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FIELD THEORETIC BAGS: SOME FORMAL CONSIDERATIONS

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ABSTRACT

We consider some formal aspects of localized quasi-classical solutions ("bags") in a field theory involving a "quark" field ψ and a scalar ("confining") field ϕ , where the latter has a quartic self-coupling and is coupled strongly to the quark field via a Yukawa coupling $f\bar{\psi}\psi\phi$. Renormalization - already coming in at the quasi-classical level through the quark "sea" contribution - and translation invariance are discussed in some detail.

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1. INTRODUCTION

The idea that hadrons can possibly be described in terms of quarks (and gluons) confined in a localized structure called "bag" has found considerable interest during the last years¹⁾⁻⁵⁾. The phenomenological aspects of this picture have been widely explored⁶⁾, most extensively for the MIT version²⁾ of the model. While various corrections - quark masses, gluon effects etc. - have been incorporated⁷⁾ in order to describe the actual splittings of the hadron multiplets, there are still some more basic aspects e.g., translation invariance, existence of a parton distribution⁸⁾ etc., which are not so well explored. A suitable framework for such more formal considerations is a quantum field theory which is susceptible of displaying bag-like structures. A model often considered^{1),3),9)-12)} is that of a spin $\frac{1}{2}$ field interacting via Yukawa coupling with a scalar field with quartic self-interaction. Depending on the special choice of the quartic coupling one can interpolate¹²⁾ between the MIT²⁾ and SLAC³⁾ versions of the bag model. Since this model is, so to say, a standard reference model we will discuss our formal developments within this model.

A systematic approach to this model field theory in connection with quasi-classical methods has been given by Friedberg and Lee¹¹⁾, however, only for the limit of a weak $\bar{\psi}\psi\phi$ coupling constant f . This restriction was imposed by the aim of these authors to make estimates of the accuracy of the quasi-classical energy which is not known in the strong coupling limit. On the other hand, in the application to bag dynamics we are faced with a strong coupling situation. Indeed, the term $f\bar{\psi}\psi\phi$ has to compensate inside the bag the large (outside) quark mass term $M\bar{\psi}\psi$. Therefore, the product $|f\sigma(x)|$, where σ is the classical expectation value of ϕ , has to become large in the bag region. We are going to consider a situation where f and σ (x inside bag) are of comparable magnitude, which we call $1/g$. This is motivated by the appearance of terms like $(\nabla\sigma)^2$ in the classical energy which is of order $1/g^2$. This assignment may not correspond to the actual situation; indeed, the limit usually considered^{8),12)} is that σ is also small in the bag region so that f has to be extremely large. We have not been able to formalize such a situation in a satisfactory way since in this case σ becomes of the order of quantum fluctuations and the usual quasi-classical methods become doubtful. Indeed the authors of Ref. 12) consider σ as a phenomenological field representing quantum effects introduced by other fields. This makes, of course, our more naive kind of formulation impossible. Nevertheless, we think that the aspects of a strong coupling situation considered here which is intermediary

between a weak coupling situation and the actual limit intended deserves a study, especially because of the two following aspects:

i) Since the classical equation of motion for the field ϕ cannot per se lead to a stable localized classical solution¹²⁾ the fermions have to come in already at the quasi-classical level. In the weak coupling limit it is consistent to consider only the valence quarks in these equations, since the quark sea enters the dynamics only in a higher order of the coupling constant. In the case of strong coupling - and in the absence of an explicit solution which would allow for more detailed estimates - we have no reason to neglect the influence of the quark sea on the quasi-classical dynamics. The consideration of these contributions leads to the necessity of considering renormalization counter terms already at the semi-classical level. The quark sea has been taken into account previously by Huang and Stump¹⁰⁾, however, the general structure of the renormalization is not transparent there, since these authors specialize their consideration to a square well potential.

ii) There is another formal aspect we consider here in more detail than has been done previously; that is the treatment of translation invariance. The quasi-classical equation leads, of course, to the appearance of zero modes in the stability equation of the scalar field corresponding to the translation invariance of the problem. We use a method developed in Refs. 13) and 14) to make use of these modes in order to construct perturbatively momentum eigenstates. The method is related to the collective coordinate method¹⁵⁾ but is more appropriate for a description in terms of quantum states.

The plan of the paper is as follows: in Section 2 we describe the classification of different terms in the Hamiltonian with respect to order of magnitude parametrized by powers of a perturbation parameter λ . We also determine the leading part of the renormalization counter terms from a consideration of the vacuum sector. In Section 3 we consider the theory in the bag sector; we discuss the semi-classical equations and the connected stability equation, leading to a formulation of the quantization procedure up to order λ^0 (the semi-classical Hamiltonian is $O(\lambda^{-2})!$). In Section 4 we discuss the translation mode and the construction of approximate momentum eigenstates. After a short outlook to higher orders in Section 5 we summarize our results in Section 6.

2. SET-UP OF THE HAMILTONIAN

As motivated in the Introduction we consider a field theory with the Hamiltonian density

$$\mathcal{H} = \psi^\dagger (-i \vec{\alpha} \vec{\partial} + \beta M) \psi + \frac{1}{2} \Pi^2 + \frac{1}{2} (\vec{\nabla} \phi)^2 + \frac{1}{g^2} U(g\phi) + f \bar{\psi} \psi \phi + \Delta \mathcal{H}_B + \Delta \mathcal{H}_F \quad (2.1a)$$

where $U(\phi)$ is a quartic polynomial

$$U(\phi) = \sum_{n=0}^4 C_n \phi^n / n! \quad (2.1b)$$

and $\Delta_B \mathcal{H}(x)$ contains the counter-terms

$$\Delta \mathcal{H}_B = \sum_{n=1}^4 \tilde{C}_n \phi^n / n! + \frac{1}{2} D (\vec{\nabla} \phi)^2 + \frac{1}{2} \left(\frac{1}{1+D} - 1 \right) \Pi^2 \quad (2.1c)$$

The relation of the $(\vec{\nabla} \phi)^2$ and Π^2 counter terms imposed by Lorentz invariance. $\Delta \mathcal{H}_F$ contains the counter terms for the fermion field and the coupling term, which do not yet become necessary at the quasi-classical level. We assume that this theory develops bag sectors characterized by a net number of excitations of the field ψ (corresponding to naive quark model expectations) in a potential given by $f\sigma(x)$ where $\sigma(x)$ is a classical expectation value of the field ϕ . Unlike the case of $1+1$ dimensional field theories with solitons, in $3+1$ dimensions the classical equation of motion for the field ϕ cannot have stable localized solutions¹⁶⁾, so the fermions have to come in already at the quasi-classical level in order to make such solutions possible. In the absence of an estimate of the realistic values for the different terms in $\mathcal{H}$ we take the usual classification for the semi-classical methods as a guideline. This has been implied already in the formulation of $U(\phi)$. For the fermion we have to choose M and $f\phi$ of order $1/g^2$ in order that these terms come in at the classical level $1/g^2$.

We see then no other way to begin a perturbation scheme than by taking the fermion kinetic energy also into account at the same level.

Since it does not contain a factor g^{-2} we introduce another parameter λ which characterizes the order in which the different parts of the Hamiltonian are taken into account. λ is the formal expansion parameter of usual perturbation theory and has to be set equal to 1 at the end (the convergence being assured - if at all - by the smallness of the terms it multiplies). For the ordering of all the other terms we let ourselves be guided by the usual coupling constant g of the (boson) loop expansion. We have then

$$H = \sum_{n=-2}^2 \lambda^n H_n + \Delta H_B + \Delta H_F \quad (2.2a)$$

where in terms of Hamiltonian densities

$$\begin{aligned} \mathcal{H}_{-2}(\vec{x}) &= U(g\phi_0)/g^2 + \psi^\dagger(-i\vec{\alpha}\vec{\partial} + \beta(M + f\phi_0))\psi(\vec{x}) \\ \mathcal{H}_{-1}(\vec{x}) &= [U'(g\phi_0)/g + f\psi^\dagger\beta\psi(\vec{x})]\tilde{\phi}(\vec{x}) \\ \mathcal{H}_0(\vec{x}) &= \frac{1}{2}\pi^2(\vec{x}) + \frac{1}{2}(\vec{\partial}\phi(\vec{x}))^2 + \frac{1}{2}U''(g\phi_0)\tilde{\phi}^2(\vec{x}) \\ \mathcal{H}_1(\vec{x}) &= U'''(g\phi_0)g\tilde{\phi}^3(\vec{x})/3! \\ \mathcal{H}_2(\vec{x}) &= U^{IV}(g\phi_0)g^2\tilde{\phi}^4(\vec{x})/4! \end{aligned} \quad (2.2b)$$

Here ϕ_0 is the vacuum expectation value of the scalar field ϕ and $\tilde{\phi}$ its quantum part. We assume that ϕ_0 is constant in space and time (an alternative possibility is discussed in Ref. 10)).

We now discuss the diagonalization of H up to order λ^0 in the vacuum sector and the choice of the base vacuum implied by this definition. At the same time we will have to fix our counter terms in lowest order.

Since ϕ_0 is (assumed to be) constant, H_{-2} is diagonalized by expanding ψ in terms of plane wave quanta corresponding to fermions and anti-fermions of mass $M + f\phi_0$. Denoting by $\overset{\circ}{b}(\vec{p})$ and $\overset{\circ}{d}(\vec{p})$ the corresponding annihilation operators we require the bare vacuum state $|0\rangle_0$ to be annihilated as

$$\dot{b}(\vec{p})|0\rangle = \dot{a}(\vec{p})|0\rangle = 0 \quad (2.3)$$

The index 0 refers to an expansion of the vacuum state $|0\rangle$ as

$$|0\rangle = \sum_{n=0}^{\infty} \lambda^n |0\rangle_n \quad (2.4)$$

The vacuum energy density in order λ^{-2} becomes now a functional of ϕ via

$$\begin{aligned} \mathcal{E}_{-2}(\phi_0) &\equiv \langle 0 | \mathcal{H}_{-2}(\phi_0) + \Delta \mathcal{H}_B(\phi_0) | 0 \rangle \\ &= -2 \int \frac{d^3 p}{(2\pi)^3} \sqrt{(M + f\phi_0)^2 + \vec{p}^2} + \frac{1}{g^2} U(g\phi_0) + \sum_{n=0}^4 \frac{\tilde{C}_n}{n!} \phi_0^n \end{aligned} \quad (2.5)$$

In order to make minimization of $\mathcal{E}_{-2}(\phi_0)$ a well-defined operation we have to choose our counter terms in such a way that $\mathcal{E}_{-2}(\phi_0)$ becomes finite. Since the first four derivatives of the fermion sea energy with respect to M diverge this implies the choice

$$\tilde{C}_n = f^n \left\{ \left(\frac{\partial}{\partial M} \right)^n \int \frac{d^3 p}{(2\pi)^3} 2 \sqrt{m^2 + p^2} \right\} \Big|_{m=M} \quad (2.6)$$

$n = 0, \dots, 4$

ϕ_0 is then determined by minimization of the finite part of $\mathcal{E}_{-2}(\phi_0)$. The coefficients of $U(\varphi)$ can then be arranged in such a way that $\mathcal{E}_{-2}(0) = 0$. This implies

$$C_0 = C_1 = 0 \quad C_2 = \mu^2 > 0 \quad (2.7)$$

Our Hamiltonian has now the following form:

$$\begin{aligned}
 \mathcal{H}_{-2} &= : \psi^\dagger (-i \vec{\alpha} \vec{\partial} + \beta M) \psi : \\
 \mathcal{H}_{-1} &= f : \psi^\dagger \beta \psi : \phi \\
 \mathcal{H}_0 &= \frac{1}{2} (\pi^2 + (\vec{\partial} \phi)^2 + \mu^2 \phi^2) \\
 \mathcal{H}_1 &= C_3 g \phi^3 / 3! \\
 \mathcal{H}_2 &= C_4 g^2 \phi^4 / 4!
 \end{aligned}
 \tag{2.8}$$

and ΔH_B is (partially) specified by (2.6); its terms $\tilde{C}_0$ and $\tilde{C}_1 \phi$ have been taken out and subsumed into the normal ordering of the fermion fields in H_{-2} and H_{-1} . We have up to now

$$H_{-2} |0\rangle_0 = 0 \tag{2.9}$$

In order to specify the properties of $|0\rangle_0$ with respect to the quantum operators connected to ϕ we have already to consider corrections to $|0\rangle_0$. We expand the field ϕ into a complete set of suitably normalized functions $\phi_q(\vec{x})$ which will be specified below as the solutions of a stability equation with eigenvalue ω_q^2 . We require then $|0\rangle_0$ to be annihilated by the corresponding annihilation operators $\hat{c}_q$, i.e.

$$\hat{c}_q |0\rangle_0 = 0 \tag{2.10}$$

Now H_{-1} creates three particle states via

$$\begin{aligned}
 H_{-1} |0\rangle_0 &= \int \frac{d^3 p d^3 p'}{(2\pi)^6 2E 2E'} f \bar{u}_s(\vec{p}) \hat{b}_s^\dagger(\vec{p}, v, \vec{p}') \hat{d}_s^\dagger(\vec{p}') \\
 &\quad \times \frac{C - \vec{p} \cdot \vec{p}'}{2\omega_{-\vec{p}-\vec{p}'}} |0\rangle_0
 \end{aligned}
 \tag{2.11}$$

The first energy correction vanishes since

$$E_{-1} = \langle 0 | H_{-1} | 0 \rangle = 0 \tag{2.12}$$

but the state $|0\rangle_0$ is corrected by

$$|0\rangle_1 = - (H_{-2})^{-1} H_{-1} |0\rangle_0 \quad (2.13)$$

Therefore we have for the correction of order λ^0

$$\begin{aligned} E_0 &= \langle 0 | H_0 | 0 \rangle_0 - \langle 0 | H_{-1} (H_{-2})^{-1} H_{-1} | 0 \rangle_0 \\ &= \tilde{C}_2 \frac{1}{2} \int \langle 0 | \phi^2(x) | 0 \rangle d^3x + D \cdot \frac{1}{2} \int \langle 0 | (\vec{\sigma} \cdot \vec{\phi}(x))^2 | 0 \rangle d^3x \\ &\quad + \langle 0 | \frac{1}{2} \int (\pi^2 + (\vec{\sigma} \cdot \vec{\phi})^2 + \mu^2 \phi^2) d^3x | 0 \rangle_0 \\ &\quad - \int \frac{d^3p}{(2\pi)^6} \frac{d^3p'}{2E 2E'} \frac{f^2}{E+E'} \text{tr} (p+M)(p'-M) \cdot \\ &\quad \cdot \int d^3x d^3x' e^{i(\vec{p}+\vec{p}')(\vec{x}-\vec{x}')} \langle 0 | \phi(\vec{x}) \phi(\vec{x}') | 0 \rangle_0 \end{aligned} \quad (2.14)$$

The last term on the right-hand side contains divergent parts of the form of the counter terms. These pieces are evaluated in Appendix A. The condition that they be compensated by the counter terms is

$$D = \left. \frac{\partial^2}{\partial k^2} I(k) \right|_{k=0} (2\pi)^3 \quad (2.15)$$

where $I(k)$ is defined by

$$I(k) = \int \frac{d^3p}{(2\pi)^6} \frac{d^3p'}{2E 2E'} \frac{f^2}{E+E'} \text{tr} (p+M)(p'-M) \quad (2.16)$$

and

$$E = \sqrt{\vec{p}^2 + M^2}, \quad E' = \sqrt{(\vec{p} + \vec{k})^2 + M^2}, \quad \vec{p}' = -\vec{p} - \vec{k}.$$

For $\tilde{C}_2$ we confirm the condition (2.6).

We now choose our set of functions $\phi_q^0(x)$ in such a way that the operator H_0 defined by

$$\begin{aligned} H_0 = & \int d^3x \left\{ \frac{1}{2} \Pi^2 + \frac{1}{2} (\nabla \phi)^2 + \frac{1}{2} \mu^2 \phi^2 \right\} \\ & - \int d^3x d^3x' \int d^3k \tilde{I}(k) e^{i\vec{k}(\vec{x}-\vec{x}')} \phi(\vec{x}) \phi(\vec{x}') \end{aligned} \quad (2.17)$$

becomes diagonal, where $\tilde{I}(k)$ is the finite part of the integral $I(k)$, i.e.,

$$\tilde{I}(k) = I(k) - I(0) - \frac{1}{2} k^2 \left. \frac{\partial^2 I(k)}{\partial k^2} \right|_{k=0} \quad (2.18)$$

In order to obtain this diagonalization the functions $\phi_q^0(\vec{k})$ have to satisfy the stability equation (corresponding to the "classical solution" ϕ_0)

$$\begin{aligned} -\nabla^2 \phi_q^0(\vec{x}) + \mu^2 \phi_q^0(\vec{x}) + \int d^3y d^3k \tilde{I}(\vec{k}) e^{i\vec{k}(\vec{x}-\vec{y})} \phi_q^0(\vec{y}) = \\ = \omega_q^2 \phi_q^0(\vec{x}) \end{aligned} \quad (2.19)$$

This equation is fulfilled for plane waves

$$\phi_q^0(\vec{x}) = e^{i\vec{q} \cdot \vec{x}}$$

and the corresponding eigenvalue ω_q^2 is given by

$$\omega_q^2 = q^2 + \mu^2 + \tilde{I}(q) \quad (2.20)$$

With this choice for the functions ϕ_q^0 E_0 is minimized by the requirement (2.10). The minimal value of E_0 is, of course, infinite and has to be compensated by a correction of the counter term $\tilde{C}_0$. Then $|0\rangle_0 + |0\rangle_1$ is the vacuum state including corrections of order λ^0 . We observe that the counter term proportional to Π^2 has not yet come into play. This is due to our non-conventional splitting of the Hamiltonian. If the free Hamiltonians of the fields ψ and ϕ appear in lowest order the energy denominator occurring in the perturbation expansion is

$$(H_{\psi}^{(0)} + H_{\phi}^{(0)} - E^{(0)})^{-1}$$

whereas here only $H_{\psi}^{(0)}$ (called H_{-2}) appears. Now ϕ couples to $\bar{\psi}\psi$ directly while Π couples only in combinations like

$$H_{\phi}^{(0)} \int \phi \bar{\psi} \psi d^3x$$

which, when applied to the vacuum contains a commutator $[\Pi^2, \phi]$ replacing ϕ by Π . Usually the expression (2.20) is of the same order as $\int \phi \bar{\psi} \psi$ but here it leads to a factor λ^2 . Indeed, since we need $(\int \Pi \bar{\psi} \psi)^2$ we have to go to order λ^4 . There an expression

$$\begin{aligned} & f^2 \int \frac{d^3p d^3p'}{(2\pi)^6 2E E'} \frac{\text{tr}(\not{p} + M)(\not{p}' - M)}{(E + E')^3} \cdot \\ & \cdot \int d^3x e^{i(\vec{p} + \vec{p}')(\vec{x} - \vec{x}')} \langle 0 | \Pi(\vec{x}) \Pi(\vec{x}', 10) \rangle \end{aligned} \quad (2.21)$$

(to be compared to the last term in (2.15) arises from a term

$$- \langle 0 | H_{-1} (H_{-2})^{-1} H_0 (H_{-2})^{-1} H_0 (H_{-2})^{-1} H_{-1} | 0 \rangle.$$

The divergence of (2.21) is then cancelled by the piece $-\frac{1}{4} D \Pi^2$ of the Π^2 counter term. Since λ is only a formal parameter and is to be set to one this asymmetry between the occurrence of the $(\nabla \phi)^2$ and Π^2 counter terms does not spoil Lorentz invariance as long as the perturbation series is convergent.

3. THE BAG SECTOR

The existence of bag sectors remains in this paper a matter of assumption. What we try here is to give a formal framework for the case that the dynamical equations (Eq. (3.5) below) have localized finite energy solutions for a suitable choice of parameter.

In order to determine the dynamical quasi-classical equations we rewrite the Hamiltonian introducing a space dependent classical expectation value $\sigma(x)$ of the field $\phi(x)$, in the form

$$\begin{aligned} \mathcal{H}_{-2} = & \frac{1}{2} (\vec{\sigma})^2 + \frac{1}{g^2} U(g\sigma) + : \psi^\dagger (-i \vec{\alpha} \vec{\sigma} + \beta (M + f\sigma)) \psi : \\ & + \langle \text{bag} | \psi^\dagger (-i \vec{\alpha} \vec{\sigma} + \beta (M + f\sigma)) \psi | \text{bag} \rangle_0 \\ & + \sum_{n=0}^4 \tilde{C}_n \frac{\sigma^n}{n!} + \frac{1}{2} D (\vec{\sigma})^2 \end{aligned} \quad (3.1a)$$

$$\begin{aligned} \mathcal{H}_{-1} = & \vec{\sigma} \vec{\sigma} \vec{\hat{\phi}} + \frac{1}{g} U'(g\sigma) \hat{\phi} + f : \psi^\dagger \beta \psi : \hat{\phi} \\ & + \langle \text{bag} | f \psi^\dagger \beta \psi | \text{bag} \rangle_0 \hat{\phi} + \sum_{n=1}^4 \tilde{C}_n \frac{\sigma^{n-1}}{(n-1)!} \hat{\phi} \\ & + D \vec{\sigma} \vec{\sigma} \vec{\hat{\phi}} \end{aligned} \quad (3.1b)$$

$$\begin{aligned} \mathcal{H}_0 = & \frac{1}{2} (\vec{\sigma} \hat{\phi})^2 + \frac{1}{2} \pi^2 + \frac{1}{2} U''(g\sigma) \hat{\phi}^2 \\ & + \frac{1}{2} \sum_{n=2}^4 \tilde{C}_n \frac{\sigma^{n-2}}{(n-2)!} \hat{\phi}^2 + \frac{1}{2} D (\vec{\sigma} \hat{\phi})^2 \end{aligned} \quad (3.1c)$$

$$\mathcal{H}_1 = (U'''(g\sigma) g + \tilde{C}_3 + \tilde{C}_4 \sigma) \hat{\phi}^3 / 3! \quad (3.1d)$$

$$\mathcal{H}_2 = (U^{IV}(g\sigma) g^2 + \tilde{C}_4) \hat{\phi}^4 / 4! \quad (3.1e)$$

The bag state $|\text{bag}\rangle_0$ is defined to be annihilated by the operators b_i and d_i corresponding to the quanta of the spin $\frac{1}{2}$ field in the external potential $f\sigma$ and of mass M , with the exception of those quantum levels which are occupied to give the hadron its correct quantum numbers (quark model assignments), i.e.

$$b_i |bag\rangle_0 = d_j |bag\rangle_0 = 0 \quad (3.2a)$$

$i, j = \text{unoccupied levels}$

$$b_k^\dagger |bag\rangle_0 = d_l^\dagger |bag\rangle_0 = 0 \quad (3.2b)$$

$k, l = \text{occupied levels}$

The normal ordering symbol $:::$ is defined correspondingly. We also define $|bag\rangle_0$ to be annihilated by the operators C_i corresponding to quanta of the shifted scalar field $\hat{\phi} = \phi - \sigma$ in an external potential to be specified below. (This implies $\sigma(x) = {}_0\langle 0 | \phi(x) | 0 \rangle_0$). Then $|bag\rangle_0$ is eigenstate of H_{-2} with eigenvalue

$$M_0 = \int d^3x \left(\frac{1}{2} (\vec{\partial}\sigma)^2 + \frac{1}{g_2} U(g\sigma) + \mathcal{V}[\sigma] \right) \quad (3.3)$$

where $\mathcal{V}[\sigma]$ is the finite part of the fermionic energy

$$\mathcal{V}[\sigma] = \int {}_0\langle bag | \psi^\dagger (-i \vec{\alpha} \vec{\partial} + \beta (M + f\sigma)) \psi | bag \rangle_0 d^3x \quad (3.4)$$

$$+ \int \left(\sum \tilde{C}_n \frac{\sigma^n}{n!} + \frac{1}{2} D (\vec{\partial}\sigma)^2 \right) d^3x$$

The fact that the local counter terms specified by the $\tilde{C}_n$ and D (which have been determined in the vacuum sector) really render also the fermion energy in the potential σ - which is a non-local functional of σ - finite, can be deduced in the framework of potential scattering by using perturbation theory. In field theory the fact is, of course, well known.

Clearly we want to minimize M_0 as a functional of σ . This leads to the equation which determines $\sigma(x)$:

$$-\vec{\partial}^2 \sigma + \frac{1}{g} U'(g\sigma(x)) + \frac{\delta \mathcal{V}[\sigma]}{\delta \sigma(x)} = 0 \quad (3.5)$$

It implies that H_{-1} reduces to

$$H_{-1} = f : \psi^\dagger \beta \psi : \hat{\phi} \quad (3.6)$$

Again the state $|bag\rangle_0$ is not sufficient in considering the theory on the λ^0 level. We have to add again the first correction

$$|bag\rangle_1 = - (H_{-2} - M_0)^{-1} H_{-1} |bag\rangle_0 \quad (3.7)$$

This leads to the energy correction of order λ^0

$$E_0 = \langle bag | H_0 - H_{-1} (H_{-2} - M_0)^{-1} H_{-1} | bag \rangle_0 \quad (3.8)$$

which can be rewritten as

$$E_0 = \langle bag | H_0 | bag \rangle_0 + \int d^3x d^3y \bar{F}(x, y) \langle bag | \phi(x) \phi(y) | bag \rangle_0 \quad (3.9)$$

with

$$\begin{aligned} \bar{F}(x, y) = & - \langle bag | : \psi^\dagger(x) \beta \psi(x) : (H_{-2} - M_0)^{-1} \\ & \times : \psi^\dagger(y) \beta \psi(y) : | bag \rangle_0 \end{aligned} \quad (3.10)$$

This is a functional of σ via H_{-2} and via the definition of $|bag\rangle_0$ and the symbol $::$. It is shown in Appendix B to be

$$T(\vec{x}, \vec{y}) = \frac{1}{2} \frac{\delta^2}{\delta \sigma(\vec{x}) \delta \sigma(\vec{y})} \int \delta b a g | \psi^+(\vec{z}) \times$$

$$\times (-i \vec{\alpha} \vec{\sigma} + \beta (M + f \sigma(\vec{z}))) \psi(\vec{z}) | b a g \rangle_0 \quad (3.11)$$

We have therefore

$$E_0 = \int \delta b a g \left\{ \frac{1}{2} \Pi^2(\vec{x}) + \frac{1}{2} (\vec{\sigma} \hat{\Phi}(\vec{x}))^2 + U''(\sigma(x)) \hat{\Phi}^2(\vec{x})/2 \right.$$

$$\left. + \frac{1}{2} \int \frac{\delta^2 U[\sigma]}{\delta \sigma(\vec{x}) \delta \sigma(\vec{y})} \hat{\Phi}(\vec{x}) \hat{\Phi}(\vec{y}) d^3 y \right\} | b a g \rangle_0 d^3 x \quad (3.12)$$

The operator whose expectation value we take here is clearly diagonalized, if $\phi(x)$ (and correspondingly $\hat{\Phi}(x)$) is expanded in terms of the eigenfunctions ϕ_i of the equation

$$-\vec{\sigma}^2 \phi_i(\vec{x}) + U''(\sigma(x)) \phi_i(\vec{x}) +$$

$$+ \int \frac{\delta^2 U[\sigma]}{\delta \sigma(\vec{x}) \delta \sigma(\vec{y})} \phi_i(\vec{y}) = \omega_i^2 \phi_i(\vec{y}) \quad (3.13)$$

This is precisely the stability equation corresponding to the solution of (3.5). Unfortunately it contains a non-local potential. In order to derive the existence of a translation mode solution we have to use the fact that $U[\sigma]$, the finite part of the fermion energy in the potential σ , does not depend on the location of $\sigma(x)$. This implies that we can write

$$U[\sigma] = \sum_{n=0}^{\infty} \frac{1}{n!} \int K_n(\vec{x}_1, \dots, \vec{x}_n) \sigma(\vec{x}_1) \dots \sigma(\vec{x}_n) d^3 x_1 \dots d^3 x_n \quad (3.14)$$

with translation and rotation invariant kernels K_n . Then one easily derives

$$\begin{aligned} \vec{\nabla}_x \int \frac{\delta^2 \mathcal{V}[\sigma]}{\delta \sigma(\vec{x}) \delta \sigma(\vec{y})} \sigma(\vec{y}) d^3 y = \\ = \int \frac{\delta^2 \mathcal{V}[\sigma]}{\delta \sigma(\vec{x}) \delta \sigma(\vec{y})} \vec{\nabla}_y \sigma(\vec{y}) d^3 y \end{aligned} \quad (3.15)$$

With this identity it follows immediately from Eq. (3.5) that the three functions

$$\vec{\varphi}_0 = \vec{\nabla} \sigma / \sqrt{N_0} \quad (3.16)$$

are zero modes of Eq. (3.13) - the three translation modes. The normalization factor N_0 is discussed further below and in Appendix C. We now expand (see Ref. 13))

$$\begin{aligned} \phi(\vec{x}) = \sigma(\vec{x}) + \vec{\nabla} \vec{\varphi}_0(\vec{x}) / \sqrt{N_0} + \sum (c_i \varphi_i(\vec{x}) + c_i^\dagger \varphi_i^*(\vec{x})) / 2\omega_i \\ \Pi(\vec{x}) = \vec{P} \vec{\varphi}_0(\vec{x}) / \sqrt{N_0} + \sum (c_i \varphi_i(\vec{x}) - c_i^\dagger \varphi_i^*(\vec{x})) / 2i \end{aligned} \quad (3.17)$$

We assume that $\sigma(x)$ is a purely radial potential, otherwise also rotational zero modes $\vec{x} \times \vec{\nabla} \sigma$ appear. With the normalization

$$\int \varphi_i^*(\vec{x}) \varphi_j(\vec{x}) d^3 x = \delta_{ij} \quad (3.18)$$

we have for the non-vanishing commutators

$$\begin{aligned} [c_i, c_j^\dagger] = 2\omega_i \delta_{ij} \quad i, j \neq 0 \\ [\mathcal{X}_k, \mathcal{P}_l] = i \delta_{kl} \quad k, l = 1, 2, 3 \end{aligned} \quad (3.19)$$

E_0 can be rewritten as

$$E_0 = \langle \text{bag} | \vec{P}^2 / 2N_0 + \sum \omega_i c_i^\dagger c_i | \text{bag} \rangle_0 \quad (3.20)$$

$$+ \frac{1}{2} \sum_{i \neq 0} \omega_i$$

which is minimized by requiring

$$c_i | \text{bag} \rangle_0 = 0 \quad (3.21a)$$

$$\vec{P} | \text{bag} \rangle_0 = 0 \quad (3.21b)$$

Now E_0 is still infinite and the coefficients $\tilde{C}_n$ have to be corrected in order to compensate the divergent parts of the scalar zero point energy in the vacuum sector. Then the first quantum correction will be given by

$$\Delta M_0 = \frac{1}{2} \sum_{i \neq 0} \omega_i - \left(\frac{1}{2} \int \frac{d^3 q}{(2\pi)^3} \omega_q \right)_{\text{vacuum}} + \text{counter terms} \quad (3.22)$$

We do not perform this renormalization here, since presumably it will not be of practical interest.

So far we have formulated the bag problem in such a way that:

- i) the bag state $|\text{bag}\rangle_0$ and its first correction $|\text{bag}\rangle_1$ are defined via (3.2) and (3.21)
- ii) the equation (3.5) for the confining potential $\sigma(x)$ is well defined (i.e., involves only finite quantities)
- iii) the stability equation (3.13) is also formulated in terms of finite quantities and the translation mode (3.16) appears explicitly.
- iv) the bag mass

$$M_0 = \int d^3x \left(\frac{1}{2} (\partial \sigma)^2 + U(g\sigma)/g^2 \right) d^3x + U[\sigma]$$

is manifestly finite and can receive only corrections of order 1 (i.e. relative order λ^2)

- v) The state defined by (3.2) and (3.21) is the lowest state in a continuum with three degrees of freedom parametrized by the eigenvalues of $\vec{P}$. We turn now to a discussion of the translation modes.

4. TRANSLATION MODES AND NON-RELATIVISTIC BAG KINEMATICS

The translation modes $\vec{\phi}_0$ are related to the confining potential via (3.16):

$$\vec{\phi}_0 = \vec{\sigma} / \sqrt{N_0} \quad (4.1a)$$

with the normalization constant

$$N_0 = \int (\vec{\sigma})^2 d^3x / 3 \quad (4.1b)$$

In Appendix C we use a virial theorem to determine N_0 . We find

$$N_0 = M_0 + \Delta \quad (4.2)$$

where Δ is defined as

$$\Delta = -\frac{2}{3} \langle \text{bag} | \vec{P}_F (H_2 - M_0) \vec{P}_F | \text{bag} \rangle - \frac{1}{3} \int (\vec{\sigma})^2 d^3x \quad (4.3)$$

Here $\vec{P}_F$ denotes the fermionic part of the momentum operator.

Let us now consider momentum eigenstates. We define

$$|\text{bag}, \vec{k}\rangle = \mathcal{P}(\vec{k}) |\text{bag}\rangle \quad (4.4a)$$

Here $\mathcal{P}(\vec{k})$ is the projection operator on momentum $\vec{k}$, formally realized as

$$\mathcal{P}(\vec{k}) = \frac{1}{(2\pi)^3} \int d^3a e^{i(\vec{P} - \vec{k})\vec{a}} \quad (4.4b)$$

where $\vec{P}$ is the momentum operator. $|\text{bag}, \vec{k}\rangle$ is a momentum eigenstate via the projection operator on the right-hand side and is to be the lowest simultaneous eigenstate of $\vec{P}$ and H (single particle state) in the bag sector. The state $|\text{bag}\rangle$ - whose relation to the previously defined

states $|bag>_0$ and $|bag>_1$ will be explained below - is not uniquely defined by the requirement imposed to $|bag, \vec{k}>$, since only its projection on momentum $\vec{k}$ will be used. It is therefore necessary to impose conditions on it to make it unique. A suitable choice are the three conditions (see Refs. 14), especially the second of those)

$$\vec{X} |bag> = 0 \quad (4.5)$$

We will use this condition and the projection operator in (4.4) to eliminate the zero modes from H^* .

At first we note that H commutes with the projection operator, so that we can write the eigenvalue condition

$$H |bag, \vec{k}> = E(\vec{k}) |bag, \vec{k}> \quad (4.6)$$

as

$$P(\vec{k}) H |bag> = P(\vec{k}) E(\vec{k}) |bag> \quad (4.7)$$

Now we use the property of the projection operator that it replaces $\vec{P}$ by $\vec{k}$, i.e.

$$P(\vec{k}) \vec{P} = P(\vec{k}) \vec{k} \quad (4.8)$$

The operator $\vec{P}$ can be decomposed as

$$\vec{P} = \vec{P} + \vec{P}_F + \vec{P}_B \quad (4.9)$$

where $\vec{P}$ is the previously introduced translation mode operator (see Eq. (3.17); it can be rewritten, using (3.16) and (3.18) as

$$\vec{P} = - \int \Pi \vec{\partial} \sigma d^3x \quad (4.10a)$$

while P_F and P_B are defined by

*) Our notation does not display the fact that also $|bag>$ depends on $\vec{k}$, though not in the two lowest orders and not as an eigenstate.

$$\vec{P}_F = - \int \psi^\dagger i \vec{\partial} \psi d^3x \quad (4.10b)$$

$$\vec{P}_B = - \int \Pi \vec{\partial} \hat{\Phi} d^3x \quad (4.10c)$$

Equation (4.8) can now be rewritten in the form

$$\mathcal{P}(\vec{k}) \mathbb{P} = \mathcal{P}(\vec{k}) (\vec{k} - \vec{P}_F - \vec{P}_B) \quad (4.11)$$

and Eqs. (4.5) and (4.11) can now be used to eliminate the translation mode operators $\mathbb{P}$ and $\mathbb{X}$ from the Hamiltonian in Eq. (4.7), leading to an equation

$$\mathcal{P}(\vec{k}) H_{\text{eff}}(\vec{k}) |b_{\text{ag}}\rangle = \mathcal{P}(\vec{k}) E(\vec{k}) |b_{\text{ag}}\rangle \quad (4.12)$$

Since P_B , generated by the replacement (4.11) contains again $\vec{P}$ and $\vec{X}$ the elimination procedure is recursive (see Ref. 14)). It is now sufficient to solve Eq. (4.12) without the projection operators in order to get a solution of Eq. (4.6). Such a solution can be constructed perturbatively, but we have to classify before the order in which we will consider the new terms in H_{eff} brought in by the elimination procedure.

In analogy to the treatment of the translation mode in the 1 + 1 dimensional scalar field theories we assign to $\vec{k}$ and $\vec{P}$ the order λ^{-1} and to $\vec{P}_B$ the order λ^0 . For $\vec{P}_F$ this analogy fails because our earlier - but presumably unavoidable - commitments concerning the order of the fermionic parts of the Hamiltonian. We assign here to $\vec{P}_F$ the order λ^0 in analogy with the pure quantum part P_B of the bosonic momentum and in view of the fact that then the first appearing divergences connected to $\vec{P}_F$ are cancelled by other terms in the same order in λ . If an explicit solution of the semi-classical equations of motion is available one can, of course, reconsider and check this assignment, e.g., by evaluating $\langle P_F^2 \rangle$. The assignment $P_F \sim \lambda^{-1}$ would not spoil the formalism.

With our assignments H_{-2} and H_{-1} receive no translation mode modifications and H_0 receives from the replacement of the $\vec{P}^2/2N_0$ term only a c-number contribution $\vec{k}^2/2N_0$. Therefore the state $|bag\rangle$ is given by

$$|bag\rangle = |bag\rangle_0 + |bag\rangle_1 + O(\lambda^2) \quad (4.13)$$

where $|bag\rangle_0$ and $|bag\rangle_1$ are the same states as defined in the previous section with the exception that the condition (3.21b) is replaced by the condition (4.6). The energy $E(\vec{k})$ is given by

$$E(\vec{k}) = M_0 + \frac{\vec{k}^2}{2N_0} + \Delta M_0 \quad (4.14)$$

This indicates a mismatch between the rest mass M_0 and the "kinetic" mass N_0 . However, this disappears if higher corrections are taken into account. In H_1 we find from the replacement of the $\vec{P}^2/2N_0$ term a contribution $\vec{k}\vec{P}_F/N_0$. This leads in order λ^4 to a contribution

$$-\frac{\vec{k}^2}{2N_0^2} \langle bag | \vec{P}_F (H_{-2} - M_0)^{-1} \vec{P}_F | bag \rangle_0 \cdot \frac{2}{3} \quad (4.15)$$

Up to a renormalization term this is equal to $\vec{k}^2\Delta/2N_0^2$ and therefore equal to the second term in a power series expansion

$$\frac{\vec{k}^2}{2M_0} = \frac{\vec{k}^2}{2N_0} \left(1 + \frac{\Delta}{N_0} + \dots \right) \quad (4.16)$$

The lacking renormalization counter term implied by the definition (4.3) of Δ can be rewritten as

$$\frac{\vec{k}^2}{2N_0^2} \left(-\frac{D}{3} \int (\vec{\sigma} \cdot \vec{\sigma})^2 dx \right) = -D \frac{\vec{k}^2}{2N_0} \quad (4.17)$$

and such a contribution arises from the counter term $-\Pi^2/2$ occurring also in order λ^4 . So it is suggested that the mismatch between M_0 and N_0 disappears perturbatively.

The elimination of $\vec{P}^2/2N_0$ introduces new kinds of terms like P_F^n and P_B^n and associated new divergences. Apart from the divergence mentioned above we have also checked that a divergent contribution (cf. the definitions below (2.16))

$$\left\{ \frac{d^3p}{(2\pi)^3 2E 2E'} \frac{t_r (p+M)(p'-M)}{(E+E')^2} \right\}_{k=0} \frac{(\vec{\nabla}\sigma)^2}{2N_0} \quad (4.18)$$

arising from the term $\langle \text{bag} | \vec{P}_F^2/2N_0 | \text{bag} \rangle_0$ in order λ^2 is cancelled in the same order. This happens because a divergence in the expression

$$\langle 0 | H_{-1} (H_{-2} - E_2)^{-1} H_0 (H_{-2} - E_2)^{-1} H_{-1} | 0 \rangle_0 \quad (4.19)$$

requires in the vacuum sector a modification of the $\tilde{C}_2$ counter term. In the bag sector then the term (4.18) is just lacking in a completeness sum leading to the corresponding divergent contribution.

So the occurrence of new terms in the effective Hamiltonian seems not to spoil renormalization. On general grounds this is to be expected since we just consider a sector of fixed momentum of a renormalizable theory. But the detailed matching of divergences may not always be obvious.

In conclusion to this section we may state that our method leads to correct non-relativistic kinematics in order λ^0 if some corrections of higher order are anticipated.

5. HIGHER CORRECTIONS

If we want to go beyond the order λ^0 - which contains already all the first quantum corrections -- a change in the arrangement of the Hamiltonian becomes necessary. This is due to the fact that $(H_{-2} - M_0)$ vanishes on all purely bosonic excitations of the state $|\text{bag}\rangle_0$ and such will be generated by our interaction terms. In order to make the inverse $(H_{-2} - M_0)^{-1}$ well defined (except on $|\text{bag}\rangle_0$) we have to include into H_{-2} also the bosonic "kinetic" Hamiltonian

$$H'_0 = \sum_{i \neq 0} \frac{c_i^\dagger c_i}{2\omega_i} \omega_i \quad (5.1)$$

(The sum does not include the translation modes). The expression (5.1) differs from H_0 by the term

$$H_0 - H'_0 = -\frac{1}{2} \int d^3x d^3y \frac{\delta^2 U[\sigma]}{\delta \sigma(x) \delta \sigma(y)} \phi(x) \phi(y) + \vec{P}^2 / 2N_0 \quad (5.2)$$

This difference remains in order λ^0 while H'_0 is promoted to order λ^{-2} . An analogous rearrangement has to be performed in the vacuum sector. This rearrangement causes, of course, a reshuffling of terms - and especially also divergences and counter terms in the perturbation series. Our occasional outlooks to higher order, made necessary in the discussion of some renormalizations, are not affected essentially, though some compensation of divergences do not occur in the same order in λ . As far as we can see the rearrangement does not cause problems. A similar rearrangement is also performed in Ref. 11).

6. SUMMARY AND CONCLUSIONS

We have given a formulation of "fermion field non-topological solitons"¹¹⁾ in a strong coupling limit which takes into account

- i) the deformation of the fermion sea energy density on the level of the quasi-classical equations and the renormalization terms associated with this feature
- ii) the translation mode which appears naturally in the stability equation and which leads to a perturbative formulation of translation invariant states.

At the same time we obtain the definition of a quantum state which can be considered as the optimal variational state that can be obtained by the ingredients of a coherent state transformation, Bogoliubov transformation of the boson and fermion field quanta and (approximate) momentum projection. In that sense our work is closely related to the formulation of Huang and Stump¹⁰⁾ who also consider the aspects i) and ii) mentioned above, though in a different way.

As compared to Friedberg and Lee¹¹⁾ we supply here the discussion of a strong coupling situation left out in their work (for reasons mentioned in the Introduction).

We are aware of the fact that our assigned orders of magnitude do not yet conform with the usual assignments met in the realistic applications of "bag field theory"; especially the classical solution $\sigma(x)$ cannot be of order λ^{-1} if the quark mass is of order λ^{-2} , since otherwise through the virial theorem (4.2) we get also hadron masses of the order of the quark mass. This difficulty is related to the fact that the hadron mass is small only due to a compensation of large $O(\lambda^{-2})$ terms. The situation in "realistic" bag field theory is admittedly more extreme than the one considered here. But we think that we do not completely miss the point by considering this less extreme situation more carefully than has been done before. It may give clues on some difficulties (e.g. parton distribution⁸⁾) and problems arising already here will not necessarily go away in the formulation of the "realistic" case. It would therefore be of interest to evaluate or estimate in realistic examples the new terms like the sea contribution to $\mathcal{V}[\sigma]$ or the correction Δ to the virial theorem, which have been brought closer to the surface in the present formulation, before going to the physically interesting limit.

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APPENDIX A

Evaluation of the λ^0 counter terms

We have to find the coefficients $\tilde{C}_2$ and D in such a way that the counter terms in (2.15) compensate the divergent parts of

$$-\int \frac{d^3 p}{(2\pi)^6} \frac{d^3 p'}{2E 2E'} \frac{f^2}{E+E'} \text{tr} (p+M)(p'-M) \cdot$$

$$\cdot \int d^3 x d^3 x' e^{i(\vec{p}+\vec{p}')(\vec{x}-\vec{x}')} \langle 0 | \phi(\vec{x}) \phi(\vec{x}') | 0 \rangle \quad (\text{A.1})$$

Since the integration in $\vec{k} = (\vec{p} + \vec{p}')$ is damped by the Fourier transform if the field expectation value decreases fast enough for $\vec{x}, \vec{x}' \rightarrow \infty$, the divergent integrations are those in the variable $(\vec{p} - \vec{p}')$; instead of the latter we may, at $\vec{k}$ fixed, also take simply $\vec{p}$. We have then to consider the integral

$$I(k) = - \int \frac{d^3 p}{(2\pi)^6} \frac{f^2}{2E 2E'} \frac{1}{(E+E')} \text{tr} (p+M)(p'-M) \quad (\text{A.2})$$

where

$$E = \sqrt{p^2 + M^2}$$

$$E' = \sqrt{(\vec{p} + \vec{k})^2 + M^2}$$

$$\vec{p}' = -\vec{p} - \vec{k}$$

At $\vec{k} = 0$ we obtain

$$I(0) = -f^2 \int \frac{d^3 p}{(2\pi)^6} \frac{p^2}{E^3} = -\frac{1}{2} \tilde{C}_2 (2\pi)^{-3} \quad (\text{A.3})$$

The first derivative with respect to k vanishes, the second derivative is still logarithmically divergent and the higher derivatives converge. The expression (A.1) contains therefore the divergent parts

$$\int d^3k I(k) \int d^3x d^3x' e^{i\vec{k}(\vec{x}-\vec{x}')} \langle 0 | \phi(\vec{x}) \phi(\vec{x}') | 0 \rangle \quad (A.4)$$

$$= -\frac{1}{2} \tilde{C}_2 \int d^3x \langle 0 | \phi^2(x) | 0 \rangle d^3x$$

and

$$\int d^3k k^2 \left(\frac{1}{2} \frac{\partial^2}{\partial k^2} I(k) \right) \bigg|_{k=0} \int d^3x d^3x' e^{i\vec{k}(\vec{x}-\vec{x}')} \langle \phi(\vec{x}) \phi(\vec{x}') | 0 \rangle \quad (A.5)$$

$$= -\frac{1}{2} D \int d^3x \langle 0 | (\vec{\partial} \phi(\vec{x}))^2 | 0 \rangle$$

so that explicitly

$$D = \frac{\partial^2}{\partial k^2} \left(\int \frac{d^3p}{(2\pi)^3} \frac{f^2}{2E 2E'} \frac{f^2}{E+E'} \text{tr}(\not{p} + M)(\not{p}' - M) \right) \bigg|_{k=0} \quad (A.6)$$

with E , E' and $\vec{p}'$ specified as below (A.2). These divergent parts have to be cancelled by the counter terms.

APPENDIX B

We will establish here the validity of Eq. (3.11)

$$F(\vec{x}, \vec{y}) = \frac{1}{2} \frac{\delta^2}{\delta \sigma(\vec{x}) \delta \sigma(\vec{y})} \int d^3z \langle \text{bag} | \mathcal{H}_{-2}^F(\vec{z}) | \text{bag} \rangle_0 \quad (3.12)$$

where $F(\vec{x}, \vec{y})$ is defined by Eq. (3.10) and where we have introduced

$$\mathcal{H}_{-2}^F(\vec{z}) = \psi^\dagger(\vec{z}) [-i \vec{z} \vec{\sigma} + \beta (M + f \sigma(\vec{z}))] \psi(\vec{z}) \quad (B.1)$$

(No normal product symbol or subtractions!) The variation of $|\text{bag}\rangle_0$ with $\delta \sigma(\vec{x})$ is given by first order perturbation theory as

$$\delta |\text{bag}\rangle_0 = - (H_{-2})^{-1} (1 - P_0) \delta H_{-2} |\text{bag}\rangle_0 \quad (B.2)$$

where P_0 is the projection into $|\text{bag}\rangle_0$. Because of this projection only the operator part of δH_{-2} contributes, i.e.,

$$\begin{aligned} & (1 - P_0) (\delta H_{-2}) |\text{bag}\rangle_0 \\ &= (1 - P_0) \int d^3z : \psi^\dagger(\vec{z}) \beta \psi(\vec{z}) : f \delta \sigma(\vec{z}) |\text{bag}\rangle_0 \end{aligned} \quad (B.3)$$

(Note the appearance of the normal product symbols!) For the first functional derivative we obtain

$$\begin{aligned} \frac{\delta}{\delta \sigma(\vec{x})} \int \langle b a g | \mathcal{H}_{-2}^F(\vec{z}) | b a g \rangle_0 d^3 z = \\ = \langle b a g | \psi^+(\vec{x}) f \beta \psi(\vec{x}) | b a g \rangle_0 \end{aligned} \quad (B.4)$$

Indeed, because of the operator $(1 - P_0)$ and the fact that $|b a g \rangle_0$ is an eigenstate of

$$H_{-2}^F = \int d^3 z \mathcal{H}_{-2}^F(\vec{z}) \quad (B.5)$$

the variation $\delta |b a g \rangle_0$ (cf., (B.2)) cannot contribute in (B.4). Now it is straightforward to take the derivative of the expression (B.4) with respect to $\delta \sigma(\vec{y})$. This yields

$$\begin{aligned} \delta^2 \int \langle b a g | \mathcal{H}_{-2}^F(\vec{z}) | b a g \rangle_0 d^3 z / \delta \sigma(\vec{x}) \delta \sigma(\vec{y}) = \\ = - \langle b a g | \psi(\vec{x}) f \beta \psi(\vec{x}) (H_{-2} - M_0)^{-1} (1 - P_0) : \psi^+(\vec{y}) f \beta \psi(\vec{y}) : | b a g \rangle_0 \\ - \langle b a g | : \psi^+(\vec{y}) f \beta \psi(\vec{y}) : (1 - P_0) (H_{-2} - M_0)^{-1} \psi^+(\vec{x}) f \beta \psi(\vec{x}) | b a g \rangle_0 \end{aligned} \quad (B.6)$$

Since H_{-2} commutes with $(1 - P_0)$ we can also use

$$\begin{aligned} \langle b a g | \psi^+(\vec{x}) f \beta \psi(\vec{x}) (1 - P_0) = \\ \langle b a g | : \psi^+(\vec{x}) f \beta \psi(\vec{x}) : \end{aligned} \quad (B.7)$$

and correspondingly for the second term in (B.6). Then (B.6) is equivalent to the equality of the right-hand side of Eqs. (3.10) and (3.11) which was to be established.

APPENDIX C
Virial Theorem

The classical confining field is determined as the solution of Eq. (3.6) which we rearrange in the form

$$-(1+D) \vec{\sigma}^2(x) + V'(\sigma(x)) + \frac{\delta \tilde{U}[\sigma]}{\delta \sigma(x)} = 0 \quad (C.1a)$$

Here

$$V'(\sigma(x)) = \frac{1}{g} U'(g\sigma(x)) + \sum_{n=1}^4 \tilde{C}_n \sigma^n(x) / (n-1)! \quad (C.1b)$$

and (cf. Eq. (B.1))

$$\begin{aligned} \tilde{U}[\sigma] &= \int d^3x \langle bag | \mathcal{H}_2(x) | bag \rangle_0 \\ &= \int d^3x \langle bag | \psi^\dagger (-i \vec{\alpha} \cdot \vec{\sigma} + \beta (M + f\sigma)) \psi | bag \rangle_0 \end{aligned} \quad (C.1c)$$

Proceeding in the usual way we multiply (C.1a) by $\sigma(x)$ and integrate over space to obtain

$$-(1+D) \int (\vec{\sigma})^2 d^3x = \int (V'(\sigma) + \frac{\delta \tilde{U}}{\delta \sigma}) \sigma d^3x \quad (C.2)$$

A second identity is obtained by applying $\sigma(x) \vec{\nabla}$ to (C.1a) and integration:

$$\begin{aligned} \frac{5}{2} (1+D) \int (\tilde{\sigma})^2 d^3x &= \\ &= 3 \int d^3x \left\{ -V'(\sigma) \tilde{\sigma} + (V(\sigma) - \tilde{C}_0) - \frac{\delta \tilde{U}}{\delta \sigma} \tilde{\sigma} \right\} \\ &\quad - \int d^3x \frac{\delta \tilde{U}}{\delta \sigma} \tilde{\sigma} d^3x \end{aligned} \quad (C.3)$$

where we have used

$$\int \sigma (\tilde{\sigma} \tilde{\sigma}) \tilde{\sigma} d^3x = \frac{5}{2} \int (\tilde{\sigma})^2 d^3x \quad (C.4)$$

(C.2) and (C.3) combine to

$$\begin{aligned} (1+D) \int (\tilde{\sigma})^2 d^3x &= -6 \int (V(\sigma) - \tilde{C}_0) d^3x \\ &\quad + 2 \int d^3x \frac{\delta \tilde{U}[\sigma]}{\delta \sigma(\tilde{x})} \tilde{\sigma}(\tilde{x}) d^3x \end{aligned} \quad (C.5)$$

The second term on the right-hand side would be equal to $-6(V[\sigma] + \tilde{C}_0)$ if $\tilde{\sigma}$ were an integral over a density local in σ . But this is not the case here and we have therefore to analyze this term. We rewrite it in the more explicit form

$$2 \int d^3x \frac{\delta}{\delta \sigma(\tilde{x})} \left(\int d^3y \phi_{\alpha\beta}(\tilde{x}, \tilde{y}) \mathcal{M}_{\alpha\beta}^F(\tilde{y}) \right) \tilde{\sigma}(\tilde{x}) d^3x \quad (C.6)$$

Using Eq. (B.4) we can rewrite (C.6) as

$$2 \int d^3x \phi_{\alpha\beta}(\tilde{x}, \tilde{x}) \psi^{\dagger}(\tilde{x}) \beta \psi(\tilde{x}) \tilde{\sigma}(\tilde{x}) d^3x \quad (C.7)$$

The term we would like to have on the right-hand side instead of (C.6) and (C.7) is

$$\begin{aligned} & -6 \int d^3x \left(\langle b_{\text{ag}} | \mathcal{H}_{-2}^F(\vec{x}) | b_{\text{ag}} \rangle_0 + \tilde{C}_0 \right) \\ & = 2 \int d^3x \vec{x} \cdot \vec{\sigma} \langle b_{\text{ag}} | \mathcal{H}_{-2}^F(\vec{x}) | b_{\text{ag}} \rangle_0 \end{aligned} \quad (\text{C.8})$$

where $\tilde{C}_0$ has been introduced in order to cancel the surface terms arising in the partial integration of the expectation value of $\mathcal{H}_{-2}(\vec{x})$. The right-hand side of (C.8) can be rewritten as

$$\begin{aligned} & 2 \int d^3x \vec{x} \cdot \vec{\sigma} \langle b_{\text{ag}} | \mathcal{H}_{-2}^F(\vec{x}) | b_{\text{ag}} \rangle_0 \\ & = 2 \int d^3x \vec{x} \cdot \langle b_{\text{ag}} | \psi^\dagger(\vec{x}) \beta \vec{\sigma} \psi(\vec{x}) | b_{\text{ag}} \rangle_0 \vec{\sigma}(\vec{x}) \\ & \quad + 2 \int d^3x \vec{x} \cdot \langle b_{\text{ag}} | \frac{1}{i} [\mathcal{H}_{-2}(\vec{x}), \vec{P}_F] | b_{\text{ag}} \rangle_0 \end{aligned} \quad (\text{C.9})$$

The first term on the right-hand side is just the second term on the right-hand side of (C.4) in the form (C.6). $\vec{P}_F$ in the second term is the fermion part of the momentum operator. The second term in Eq. (C.8) can be rewritten as

$$\begin{aligned} & 2 \int d^3x x^k \langle b_{\text{ag}} | \frac{1}{i} [T_{-2}^{00}(\vec{x}), P_F^k] | b_{\text{ag}} \rangle_0 \\ & = \frac{2}{i} \langle b_{\text{ag}} | [M_{-2}^{k0}, P_F^k] | b_{\text{ag}} \rangle_0 \end{aligned} \quad (\text{C.10})$$

Here M_{-2}^{k0} is the fermion part of the boost operator in order λ^{-2} . Using the identity

$$[M_{-2}^{k_0}, H_{-2}] = i P_F^k \quad (C.11)$$

which holds exactly (i.e., without neglecting higher order terms!) we find

$$(H_{-2} - M_0) M_{-2}^{k_0} |b_{ag}\rangle_0 = i P_F^k |b_{ag}\rangle_0$$

or

$$M_{-2}^{k_0} |b_{ag}\rangle_0 = (H_{-2} - M_0)^{-1} i P_F^k |b_{ag}\rangle_0 \quad (C.12)$$

Therefore

$$\begin{aligned} \frac{2}{i} \langle b_{ag} | [M_{-2}^{k_0}, P_F^k] | b_{ag} \rangle_0 \\ = -4 \langle b_{ag} | P_F^k (H_{-2} - M_0)^{-1} P_F^k | b_{ag} \rangle_0 \end{aligned} \quad (C.13)$$

So (C.5) may be rewritten as

$$\begin{aligned} (1+D) \int (\vec{\nabla} \sigma)^2 d^3x = -6 \left(\int V(\sigma) d^3x + U[\sigma] \right) \\ + 4 \langle b_{ag} | \vec{P}_F (H_{-2} - M_0)^{-1} \vec{P}_F | b_{ag} \rangle_0 \end{aligned} \quad (C.14)$$

To the logarithmic divergent counter term on the left-hand side there corresponds a logarithmic divergent part of the second term on the right-hand side. From (C.14) and the definition of M_0 Eq. (3.3) we find

$$\int (\vec{\nabla} \sigma)^2 d^3x = 3 [M_0 + \Delta] \quad (C.15)$$

with

$$\Delta = -\frac{2}{3} \langle b_{ag} | \vec{P}_F (H_{-2} - M_0)^{-1} \vec{P}_F | b_{ag} \rangle_0 - \frac{\tilde{D}}{3} \int (\vec{\nabla} \sigma)^2 d^3x \quad (C.16)$$

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