - \frac{1}{4} \bar{\psi} \cdot \gamma \phi \gamma \cdot \psi + \bar{c}^* \phi c \right\}} (\det \phi)^{-\frac{1}{2}} \quad (4.3)$$

where the extra factor of $(\det \phi)^{-\frac{1}{2}}$ comes about because

$$A \int [df] e^{\left\{ \frac{1}{4} \bar{f} \phi f \right\}} (\det \phi)^{\frac{1}{2}} = 1 \quad (4.4)$$

for some normalization constant A. This factor of $(\det \phi)^{-\frac{1}{2}}$ is customarily dropped on the grounds that it is a constant but Nielsen has noted that it, too, can be exponentiated by means of a Majorana spin $\frac{1}{2}$ ghost¹¹⁾. Thus we obtain an effective Lagrangian

$$\mathcal{L}_{\text{eff}} = \mathcal{L} - \frac{1}{4} \bar{\psi} \cdot \gamma \phi \gamma \cdot \psi + \bar{c}^* \phi c + \hat{N} \phi N \quad (4.5)$$

N is the "Nielsen ghost" (the caret indicates Majorana conjugation). This ghost serves to cancel the two additional spin $\frac{1}{2}$ modes in the ψ_μ propagator. In fact, for the gauge choice (3.1) both the extra spin $\frac{1}{2}$ mode and the Nielsen ghost decouple independently so that an actual cancellation between them is not apparent, and the additional ghost N is not necessary. But although extra modes in the propagator may decouple, they will still contribute to the partition function at finite temperature, so extra ghosts would appear to be necessary in this case¹¹⁾.

By analogy one would expect to have an additional real scalar ghost for the covariant quantization of the $B_{\mu\nu}$ field, but the analogous mechanism is absent. This remains a puzzling discrepancy which the author has not yet been able to resolve.

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FIGURE CAPTIONS

- Fig. 1 : Diagrammatic representation of Eq. (2.2). The solid line is a gauge line, while the dashed line is a ghost line. The slash indicates that the propagator on that line has been amputated.
- Fig. 2 : Diagrammatic representation of Eq. (2.5), shows the decoupling of longitudinal gauge lines.
- Fig. 3 : Proof of two particle unitarity. All external lines are on shell and have physical polarizations.
- Fig. 4 : Diagrammatic representation of Eq. (1.9).

FIG. 1

FIG. 2

(a)

(b)

(c)

FIG. 3

FIG. 3

FIG. 4