MULTIPARTICLE REACTIONS ON DEUTERONS

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A B S T R A C T

The deuteron is a weakly bound system with a rather large internucleon distance. This allows one to utilize the would-be spectator nucleon as a probe of how the formation of the secondary particles produced on the first nucleon proceeds. The main features of the rescattering events, when both nucleons take part in the interaction, turn out to be very sensitive to the multiple production mechanisms. In this paper we develop the multiperipheral parton model approach to the rescatterings in the multiple production off deuterons. We discuss :

- i) the single-particle inclusive spectra in the rescattering events ;
- ii) the influence of the rescatterings on the two-particle correlations ;
- iii) the multiplicity distributions and the KNO scaling in the rescattering events ;
- iv) the transverse momentum distributions in the rescattering events.

The results obtained suggest that a study of the rescattering events provides us with a unique possibility to get information on the space-time picture of the collision processes at high energies. The experimental data available on the subject give strong evidence for the existence of the large formation lengths of the secondary particles.

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1. INTRODUCTION

The last few years have been marked by increased interest in the multi-particle production off nuclei. The basic observation is that in almost all the approaches to the multiple production on the current theoretical market the time scales and the longitudinal distances characteristic of the high energy interactions turn out to be proportional to the momenta of the secondary particles. This was emphasized especially in the remarkable papers by Kancheli and Matinyan ¹⁾, Kancheli ²⁾ and Gottfried ³⁾. It is expected that something new could happen as these longitudinal distances rise and become comparable to the radii of the nuclei. This is one of the main points of attraction of the particle-nucleus interactions.

During the seventies, a number of theoretical approaches to the particle-nucleus interactions were proposed. While all the authors agree that attention should be paid to the growth of the interaction time scales, these approaches differ drastically in what concerns the rescatterings of the secondary particles into "status nascendi" in the nuclear matter [a thorough review on the subject is contained in "Multiparticle Production on Nuclei at very high Energies" ⁴⁾⁻⁸⁾, see also the earlier papers ⁹⁾]. The present authors' preference goes to the multiperipheral parton model (MPPM) put forward by Kancheli ²⁾ and developed further in a series of papers ¹⁰⁾⁻¹²⁾. This MPPM proved to be capable of quantitative description of all the main features of the experimental data available. Unfortunately, the quality of these data leaves much to be desired, and in fact all model builders could claim to be at least not at variance with these data, see Refs. ⁴⁾⁻⁸⁾. The problem is that the main part of these data are on collisions with the emulsion nuclei, where an unambiguous identification of the target nucleus is impossible. The second problem is that in the treatment of the data with the heavy nuclei certain nuclear physics complications are inevitable.

A unique target, whose properties are well understood, is a deuteron. The internucleon distances in the deuteron are large enough,

$$\langle r_d^2 \rangle^{1/2} \approx 2 F$$

so that the deuteron could be treated as a system of two free nucleons. The incident particle hitting a deuteron interacts with one of its nucleons just in the same way as with a free one. The secondary particles produced in this interaction could rescatter on the would-be-spectator-nucleon. The probabilities of these rescatterings depend on how the formation of the secondary

particles proceeds. Thus, the rescattering events (RE) in the multiparticle reactions on deuterons provide us with a unique possibility of shedding light on the multiproduction process.

This paper is devoted to the presentation of the MPPM predictions for the RE in multiparticle reactions on deuterons ^{*)}. We compare these predictions with the recent data on particle-deuteron interactions and point out the experiments which seem to be most interesting in the light of the current theoretical ideas on the subject.

The essence of the MPPM approach to the multiparticle production at high energies lies in the formation length concept. It means that the secondary hadrons of momenta k are present in the final state only at distances Δz exceeding their formation lengths l_f ,

$$\Delta z \gtrsim l_f \quad (1)$$

from the target position. In the MPPM the formation lengths l_f turn out to be proportional to the momenta k of the secondary particles :

$$l_f \approx k/m^2 \quad (2)$$

where m is some characteristic mass. This can be understood as follows. The uncertainty relation tells us that the production of the slow particle is characterized by the time scale

$$\tau \approx 1/\Delta E \quad (3)$$

where ΔE is an energy typical for strong interactions. Owing to the short-range order in the rapidity space, this energy ΔE should not depend on the colliding particles, and $\Delta E \approx m$. When the hadron produced is moving, the relativistic time dilation should be taken into account, so that we arrive at Eq. (2).

Let us now turn to the multiple production on deuterons. Let particle "b" be produced on the first of the nucleons in the deuteron. The probability w_r of its rescattering on the second nucleon is equal to

$$w_r = \sigma_{in}(bN) \cdot \langle 1/4\pi r_d^2 \rangle \quad (4)$$

*) A brief account of the MPPM approach to the multiple production off deuterons was published previously ¹³⁾.

provided that the corresponding formation length ℓ_f exceeds the characteristic internucleon distance r_c , and w_r vanishes otherwise. Here r_c is some distance

$$\langle 1/r_d^2 \rangle^{-1/2} \leq r_c \leq \langle r_d^2 \rangle^{1/2}, \quad r_c \approx 1F \quad (5)$$

Thus, the fraction of the RE will be given by the equation

$$P_{RE} = \sum_b \langle n_b(\ell_f \leq r_c) \rangle G_{in}(bN) \langle 1/4\pi r_d^2 \rangle \quad (6)$$

where $\langle n_b(\ell_f \leq r_c) \rangle$ denotes the mean multiplicity of secondaries with $\ell_f \leq r_c$. In the simplest version of the MPPM this condition corresponds to the momenta

$$k \leq k_c = r_c m^2 \quad (7)$$

These secondary interactions lead to excess multiplicity in the rescattering events,

$$\delta n \approx n_{\pi N}(k \approx k_c) - 1 \quad (8)$$

Here we subtracted unity, since we borrowed the rescattered particle from those produced on the first nucleon.

Evidently, in the simple-minded MPPM the spectra of fast secondary hadrons with the momenta $k \gtrsim k_c$ are not disturbed by rescatterings, whereas the yields of the lower momentum particles will be approximately doubled as compared to those in the collisions with the free nucleons, see Fig. 1. Thus, the following behaviour of the normalized inclusive spectrum R_y is expected to take place (see Fig.2) :

$$R_y = \left(\frac{dn}{dy} \right)_{RE} / \left(\frac{dn}{dy} \right)_{hN} = \begin{cases} 1, & k > k_c \\ 2, & k < k_c \end{cases} \quad (9)$$

A more refined analysis demonstrates that in the transition region of the momenta $k \approx k_c$ there should be a shallow dip where $R_y < 1$ (see below).

These are all the essential features of the RE in the particle-deuteron inelastic collisions implied by the growth of the formation lengths at high energies. The further presentation is organized as follows. In Section 2, we present a more detailed account of the MPPM approach to the rescatterings. We start with the simple-minded model which corresponds to the so-called fan diagram dominance approximation. Note that there is a formal correspondence between the rescattering diagram of Fig. 1 and the fan diagram of the Reggeon field theory, see Fig. 3 for an illustration. The contribution of this fan diagram is proportional to some effective triple-Pomeron coupling G_{eff} . It is well known that the triple-Pomeron coupling G_{PPP} , which determines the inelastic diffraction cross-section in the large mass region, is very small. A common assumption is that these two couplings are identical: $G_{\text{eff}} = G_{\text{PPP}}$ ¹⁴⁾. Should this be the case, the fan diagram contribution to the particle-nucleus amplitudes could be safely neglected, and the eikonal-type rescatterings should give the dominant contribution to the particle-nucleus interaction amplitudes ¹⁵⁾⁻¹⁸⁾. Just to get an impression of what is the difference between the predictions of these two limiting cases - fan diagram dominance and eikonal diagram dominance - we present in Section 3 the eikonal model (EM) predictions for the RE. In the Appendix we shall demonstrate that in fact the basic motivations for the eikonal diagram dominance are groundless: G_{eff} is not small and does not coincide with G_{PPP} . Nevertheless, the best way to resolve this controversy is to formulate the quantitative predictions of the competing approaches and to compare them with the data. It turns out that the EM predictions for the excess multiplicities are at sharp variance with the data.

In the fan diagram dominance approximation of Section 2 it was implicitly assumed that all the secondary particles were elementary ones. In Section 4 we discuss the modifications of this simple approximation due to the composite structure of hadrons. We adopt here the additive quark model. Note that the experimental data on the multiple production off heavy nuclei give strong evidence for this variant of the MPPM ¹⁹⁾. Therefore, these are the additive quark model predictions to be compared with the data on the RE.

In Sections 2, 3 and 4 the single-particle spectra are discussed. For a deeper insight into the rescattering mechanisms one should study two-particle correlations as well. In Section 5 we discuss the influence of the rescatterings on the two-particle correlations. Our main conclusion is that rescatterings tend to reduce the correlation function

$$R_2(y_1, y_2) = \frac{d^2n}{dy_1 dy_2} \cdot \left(\frac{dn}{dy_1} \frac{dn}{dy_2} \right)^{-1} - 1$$

The negative contribution to the correlation function, δR_2 , is found to be rather large : $\delta R_2 \simeq -(0.1 \pm 0.3)$. The rapidity dependence of δR_2 turns out to be very sensitive to the rescattering mechanism.

Obviously, the smaller the correlations are the narrower the multiplicity distributions should be. In Section 6 we present our predictions for the multiplicity distributions in the RE. The distortions of the multiplicity distributions in RE compared to the multiplicity distributions in hadron-nucleon collisions are found to be rather large. KNO scaling is shown to break in the RE, but the breaking effects turn out to be completely negligible numerically.

In Section 7 we discuss the influence of the rescatterings on the transverse momentum distributions. We find that rescatterings tend to increase the mean transverse momenta, but numerically the effects found are very small.

In conclusion, we give a brief summary of the results obtained and a list of our hopes for future experiments on the subject.

2. RESCATTERINGS IN THE FAN DIAGRAM DOMINANCE APPROXIMATION

As was explained in the Introduction, the rescatterings we deal with are in all respects reminiscent of the rescatterings within a thick target. The only parameter which determines the probability of the rescatterings is a formation length : there is a strong suppression of this probability at distances from the target nucleon smaller than a formation length, and the secondary hadrons behave as free hadrons at larger distances. To a good approximation the formation of the secondary hadrons could be viewed as a step-like process [see, for example, the review ⁶⁾ and original paper ¹¹⁾]. Thus, the probability of the rescattering will be given by the equation ^{*}) :

$$w_N^b(y) = \sigma_{in}(bN) \int_0^\infty dr |\psi_d(r)|^2 \theta(r - d e^y) \quad (10)$$

Here $\psi_d(r)$ is a deuteron wave function, and a step function in the integrand accounts for the formation lengths l_f :

$$l_f = k/m^2 = d \exp(y) \quad (11)$$

*) This equation, and the whole probabilistic picture the MPPM is crucially based upon, is almost self-evident when one starts with an analysis of the space-time picture of the multiparticle reactions. It is less trivial to arrive at this equation in the field-theoretic approach. A straightforward, but lengthy, derivation of Eq. (10) in the field-theoretic language is presented in the Appendix.

Here we introduce a convenient variable

$$y = \log(k/\mu_\pi) \quad (12)$$

to be called, somewhat deliberately, a rapidity.

It follows from Eq. (10) that a fraction of the RE is equal to

$$P_{RE} = \sum_b \int_0^\infty dy w_r^b(y) \frac{dn_b(Y,y)}{dy} \quad (13)$$

Here $Y = \log(p/\mu_\pi)$ is the projectile's rapidity, and $dn_b(Y,y)/dy$ denotes the inclusive spectra of the secondary particles produced off the first nucleon. Note that to a great extent the spectra $dn_b(Y,y)/dy$ are universal and that the formation of the final states is also similar in all high energy inelastic collisions. As a result, fractions of the RE should not depend on the value of the interaction cross-section of the incident particle, and should be approximately the same for the incident hadrons, photons, and leptons.

The rescatterings lead to the production of the additional particles, resulting in the excess multiplicity in the RE over the mean multiplicity in the particle-nucleon interactions. Simple considerations lead to the equation

$$\langle \delta n \rangle = \frac{1}{P_{RE}} \sum_b \int_0^Y dy w_r^b(y) \frac{dn_b(Y,y)}{dy} \langle n_{bN}(y) \rangle - 1 \quad (14)$$

Here $\langle n_{bN}(y) \rangle$ denotes the mean multiplicity in bN interactions, and we subtracted unity since the rescattered particle b was borrowed from the shower produced on the first nucleon.

The next, and even more interesting, quantity is a normalized inclusive spectrum of secondary particles in the RE :

$$R_y^i = \frac{dn_{RE}(hd \rightarrow i + \dots)}{dy} / \frac{dn(hN \rightarrow i + \dots)}{dy} \quad (15)$$

After some manipulations, one easily arrives at the following expression for R_y^i :

$$R_y^i = (1 - w_r^i(y)/P_{RE}) + \frac{1}{P_{RE}} \frac{dn_i^{hN}(Y,y)}{dy} \cdot \sum_b \int_y^Y dy_1 w_r^b(y_1) \frac{dn_b^{hN}(Y,y_1)}{dy_1} \cdot \frac{dn_i^{bN}(y_1,y)}{dy} \quad (16)$$

Here the first term corresponds to a flux of particles produced on the first nucleon multiplied by the probability to escape from rescattering. The second term accounts for the production of particles by the rescattered particles. Note that the resulting spectra in the RE satisfy the energy momentum conservation, provided the input spectra used satisfy the energy momentum conservation sum rules.

Should all the relevant spectra be known, it would be an easy task to compute all the quantities of interest. The problem is that, despite all the thick compilations available, little is known even about such fundamental spectra as the spectra of pions ! What is measured experimentally is a sum of the spectra of prompt pions and of decay products of resonances. Long ago it was predicted by Levin and Ryskin ²⁰⁾ and by Anisovich and Shekhter ²¹⁾ that resonances should constitute a majority of prompt particles. According to the recent data, there is indeed a copious production of resonances and enthusiastic people could claim that there are no prompt pions at all ²²⁾. Still there are no detailed data on the spectra of resonances at our disposal to be used as an input in Eqs. (13), (14) and (16).

Fortunately enough, we have learnt from our previous experience with the MPPM model description of multiple production on heavy nuclei that all ambiguities tend to cancel in the relative quantities ^{6),19)}. The normalized spectra R_y and the normalized multiplicities $R_A = \langle n \rangle_A / \langle n \rangle_N$ are in fact model-independent quantities. The only free parameter we are left with is a mass m which determines a scale for the formation lengths. In the fan diagram dominance approximation this mass should be taken equal to ¹¹⁾:

$$m^2 \approx 2 (\text{GeV})^2 \quad (17)$$

This value of m^2 corresponds to the characteristic momentum $k_0 \approx (3-5) \text{ GeV}$.

To get an impression of the detailed form of the normalized spectra in RE, let us start with a simple case of the plateau-like spectrum :

$$\frac{dn(Y,y)}{dy} = \theta(Y-y)\theta(y) \quad (18)$$

with quantum numbers of secondary particles neglected. The height of the plateau is chosen to satisfy the energy momentum conservation. It turns out that in this case $w_r(y)P_{RE}$ could be reasonably well approximated by the linear function of y (see Fig.4) :

$$w(y)/P_{RE} = \frac{2}{y_c} \left(1 - \frac{y}{y_c}\right) \theta(y_c - y) \quad (19)$$

where $y_c \simeq 5.5$. Then it follows from Eq. (16) that

$$R_y = 1 - \frac{2}{y_c} \left(1 - \frac{y}{y_c}\right) \theta(y_c - y) + \left(1 - \frac{y}{y_c}\right)^2 \theta(y_c - y) \quad (20)$$

As was expected, $R_y = 1$ for $y > y_c$, and $R_y \rightarrow 2$ as $y \rightarrow 0$. There is a shallow dip at

$$y_{min} = y_c - 1 \quad (21)$$

with a value of R_y

$$(R_y)_{min} = 1 - 1/y_c^2 \quad (22)$$

and with a width of the dip

$$\Delta y = 2 \quad (23)$$

More sophisticated calculations with the empirical spectra of charged particles used as an input result in essentially this same behaviour of R_y except for the region of the very low momenta, $k \lesssim 0.5$ GeV/c, see Fig. 5.

Evidently the excess multiplicity δn will be small, since all re-scattered particles have low energies. Taking all secondary particle as pions we obtained

$$\langle \delta n_{ch} \rangle = 1.75 \quad (24)$$

Note that in the fan diagram dominance approximation an excess multiplicity does not depend either on the energy or on the quantum numbers of the incident particles.

The only quantity sensitive to the prompt multiplicity is a fraction of the RE. Should all observed pions have been prompt ones, we would have obtained, from Eq. (13),

$$P_{RE} = 24\% \quad (25)$$

No doubt this is an overestimate of P_{RE} . According to the recent data, the yields of pseudoscalar and vector mesons are of comparable value ²²⁾, and the prompt multiplicity is approximately lower by a factor of ~ 1.6 than the measured one. Thus, a fraction of the RE reduces to

$$P_{RE} \approx 15\% \quad (26)$$

Let us discuss now the experimental situation. There are two signatures for the RE. The most clear-cut one is the presence in the final state in the forward hemisphere of two protons with momenta exceeding the Fermi momenta in the deuteron. These two-proton events occur when one of the interactions is followed by a charge exchange $n \rightarrow p$. Fortunately, the probability of $n \rightarrow p$ charge exchange is high enough, $P_{n \rightarrow p} \simeq (0.3-0.4)$, so that two-proton events should constitute at least one-fourth of the RE and $\sim 4\%$ of the whole sample of interactions. Note that these recoiled protons could not be confused with the protons from $p\bar{p}$ pairs produced in the central region. Still there are no comprehensive analyses of the RE performed in such a way.

The second approach is based on the analysis of the topological cross-sections. Evidently, collisions with protons lead to the even-topology events, while those with neutrons lead to the odd-topology final states. In collisions with deuterons even-topology events consist of :

- i) hp collisions with or without rescatterings on neutrons;
- ii) hn collisions with a visible spectator proton;
- iii) hn collisions with rescatterings on the proton.

Odd-topology events come from hn collisions without rescatterings and with an invisible spectator proton. Thus, rescatterings lead to the enrichment of the even-topology sample and to the abundance of the number of proton-like interactions compared to the number of neutron-like interactions. Using a subtraction procedure, one could get some information on the RE. So far all the data were analyzed in this way.

The main experimental results are as follows. The fraction of the RE was found to be $P_{RE} \simeq 14\%$ for nd interactions and $P_{RE} \simeq 17\%$ in pd interactions ²³⁾⁻³⁰⁾ in good agreement with our estimate for P_{RE} given by Eq. (26). The excess multiplicities estimated in experimental papers ^{23),29),30)} via the subtraction procedure are presented in Fig. 6. They are consistent with our estimation, $\delta n_{ch} = 1.75$. So far there have been no attempts to measure the inclusive spectra in the two-proton events. The only attempt to evaluate the spectra in the RE was undertaken elsewhere ²⁸⁾. The spectrum of the γ quanta have been obtained by subtraction of the spectrum in the neutron-like events from the spectrum in the proton-like events :

$$R_y^\gamma = \frac{1}{P_{RE}} \cdot \frac{\sigma(\pi^- p \rightarrow \gamma + \dots) - \sigma(\pi^- n \rightarrow \gamma + \dots)}{\sigma(\pi^- p \rightarrow \gamma + \dots) + \sigma(\pi^- n \rightarrow \gamma + \dots)} \quad (27)$$

Presumably, this subtraction procedure is more reliable for the spectrum of π^0 mesons, and, hence, for the spectrum of γ quanta, than for the spectra of charged particles. The resulting spectrum agrees well with our predictions (the broken curve in Fig. 5), but the error bars are too large for us to be satisfied by this agreement. The spectra of π^- mesons in proton-like interactions in $\pi^- d$ collisions at 21 GeV/c also show an excess compared with the spectra in true $\pi^- p$ interactions at 16 GeV/c in the backward hemisphere, in perfect agreement with our prediction ³⁰⁾.

3. RESCATTERINGS IN THE EIKONAL MODEL

As is demonstrated schematically in Fig. 3, there is a formal correspondence between the rescattering diagram of Fig. 1 and the fan diagram of the Reggeon field theory, where an interaction of Pomerons is described by some effective triple-Pomeron coupling G_{eff} . A never disputed assumption is that this triple-Pomeron coupling is the same coupling G_{PPP} which determines the inelastic diffraction cross-section, see Fig. 7. It is well known that the latter coupling is small and there are statements that owing to the smallness of G_{PPP} the fan diagrams give a negligible contribution to the rescattering process. Had this been the case, the eikonal-type diagrams would give a dominant contribution to the rescatterings ¹⁵⁾⁻¹⁸⁾.

In the eikonal model the starting point is Glauber's multiple scattering expansion for the particle-nucleus forward scattering amplitude. Here the projectile is supposed to undergo successive elastic scatterings on the target

nucleons, see Fig. 8. Once the elastic scattering amplitude is known, one could construct production amplitudes via the substitution of Pomerons by multiperipheral showers ³¹⁾, as is demonstrated schematically in Fig. 9. One quickly discovers that Glauber's double-scattering amplitude is not relevant to the problem at all, since it has no discontinuities corresponding to the inelastic scattering on both nucleons, see Fig. 10. It is not a surprise since Glauber's amplitude is just a typical Amati-Fubini-Stanghellini (AFS) amplitude.

In the Reggeon field theory this trouble is circumvented by the substitution of the planar AFS two-particle-two-Pomeron vertex of Fig. 8 by the non-planar Mandelstam vertex of Fig. 11. The double-scattering diagram with the latter vertex has the desired discontinuity, see Fig. 11. The next step is an assumption that the Pomeron-particle scattering amplitude M_2 of Fig. 11 has normal analyticity properties typical of particle-particle scattering amplitudes. The basic observation is that the relevant quantity, the double-scattering amplitude, is proportional to a dispersion integral over the discontinuity of M_2 in the Pomeron-particle energy s' . The path of the integration is shown in Fig. 12. After the deformation of this contour, as shown in Fig. 12, one gets the pole contribution and a cut contribution which corresponds to the multiparticle intermediate states. It could be easily verified that the pole term leads to the convolution of the two projectile-nucleon amplitudes just of the form given by Glauber's theory. Therefore, in what concerns the multiparticle final states of interest the pole term in M_2 should be taken to correspond to the double successive inelastic scattering of the projectile ³²⁾. The resulting structure of the final states can be analyzed using the formalism of the preceding Section. One should simply put

$$w_r(y) = \sigma_{aN} < 1/4\pi r_d^2 > \quad (28)$$

for the leading particles and

$$w_r(y) = 0 \quad (29)$$

for all other secondary particles.

As was noted above, parametrization dependent effects tend to cancel in the relative quantities. Let us neglect, for the moment, the contributions of the inelastic intermediate states and let us use as an input the simplest parametrizations for the spectra compatible with energy momentum conservation. We choose somewhat deliberately parametrizations

$$L(Y, y) = \exp(y - Y) \quad (30)$$

and

$$S(Y, y) = 1 - \exp(y - Y) \quad (31)$$

for the spectra of the leading (L) and secondary (S) particles, respectively. These formulae describe the observable spectra reasonably well. From Eqs. (16), (28) and (29) it follows then that

$$R_y^L = Y - y \quad (32)$$

$$R_y^S = 2 - (Y - y) / (\exp(Y - y) - 1) \quad (33)$$

for the normalized spectra of the leading and secondary particles, and that for the integral spectrum of all particles

$$R_y^{ch} = 2 [1 - \exp(y - Y)] \quad (34)$$

Note that both the parametrization for $L(Y, y)$ used and the resulting expression for R_y^L are reliable only in the fragmentation region. This striking difference in the momentum dependences of R_y^S and R_y^L and the fact that $R_y^{ch} \rightarrow 0$ at $y \rightarrow Y$ have been overlooked in some papers¹⁵⁾⁻¹⁷⁾. Obviously, there is no plateau with $R_y^{ch} = 1$ in the rapidity interval $Y - \Delta \leq y \leq Y - \delta$ where $\Delta = (2+3)$, $\delta \ll 1$, as was claimed by Lehman in his recent paper on the rescatterings in the eikonal model (EM)¹⁸⁾. More detailed calculations with the empirical spectra used as an input lead to essentially this same momentum dependence of the relative spectra, see Fig. 13.

In the more realistic approach, one should take into account inelastic intermediate states as well. In this case, the leading particle is either a few-particle system or some nucleon resonance. The simplest way to account for these states is as follows. Let us compute the spectra of the leading and secondary particles as was done above. Once inelastic rescatterings take place this effective leading particle is some superposition of the true leading particle and of resonance states with the relative fractions given by

$$\alpha_L = \frac{\sigma_{el}}{\sigma_{el} + \sigma_D} \quad \text{and} \quad \alpha_R = \frac{\sigma_D}{\sigma_{el} + \sigma_D} \quad (35)$$

respectively. Here σ_D is a part of the inelastic diffraction cross-section with a longitudinal momentum transfer cut-off $q_L \leq 1/r_d$.

Let $R_L(Y, y)$ and $R_S(Y, y)$ be the spectra of the leading and secondary hadrons in the decays of the diffractively excited states. Then the resulting spectra will be given by the questions :

$$\frac{dn_L}{dy} = \frac{dn_L^{eff}}{dy} \cdot \alpha_L + \alpha_R \int_{y_{min}}^{y_{max}^L} d\tilde{y} \frac{dn_L^{eff}}{d\tilde{y}} R_L(\tilde{y}, y) \quad (36)$$

$$\frac{dn_S}{dy} = \frac{dn_S^{eff}}{dy} + \alpha_R \int_{y_{min}}^{y_{max}^S} d\tilde{y} \frac{dn_S^{eff}}{d\tilde{y}} R_S(\tilde{y}, y) \quad (37)$$

Here $y_{max}^{L,S}$ and $y_{min}^{L,S}$ stand for the boundary rapidities of the decay products in decays of the excited states.

It turns out that the inclusion of the inelastic intermediate states has almost no influence on the normalized spectra of the leading particles. As for the secondary particles, there are some qualitative effects. All decay products have an apparent absorption cross-section equal to that of the parent resonance. Thus, inelastic diffraction leads to the apparent absorption of pions with the values of x ,

$$x = k/p \approx (1 - m_p^2/m_{N^*}^2)/2 \quad (38)$$

But numerically the effect turns out to be negligible even with the grossly over-estimated inelastic diffraction cross-section $\sigma_D = \sigma_{el}$, see Fig. 13.

The experimental values of R_y^Y presented in Fig. 5 disagree with the EM prediction : the experimental values of R_y^Y are systematically lower than theoretical ones. The next piece of evidence against the EM comes from the data on the excess multiplicities. In the EM the excess particles are produced by the leading particle, so that

$$\langle \delta n_{ch} \rangle \approx \langle n_{hN}(E_{eff} = E(1-K_{in})) \rangle - 1 \quad (39)$$

Here K_{in} stands for the mean inelasticity, $K_{in} \simeq 0.5$. A steep rise of the excess multiplicity at the high energies predicted by Eq. (39) sharply contradicts the experimental data shown in Fig. 6. The absolute value of δn_{ch} could be reduced slightly when inelastic intermediate states are taken into account: one should subtract the mean multiplicity of the charged particles in the diffractively produced state instead of unity. But this is of little help, since it does not influence the rate of growth of δn_{ch} ^{*)}. Thus, there are theoretical arguments and direct experimental evidence against the eikonal rescattering dominance.

4. RESCATTERINGS IN THE ADDITIVE QUARK MODEL

In the fan diagram dominance approximation, as presented above, an implicit assumption was that all particles are elementary. A somewhat more complicated picture of the interaction emerges as one takes into account the composite nature of hadrons ^{12),34)}.

To take an example, let us compare interactions of protons and deuterons with protons and heavy nuclei. Let energies be high enough so that the formation lengths are much larger than the dimensions of the nuclei. Had this been the case, the fragmentation spectra in proton-proton and proton-nucleus interactions would be identical. But this would not be the case for deuterons. In deuteron-proton collisions an impulse approximation holds and in any collision one of the nucleons is a spectator. Therefore, a typical final state consists of the newly produced hadrons plus this spectator nucleon in the fragmentation region. The situation changes drastically when the target is a heavy nucleus. In this case a collision of one of the deuteron's nucleons with a nucleus implies that the would-be-spectator-nucleon will interact as well. As a result, for the black and large nuclei, there will be no spectator nucleons at all, and the yields of the secondary particles in the central region will be doubled as compared to those in deuteron-proton collisions. Thus, a composite structure of hadrons leads to the apparent absorption of the leading particles despite the fact that formation lengths are larger than the dimensions of the target nuclei ^{12),34)}.

*) A very closely related approach to the RE in the EM framework was presented in the recent paper by Baker et al. ³³⁾. The absolute value of the excess multiplicity given by these authors is somewhat smaller than calculated in the present paper, whereas the energy dependences of δn_{ch} are the same. The origin of the discrepancy in the absolute value of δn_{ch} is unclear. In any case, the steep rise of δn_{ch} is excluded by the data.

In the additive quark model mesons are considered as deuteron-like systems and baryons as tritium-like systems made of constituent quarks. The formation of the final states is now a two-step process. The first one is a formation of the constituent quarks from the quark-partons, and the second one is a formation of hadrons from these constituent quarks. The resulting interaction picture is a hybrid of the fan diagram dominance and eikonal diagram dominance models :

$$w_r(y) = \sigma_{qN} < 1/4\pi r_d^2 > \quad (40)$$

for the spectator quarks, and

$$w_r(y) = \sigma_{qN} \int_0^\infty dr |\psi_d(r)|^2 \theta(r - d_q \cdot e^y) \quad (41)$$

for the secondary quarks produced in the first collision. Here σ_{qN} denotes the quark-nucleon interaction cross-section, given in the additive quark model by the relation

$$\sigma_{qN} \approx \frac{1}{2} \sigma_{\pi N}^{in} \approx \frac{1}{3} \sigma_{NN}^{in} \quad (42)$$

To go further, one needs spectra of quarks. Fortunately, none of the quantities of interest is sensitive to the detailed form of these spectra. Let us start with the fractions of the RE. When we take into account rescatterings of the spectator quarks, we obtain

$$P_{RE}(\pi d) = [\langle n_q(k \leq k_c) \rangle + 1] \sigma_{qN} < 1/4\pi r_d^2 > \quad (43)$$

$$P_{RE}(pd) = [\langle n_q(k \leq k_c) \rangle + 2] \sigma_{qN} < 1/4\pi r_d^2 > \quad (44)$$

A quantity of interest is the product $\langle n_q(k \leq k_c) \rangle \cdot \sigma_{qN}$. Note that a prompt multiplicity of quarks is twice a prompt multiplicity of mesons. This factor of two cancels the one half in the relation (42). Thus, one has

$$\langle n_q(k \leq k_c) \rangle \sigma_{qN} = \langle n_{\text{prompt mesons}}(k \leq 2k_c) \rangle \sigma_{\pi N} \quad (45)$$

In the additive quark model the mass m which determines the scale for the formation lengths, should be taken smaller than that in the fan diagram dominance model. According to the analysis of the data on multiparticle production off heavy nuclei, the best fit is provided by ¹⁹⁾

$$m^2 \approx 0.7 (\text{GeV})^2 \quad (46)$$

This value of m^2 results in

$$P_{RE}(\pi d) \approx 15\% \quad (47)$$

and

$$P_{RE}(pd) = P_{RE}(\pi d) + 2.5\% \approx 17.5\% \quad (48)$$

in perfect agreement with the experimental data. Here we assumed a universality of $\langle n_q(k < k_c) \rangle$ in πN and NN interactions.

Of all rescatterings relative contributions of the spectators are equal to

$$Z(\pi d) = \frac{1}{\langle n_q(k < k_c) \rangle + 1} \quad (49)$$

and

$$Z(pd) = \frac{2}{\langle n_q(k < k_c) \rangle + 2} \quad (50)$$

Interactions of the secondary quarks lead to the energy-independent excess multiplicity, while in the collisions of the spectators the excess multiplicities have an energy dependence identical to that in the particle-nucleon interactions. This latter contribution leads to the slow growth of the excess multiplicities of quarks :

$$\langle \delta n_q \rangle = (1-\eta) \langle n_q(qN, k_q < k_c) \rangle + \eta \langle n_q(qN, k_q = E/N_q) \rangle - 1 \quad (51)$$

where N_q is the number of the constituent quarks in the incident hadron : $N_q = 2$ for the incident mesons and $N_q = 3$ for the incident baryons. To go further, we assume that the multiplicity of the prompt mesons is half of the multiplicity of quarks ²¹⁾. The excess multiplicities estimated in this way are presented in Fig. 6. They agree well with the experimental values of δn_{ch} .

Interactions of the spectator quarks lead to the apparent absorption of particles produced in the fragmentation region. Let the normalized spectra of quarks R_y^{qi} have been calculated using Eqs. (16), (40) and (41). In the additive quark model hadrons are formed via fusion of quarks. Far away from the fragmentation region this fusion is obligatory owing to the confinement of quarks : each neighbouring $q\bar{q}$ pair forms some meson with a probability equal to unity. Thus, the spectrum of the prompt mesons will be similar to that of the quarks with the momentum rescaled by a factor of two :

$$\frac{dn_h}{dy} = \frac{1}{2} \frac{dn_q}{dy}, \quad K_h = K_q + K_{\bar{q}} \approx 2K_q \quad (52)$$

This rescaling of the momentum is compensated after the decays of the resonances are taken into account. Thus, far away from the projectile fragmentation region we obtain :

$$R_y^h = R_y^q \quad (53)$$

Spectator quarks carry approximately half of the momentum of the incident meson and two thirds of the momentum of the incident baryons. Therefore, fast hadrons are formed by fusion of these spectator quarks with one of the newly produced quarks. As a result, the yields of the projectile fragments off the nucleus will be proportional to the yields of the spectator quarks. Here we follow Anisovich and Shekhter who proposed considering a fusion as a statistical process ²¹⁾.

Let us consider the production of a spectator meson when a spectator quark is one of its constituent quarks. Then for such mesons one has ³⁴⁾

$$R_y^h \approx R_y^{sp} \quad (54)$$

Fast secondary baryons are produced when two of their constituent quarks are spectators. Thus, for secondary baryons one has ³⁴⁾

$$R_y^h \approx (R_y^{sp})^2 \quad (55)$$

In terms of the parameter η introduced above, Eqs. (53), (54) and (55) correspond to

$$R_y^h = 1 + \eta \quad (56)$$

in the plateau region, and to

$$R_y^h = 1 - \eta \quad (57)$$

for the fast leading particles with the rapidities $y \rightarrow Y$. In order to get a normalized spectrum for all particles one should take a combination of the form

$$\frac{dn}{dy} = R_y^q S(Y, y) + (R_y^{sp})^p L(Y, y) \quad (58)$$

where p is the number of spectator quarks in the leading hadron : $p=1$ in nd collisions and $p=2$ in pd collisions. The normalized spectra of charged hadrons calculated in this way are presented in Fig. 14 and are compared with the data on R_y^V in Fig. 5. The agreement is good, but much more accurate data are needed in order to choose between the additive quark model and the fan diagram dominance model.

5. TWO-PARTICLE CORRELATIONS IN THE RESCATTERING EVENTS

For a deeper insight into the production mechanism one should study two-particle distributions alongside single-particle ones. Here we shall focus our attention on the influence of the rescatterings on the two-particle correlation function :

$$R_2(y_1, y_2) = \frac{d^2n}{dy_1 dy_2} / \frac{dn}{dy_1} \frac{dn}{dy_2} - 1 \quad (59)$$

Let us recall first of all the basic features of the correlations in, say, proton-proton collisions. Here one has rather large positive correlations in the central region, where $R_2(y_1, y_2) \approx 0.6$, and negative correlations near the boundaries of the phase space ³⁵⁾. This behaviour of the correlations is well understood in terms of the multiladder production processes and the kinematical constraints ³⁶⁾. Let $dn^{(1)}/dy$ and $dn^{(2)}/dy_1 dy_2$ be a single- and a two-particle inclusive spectrum, respectively, in a single-ladder production. It is easily seen that in the central region the single-particle spectrum is given by the equation

$$\frac{dn}{dy} = \langle j \rangle \frac{dn^{(1)}}{dy} \quad (60)$$

where $\langle j \rangle$ is the mean number of ladders produced. The two-particle cross-section is given by the equation

$$\frac{d^2n}{dy_1 dy_2} = \langle j(j-1) \rangle \frac{dn^{(1)}}{dy_1} \frac{dn^{(2)}}{dy_2} + \langle j \rangle \frac{d^2n^{(1)}}{dy_1 dy_2} \quad (61)$$

Here the first term describes a contribution to the two-particle cross-section when two particles are produced in two different ladders, and the second one corresponds to the case when both particles are from the same ladder. Thus, we have ³⁶⁾

$$R_2(y_1, y_2) = \frac{\langle j^2 \rangle - \langle j \rangle^2}{\langle j \rangle^2} + \frac{R_2^{(1)}(y_1, y_2)}{\langle j \rangle} \quad (62)$$

where $R_2^{(1)}(y_1, y_2)$ is a correlation within one ladder.

Let one of the particles have rapidity $y \rightarrow Y$. Owing to the energy-momentum conservation, such a particle could be produced only in the one-ladder process, since in the m ladder process a boundary rapidity is equal to $Y_m \simeq Y - \ln m$. Therefore, once $y_1 \rightarrow Y$, the number of ladders in Eq. (61) should be put equal to unity. There is a short-range order in rapidities within one ladder and for $y_1 - y_2 \gg 1$ one has

$$R_2(y_1 \simeq Y, y_2) \simeq \frac{1}{\langle j \rangle} - 1 < 0 \quad (63)$$

When both particles are in the fragmentation region, $y_1 \simeq y_2 \simeq Y$, the two-particle cross-section vanishes completely for rapidities

$$e^{y_1} + e^{y_2} \simeq e^Y \quad (64)$$

so that at such rapidities $R_2(y_1, y_2) \equiv -1$.

The energy-momentum conservation constraints should reveal themselves most clearly when the variables $y = \ln e/\mu_\pi$ or $y = \ln k/\mu_\pi$ have been used. They are somewhat less striking in the spectra in the true rapidity $y = \ln[(e+p_z)/(e-p_z)]$. In any case, the finite-energy effects in the correlations are large in a wide region of rapidities.

Let us turn now to our rescattering problem. Had the momentum of the rescattered particle been fixed, we would have had a typical two-ladder production process at the momenta below p_c and a typical one-ladder production at higher momenta. In this one-ladder part of the spectrum both the single- and two-particle spectra will be identical to that on a hydrogen target. As for the region of the momenta lower than, but close to, p_c we are effectively near the phase space boundary with a negative two-particle correlation function. Thus, rescatterings lead to a negative contribution to the correlations ³⁷⁾.

This contribution to the correlation function could be estimated as follows. Let us take a simple-minded model with an uncorrelated particle production in the particle-nucleon collisions. For the single- and two-particle spectra in hN collisions we take the simplest parametrizations, with energy-momentum conservation explicitly taken into account :

$$\frac{dn^{(1)}}{dy} = \theta(Y-y)\theta(y) \quad (65)$$

and

$$\frac{d^2n^{(1)}}{dy_1 dy_2} = \theta(y) \cdot \theta(e^Y - e^{y_1} - e^{y_2}) \quad (66)$$

We build up the spectra in the RE as follows. A particle with rapidity y_c corresponds to the gap in the rapidity spectrum in the interval $[y_o^*-1, y_o^*]$, where, by virtue of energy momentum considerations, $y_o^* = y_c + 1 - \ln(e-1)$. The resulting single-particle spectrum takes the form :

$$\frac{dn(y_c)}{dy} = \theta(y)\theta(Y-y) [\theta(y-y_o^*) + \theta(y_o^*-1-y) + \theta(y_c-y)] \quad (67)$$

Here the first two terms in the square bracket correspond to the unscattered particles produced in the first interaction, and the last one corresponds to the particles produced by the rescattered particle.

The two-particle spectrum takes the form :

$$\begin{aligned} \frac{d^2 n(y_c)}{dy_1 dy_2} = & \theta(y) \theta(e^Y - e^{y_1} - e^{y_2}) \cdot \\ & \cdot [\theta(y_1 - y_0^*) \theta(y_2 - y_0^*) + \theta(e^{y_0^* - 1} - e^{y_1} - e^{y_2}) \\ & + \theta(e^{y_c} - e^{y_1} - e^{y_2}) + 2 \theta(y_c - y) \theta(y_0^* - 1 - y)] \end{aligned} \quad (68)$$

Here the first and the second terms in the square brackets correspond to the case when both particles are taken from the first ladder. The third term corresponds to the case when both particles are newly produced ones. The last one corresponds to the case when one of the particles is from the first ladder and the second is a newly produced one. Both single- and two-particle spectra should be averaged over y_c with the weight function $w_r(y_c) P_{RE}^{-1}$.

The resulting correlation plot is presented in Fig. 15. The "natural" value for the correlation function is $R_2(y_1, y_2) = 0$ everywhere except in the shadowed area where $e^{y_1} + e^{y_2} \geq e^Y$ and $R_2(y_1, y_2) = -1$. We see that rescatterings tend to reduce this "natural" value for the correlation function in approximately half of the (y_1, y_2) plane. At the momenta $k_1, k_2 \lesssim 10$ GeV/c the negative contribution to the correlation function is rather large: $\delta R_2 \approx -(0.1-0.15)$. Note that the effects are maximal at the rapidities where rescattering takes place.

In the EM the "natural" correlation function is zero as well. Here rescatterings take place in the fragmentation region. Thus, one expects negative correlations in the fragmentation region. The effects, estimated in the same way as was done above, turn out to be rather large, see Fig. 16.

The most interesting case is that of the additive quark model. In the EM we deal with the typical two-ladder production with j fixed: $j=2$. The fan diagram dominance approximation corresponds mainly to the single-ladder production modified by the rescattering at low momenta. The additive quark model corresponds to the superposition of these two mechanisms. As a result, the correlation function contains a term $\langle j^2 \rangle \cdot \langle j \rangle^{-2} - 1$ connected with the dispersion in the number of the ladders: $j=2$ in the fraction η of the RE and $j=1$ in the fraction $1-\eta$ of the RE. Thus, a "natural" value for the correlation function is

$$R_2 = \frac{(1-z) + 4z}{(1+z)^2} - 1 = \frac{2(1-z)}{(1+z)^2} \approx \begin{cases} 0.11, & \pi d \\ 0.13, & pd \end{cases} \quad (69)$$

Rescatterings take place both in the fragmentation region and in the central region, resulting in negative contributions to the correlation function in both regions, see Fig. 17. We see that in this case also at the momenta $k_1, k_2 \lesssim 3 \text{ GeV}/c$ the correlation function is reduced by $\delta R_2 \simeq -(0.1-0.15)$ from its "natural" value given by Eq. (69).

In the more realistic approach to the two-particle distributions, one should take into account correlations in the input particle-nucleon interactions. These correlations could be treated as a term proportional to $R_2^{(1)}(y_1, y_2)$ in the basic equation (62). Still we have not succeeded in solving this problem completely. For the coinciding rapidities $y_1 = y_2 = y$ a more or less reliable estimate could be made as follows. An experimentally measured two-charged-particle correlation function $R_2^{NN}(y, y)$ is reasonably well fitted by the function ³⁵⁾

$$R_2^{NN}(y, y) \approx \frac{0.6}{\Delta^2} \left(\frac{Y}{2} + \Delta - y \right) \left(y - \frac{Y}{2} + \Delta \right) \quad (70)$$

at all rapidities except the fragmentation region $y \rightarrow Y$. In the 200 GeV pp interactions $\Delta \simeq 2$. At coinciding rapidities a rough estimate for the mean number of the ladders is

$$\langle j \rangle \approx R_y \quad (71)$$

Thus, we obtain an additional contribution to the correlation function in the RE :

$$\delta R_2^{NN} \approx -R_2^{NN}(y, y) \frac{R_y - 1}{R_y} \quad (72)$$

This contribution also tends to reduce the correlation function in the RE. The resulting values of $\delta R_2^{NN}(y, y)$ are presented in Fig. 18. In the fan diagram dominance approximation δR_2^{NN} turns out to be small, since R_y

differs from unity only at rapidities where $R_2^{NN}(y,y)$ is numerically small. In this case $\delta R_2^{NN} \simeq -0.04$ in the rather restricted domain of the rapidities. In the additive quark model $\delta R_2^{NN} \simeq -(0.1+0.15)$ at almost all the rapidities of interest. In the eikonal diagram dominance model $\delta R_2^{NN} \simeq -\frac{1}{2}R_2^{NN}(y,y) \simeq -(0.2+0.3)$.

In rough approximation the rescattering induced correlations calculated above and presented in Figs. 15, 16 and 17, and the contribution to the correlation function given by Eq. (72) are additive. The resulting over-all picture is the following one. The quantity of interest

$$\delta R_2^{RE}(y_1, y_2) = R_2^{RE}(y_1, y_2) - R_2^{NN}(y_1, y_2) \quad (73)$$

turns out to be very similar in the fan diagram dominance and in the additive quark models : $\delta R_2^{RE} \simeq 0$ in the central region, and $\delta R_2^{RE} \simeq -(0.1+0.2)$ in the backward hemisphere. In the additive quark model, however, there is also a decline of $R_2^{RE}(y_1, y_2)$ in the projectile fragmentation region. In the EM δR_2^{RE} is negative and large both in the central region and in the projectile fragmentation region.

Unfortunately these predictions are unlikely to be tested in the near future, since experiments with very high statistics are needed in order to have a large sample of the RE.

6. MULTIPLICITY DISTRIBUTIONS IN THE RESCATTERING EVENTS

The larger are the correlations the wider are the multiplicity distributions. In the previous Section we learnt that the correlations in the RE events are smaller than those in the particle-nucleon collisions. Thus, an expectation is that multiplicity distributions in the RE should be narrower than those in particle-nucleus interactions. This could be easily understood as follows.

The multiplicity distributions in the particle-nucleon collisions are known to follow the KNO scaling :

$$w_n = \frac{1}{\langle n \rangle} \psi\left(\frac{n}{\langle n \rangle}\right) \quad (74)$$

Let $\langle n \rangle$ be very large, so that n could be considered as a continuous variable. Then in terms of the variable $z = n/\langle n \rangle$ one has

$$\frac{dw}{dz} = \psi(z) \quad (75)$$

The scaling function $\psi(z)$ could be well fitted by a function of the form :

$$\psi(z) \approx 2a^2 z e^{-a^2 z^2} \quad (76)$$

In the EM the multiplicity distribution function is given by a convolution of the form :

$$\begin{aligned} w_n^{RE} &= \sum_{\substack{n_1+n_2=n \\ n/\langle n \rangle}} \frac{1}{\langle n \rangle} \psi\left(\frac{n_1}{\langle n \rangle}\right) \cdot \frac{1}{\langle n \rangle} \psi\left(\frac{n_2}{\langle n \rangle}\right) = \\ &= \int_0^{n/\langle n \rangle} dx \psi(x) \psi\left(\frac{n}{\langle n \rangle} - x\right) \end{aligned} \quad (77)$$

Evidently, $\langle n_{RE} \rangle = 2\langle n \rangle$, and after introducing the variable $z = n/\langle n_{RE} \rangle$ we arrive at the equation :

$$\psi_{RE}(z) = 2 \int_0^{2z} dx \psi(x) \psi(2z-x) \quad (78)$$

Firstly, we see that multiplicity distributions in the RE obey the KNO scaling. Secondly, it follows from Eq. (78) that at small multiplicities, $z \ll 1$,

$$\psi_{RE}(z) \sim z^3 \quad (79)$$

and at large multiplicities, $z \gg 1$,

$$\psi_{RE}(z) \sim \exp[-2a^2 z^2] \quad (80)$$

Thus, in the RE, the KNO function should be a steeper function of z than $\psi(z)$, both at small and at large multiplicities.

This steepening of the KNO function takes place in all models. In the fan diagram dominance approximation one should convolute two KNO functions with unequal mean multiplicities $\langle n_1 \rangle$ and $\langle n_2 \rangle$. At small multiplicities $\psi(z)$ is again proportional to z^3 , and at large multiplicities

$$\Psi_{RE}(z) \sim \exp \left[- \frac{(\langle n_1 \rangle + \langle n_2 \rangle)^2}{\langle n_1 \rangle^2 + \langle n_2 \rangle^2} \cdot a^2 z^2 \right] \quad (81)$$

The factor $(\langle n_1 \rangle + \langle n_2 \rangle)^2 / (\langle n_1 \rangle^2 + \langle n_2 \rangle^2)$ is larger than unity and this function falls more rapidly at large z than $\Psi(z)$ does. Note that in this case $\langle n_1 \rangle$ is a rising function of the energy, while $\langle n_2 \rangle$ does not depend on the energy. As a result, in the fan diagram dominance approximation, there is no exact KNO scaling in the RE. Nevertheless, at all realistic energies deviations from the exact KNO scaling turn out to be completely negligible.

To be more precise, one should compute the multiplicity distributions in the RE using the following equation :

$$w_n^{RE} = \frac{1}{P_{RE}} \sum_b \int_0^Y dy \frac{dn_b(Y,y)}{dy} w_n^b(y) \sum_{n_1+n_2-1=n} w_{n_1}^{hN}(\langle n(Y) \rangle) w_{n_2}^{bN}(\langle n(y) \rangle) \quad (82)$$

In the additive quark model we make an assumption that each prompt quark corresponds to one final meson, see the discussion in Section 4. At low energies, where mean multiplicities are small, we use the modified KNO scaling:

$$w_n = \frac{1}{\langle n-\alpha \rangle} \Psi(z'), \quad z' = (n-\alpha) / \langle n-\alpha \rangle \quad (83)$$

Finally, we make a plausible assumption of the universality of $\Psi(z')$ for all projectiles. The resulting functions $\Psi_{RE}(z)$ turn out to be very close to each other in all three models. Those presented in Fig. 19 were calculated in the additive quark model framework. The experimental multiplicity distribution in the RE plotted in Fig. 19 was obtained in Ref. 23) from the analysis of the two-proton events. This multiplicity distribution shows some evidence for the predicted steepening of the KNO function in the RE. This prediction is one that could be checked experimentally in the easiest way and more data on the multiplicity distributions in the RE are urgently needed.

7. TRANSVERSE MOMENTUM DISTRIBUTIONS IN THE RESCATTERING EVENTS

We complete our discussion of the rescatterings in the multiple production on deuterons by a study of the influence of the rescatterings on the transverse momentum distributions. Evidently, rescatterings tend to increase the transverse momenta of the secondary particles. Let $\langle q_t \rangle$ be the mean transverse momentum in the ordinary hN collisions and $k_{\perp c}$ be the transverse momentum of the particle which undergoes a second collision. Then the transverse momenta of the newly produced particles will contain an additional component connected with $k_{\perp c}$

$$\langle k_{\perp}^2 \rangle = \langle q_{\perp}^2 \rangle + x^2 k_{\perp c}^2 \quad (84)$$

Here x is a fraction of the longitudinal momentum k_c carried by the newly produced particle.

In the practical calculations one should take into account the dependence of the transverse momenta $\langle q_t \rangle$ on the longitudinal momentum - the so-called sea-gull effect. This dependence could be well described by the simple parametrization

$$\langle q_t(Y, y) \rangle = q_t^0 + q_t^1 [e^{-y}(1-e^{-Y}) + e^{y-Y}(1-e^{y-Y})] \quad (85)$$

where

$$q_t^0 \approx q_t^1 \approx 0.3 \text{ (GeV/c)}$$

To get a quick estimate of the magnitude of the expected effects, one could use a Gaussian form of the input spectra :

$$\frac{dn}{dy d^2 \vec{k}_{\perp}} = \frac{dn(Y, y)}{dy} \cdot \frac{1}{\pi \langle q_t^2(Y, y) \rangle} \cdot \exp \left[-\frac{\vec{k}_{\perp}^2}{\langle q_t^2(Y, y) \rangle} \right] \quad (86)$$

When transverse momenta are taken into account, the basic Equation (16) for the spectra in the PE should be modified as follows :

$$\frac{d^3 n_{RE}(Y, y, \vec{k}_\perp)}{dy d^2 \vec{k}_\perp} = \frac{d^3 n_{hN}(Y, y, \vec{k}_\perp)}{dy d^2 \vec{k}_\perp} \left(1 - \frac{w_r(y)}{P_{RE}}\right) + \frac{1}{P_{RE}} \int_y^Y dy_1 w_r(y_1) \int d^2 \vec{p}_\perp \frac{dn(Y, y_1, \vec{p}_\perp)}{dy_1 d^2 \vec{p}_\perp} \cdot \frac{dn(y_1, y, \vec{k}_\perp - \vec{p}_\perp e^{y-y_1})}{dy d^2 \vec{k}_\perp} \quad (87)$$

In the Gaussian approximation the integral over the transverse momenta in Eq. (87) could be taken explicitly, and we obtain :

$$\begin{aligned} \frac{d^3 n_{RE}(Y, y, \vec{k}_\perp)}{dy d^2 \vec{k}_\perp} &= \frac{d^3 n_{hN}(Y, y, \vec{k}_\perp)}{dy d^2 \vec{k}_\perp} \left(1 - \frac{w_r(y)}{P_{RE}}\right) \\ &+ \frac{1}{P_{RE}} \cdot \int_y^Y dy_1 w_r(y_1) \frac{dn(Y, y_1)}{dy_1} \cdot \frac{dn(y_1, y)}{dy} \cdot \\ &\cdot \frac{1}{\pi x^2(Y, y_1, y)} \cdot \exp \left[-\frac{\vec{k}_\perp^2}{x^2(Y, y_1, y)} \right] \end{aligned} \quad (88)$$

where

$$x^2(Y, y_1, y) = \langle q_{t^2}^2(y_1, y) \rangle + e^{2(y-y_1)} \langle q_{t^2}^2(Y, y) \rangle \quad (89)$$

The resulting mean transverse momenta in the RE are presented in Fig. 20. Evidently, the effects found are negligible. Even these tiny effects are in fact overestimated. When prompt particles are resonances, the transverse momenta of pions are given by the equation ²⁰⁾ :

$$\langle q_{t^2}^2 \rangle_\pi \approx \langle q_{t^2}^2 \rangle_R + \langle x_\pi^2 \rangle \langle q_{t^2}^2 \rangle_R \quad (90)$$

Here the first term corresponds to the momentum of the pions in the decays of resonances, and the second term describes the transfer of the parent resonance's transverse momentum to pions. In this equation x_π is a fraction of the longitudinal momentum of the resonance carried by the pion [compare this with Eq. (84)]. Evidently, rescatterings influence only the last term in Eq. (90), which gives approximately one half of $\langle q_{t^2}^2 \rangle_\pi$.

Our conclusion is that rescatterings have negligible influence on the transverse momentum distributions.

8. SUMMARY AND CONCLUSIONS

The results obtained suggest that the RE provide us with a unique possibility of studying the space-time picture of the inelastic collision processes. The recoil protons in the elastic and inelastic interactions populate a very distinct part of the phase space. Thus, the presence in the final states of two protons with large longitudinal momenta $k_z \gtrsim (0.2+0.3)$ GeV/c, is an unambiguous signature for the double inelastic interactions. We have seen that the structure of the final states in the RE is rather sensitive to the formation process. Even the present data rule out the eikonal rescattering dominance model. The low excess multiplicities observed give strong evidence for the existence of large formation lengths of the secondary particles. All the data available are in perfect agreement with the predictions of the additive quark model.

The most sensitive tests of the approach proposed are provided by single-particle spectra and two-particle correlations. Unfortunately, to get a reasonable precision in the single-particle spectra, one should have at least hundreds of clear-cut RE, and at least a few thousands of the RE to look for the influence of the rescatterings on the two-particle correlations. But even the integral quantities, the excess multiplicities, and the multiplicity distributions in the RE, are sensitive enough to the production mechanism. Note that in the coherent tube model ⁸⁾, where RE are supposed to be similar to hN interactions at rescaled energy, $E \rightarrow 2E$, the multiplicity distributions in the RE should be identical to those in hN interactions. More detailed data on the multiplicity distributions in the RE, based on the two-proton signature selection, are urgently needed to test the predicted narrowing of the KNO scaling function in the RE.

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A P P E N D I X

We shall present here a field theoretical derivation of the formalism presented in Section 2.

We are interested in the cross-section of the particle-deuteron collisions of the kind where one of the secondary particles produced on the first of two nucleons undergoes an inelastic collision with the second one. The amplitude of this process is given by the diagram of Fig. 1. For the sake of simplicity we use here multiperipheral diagrams. Later on, it will be evident that all the results are general enough and are not the exclusive feature of the the multiperipheral model.

The rescattering cross-section we are interested in is given by the product of the amplitude of Fig. 1 and its complex conjugate. After summation over the phase space of particles, whose propagators are denoted by crossed lines, we obtain the following integral representation for the rescattering cross-section (see Fig. 21) :

$$\sigma_{RE} = \frac{1}{2S} \int \frac{d^4 k}{(2\pi)^4 i} \cdot \frac{d^4 r}{(2\pi)^4 i} \cdot \frac{d^4 q}{(2\pi)^4 i} \cdot \frac{A_6(p, \frac{1}{2}p_d + r, k, q)}{(k^2 - \mu^2 + i\varepsilon)((k+q)^2 - \mu^2 - i\varepsilon)} \times$$

$$\times \frac{A_4^{in}(k+q, \frac{1}{2}p_d - r, r) \Gamma_d(p_d, q) \Gamma_d(p_d, q - r)}{[(\frac{1}{2}p_d + r)^2 - m^2 + i\varepsilon][(\frac{1}{2}p_d - r)^2 - m^2 + i\varepsilon][(\frac{1}{2}p_d - q + r)^2 - m^2 - i\varepsilon][(\frac{1}{2}p_d + q - r)^2 - m^2 - i\varepsilon]} \quad (A.1)$$

All the notations here are in fact self-explanatory. $A_6(p, p_d/2 + r, k, q)$ denotes the discontinuity of the non-forward six-point amplitude, $A_4^{in}(k+q, p_d/2 - r, r)$ denotes the inelastic part of the discontinuity of the non-forward elastic scattering amplitude. An important point is that three of the six propagators in the integrand, denoted in Fig. 21 by double-crossed lines, come from the complex conjugate amplitude. Therefore they should be taken with the wrong sign ie prescription.

Let us start with the amplitude $A_4^{in}(k+q, p_d/2 - r, r)$. The momenta q and r are cut off by the deuteron wave function and are small compared with the typical momentum transfers in the strong interactions. Thus, the nucleons could be taken on their mass shell. The mass variables for the rescattered mesons are

$$\begin{aligned}\mu_1^2 &= k^2 \\ \mu_2^2 &\approx k^2 + 2kq\end{aligned}\tag{A.2}$$

Both particles are close to the mass shell only provided

$$|kq| \lesssim \mu^2\tag{A.3}$$

A commonly accepted assumption is that soft scattering amplitudes vanish when external particles are far off the mass shell. Thus we have

$$A_4^{in}(k+q, \frac{1}{2}P_d-r, q) \approx 4|\vec{k}|m\mathcal{G}_{in}\theta(\mu^2 - |kq|)\tag{A.4}$$

Next let us look at the q dependence of the six-point amplitude $A_6(p, p_1, k, q)$. In the multiperipheral $\lambda\phi^3$ theory this amplitude is given by the box diagram of Fig. 22 :

$$\begin{aligned}A_6(p, p_1, k, q) &= \lambda^2 \int \frac{d^4x}{(2\pi)^4} G_1(x) G_2(x) G_3(k-x) G_4(k-x+q) \times \\ &\times \mathcal{F}_1(s_1, t_1=0, x^2, x^2) \mathcal{F}_2(s_2, t_2=q^2, (k-x)^2, (k-x+q)^2)\end{aligned}\tag{A.5}$$

It could easily be shown that the main contribution to this integral comes from the domain of the momenta $x \approx k/2$. Therefore, all the mass variables could be kept close to the mass shell only provided condition (A.3) holds. Note that at $q=0$ this six-point amplitude is just a six-point amplitude for the single-particle inclusive spectrum :

$$d\mathcal{G}_{in} \approx \frac{1}{2s} A_6(p, p_1, k, 0) \frac{d^3\vec{k}}{2k_0 (2\pi)^3}\tag{A.6}$$

Thus, we arrive at the following equation :

$$A_6(p, p_1, k, q) \approx k_0 \frac{d^3\mathcal{G}_{in}}{d^3\vec{k}} \cdot 32\pi^3 s \theta(\mu^2 - |kq|)\tag{A.7}$$

To go further, we note that in the non-relativistic limit, q_0 is small, $q_0 \simeq q_z^2/2m \ll q_z$, and could be neglected everywhere but in the propagators of the nucleons. This allows one to perform the integrations over q_0 and r_0 explicitly. The result is

$$\int \frac{d^4 r}{(2\pi)^4 i} \frac{d^4 q}{(2\pi)^4 i} \frac{\Gamma_d(P_d, r) \Gamma_d(P_d, q-r)}{\left[\left(\frac{1}{2} P_d + r \right)^2 - m^2 + i\varepsilon \right] \left[\left(\frac{1}{2} P_d - r \right)^2 - m^2 + i\varepsilon \right] \cdot \left[\left(\frac{P_d}{2} - q + r \right)^2 - m^2 - i\varepsilon \right]} \times$$

$$\times \frac{1}{\left[\left(\frac{1}{2} P_d + q - r \right)^2 - m^2 - i\varepsilon \right]} = \quad (A.8)$$

$$\int \frac{d^3 \vec{r}}{2m(2\pi)^3} \frac{d^3 \vec{q}}{2m(2\pi)^3} \frac{\Gamma_d(\vec{r}) \Gamma_d(\vec{q} - \vec{r})}{(\Delta^2 + \vec{r}^2) (\Delta^2 + (\vec{r} - \vec{q})^2)}$$

An important observation is that the integral over the momentum $\vec{r}$ is just an integral for the deuteron electromagnetic form factor ³⁸⁾:

$$\int \frac{d^3 \vec{r}}{2m(2\pi)^3} \frac{\Gamma_d(\vec{r}) \Gamma_d(\vec{q} - \vec{r})}{(\Delta^2 + \vec{r}^2) (\Delta^2 + (\vec{r} - \vec{q})^2)} = \mathcal{P}_{EM}(\vec{q}^2) \quad (A.9)$$

Next, let us perform an integration over k_0 in the integral

$$\mathcal{J}(q) = \int \frac{dk_0}{2\pi i} \frac{1}{(k^2 - \mu^2 + i\varepsilon) (k^2 - \mu^2 - 2kq - i\varepsilon)} \quad (A.10)$$

One should integrate along the real axis of k_0 over the domain of k_0 where $|k^2|$ is bounded: $|k^2| \lesssim \mu^2$. We have:

$$\mathcal{J}(q) = \int \frac{dk_0}{2\pi i} \frac{1}{2kq} \left[\frac{1}{k^2 - \mu^2 - 2kq - i\varepsilon} - \frac{1}{k^2 - \mu^2 + i\varepsilon} \right]$$

$$= \frac{1}{2k_0} \cdot \frac{1}{2\vec{k}\vec{q}}$$
(A.11)

Note the importance of the correct $i\varepsilon$ prescriptions. Had the diagram of Fig. 21 been taken as a usual Feynman diagram with the usual $i\varepsilon$ prescription, both poles would be on the same side of the integration path. Thus, the

contribution of the diagram of Fig. 21 to the forward scattering amplitude, taken not as a unitary diagram, but as a usual Feynman diagram, vanishes. This result is almost self-evident : once there has been an inelastic collision on the first nucleon, the subsequent interactions of the secondary particles do not influence the absolute value of the cross-section.

Combining all the equations, we arrive at the integral for the rescattering cross-section σ_{RE} of the form

$$\sigma_{RE} = \int \frac{d^3 \vec{k}}{k_0} \left(k_0 \frac{d\sigma_{in}^{hN}}{d^3 \vec{k}} \right) \sigma_{bN}^{in} \int \frac{d^3 \vec{q}}{(2\pi)^3 (\vec{q} \cdot \vec{n})} \rho(\vec{q}^2) \theta(\mu^2 - |\vec{k} \vec{q}|) \quad (A.12)$$

Here $\vec{n}$ is a unit vector in the direction of $\vec{k}$: $\vec{n} = \vec{k}/|\vec{k}|$. Let us replace $\rho(\vec{q}^2)$ by its integral representation

$$\rho(\vec{q}^2) = \int d^3 \vec{r} |\psi_d(\vec{r})|^2 e^{i \vec{q} \cdot \vec{r}} \quad (A.13)$$

Then we arrive at

$$\sigma_{RE} = \int \frac{d^3 \vec{k}}{k_0} \left(k_0 \frac{d\sigma_{in}^{hN}}{d^3 \vec{k}} \right) \sigma_{bN}^{in} \int \frac{d^3 \vec{q}}{(2\pi)^3 (\vec{q} \cdot \vec{n})} \int d^3 \vec{r} e^{i \vec{q} \cdot \vec{r}} |\psi_d(\vec{r})|^2 \theta(\mu^2 - |\vec{k} \vec{q}|) \quad (A.14)$$

The integration over $d^2 \vec{q}_\perp$ results in $\delta(\vec{r}_\perp)$, and we obtain

$$\sigma_{RE} = \int \frac{d^3 \vec{k}}{k_0} \left(k_0 \frac{d\sigma_{in}^{hN}}{d^3 \vec{k}} \right) \sigma_{bN}^{in} \int \frac{dq_z}{2\pi q_z} e^{i q_z z} \int dz |\psi_d(z, \vec{r}_\perp=0)|^2 \times \theta(\mu^2 - |\vec{k} \vec{q}|) \quad (A.15)$$

Here the z axis is chosen to be parallel to the momentum $\vec{k}$. Not surprisingly we see that rescattering takes place only provided the nucleons are aligned along the momentum of the secondary particle.

Our final observation is that the function

$$y(z) = \int \frac{dq_z}{2\pi q_z} e^{i q_z z} \theta(\mu^2 - |\vec{k} \vec{q}|) \quad (A.16)$$

is similar to the step function $\Theta(z - |\vec{k}|/\mu^2)$. There are two cut-off factors in the integrand: the condition $|q_z| \lesssim \mu^2/|\vec{k}|$, and the oscillations of the exponent, leading to $|q_z| \lesssim 1/z$. Provided that $z \lesssim |\vec{k}|/\mu^2$ - the former cut-off is an important one - one could neglect the phase of the exponent, and the integral vanishes: $\varphi(z) \simeq 0$ for $z \lesssim |\vec{k}|/\mu^2$. In the reverse case of $z > |\vec{k}|/\mu^2$ one could drop the cut-off condition $|\vec{k}q| \lesssim \mu^2$ altogether, and the integration limits could be extended to infinity:

$$\varphi(z) = \int_{-\infty}^{+\infty} \frac{dq_z}{2\pi q_z} e^{iq_z z} \quad (\text{A.17})$$

But this is an integral representation of the step function $\Theta(z)$. Thus, $\varphi(z) \simeq 1$ at $|z| \gtrsim |\vec{k}|/\mu^2$. This completes our proof of Eq. (13).

A few comments are in order. Firstly, we started with the full amplitude apparatus of field theory, and arrived at the probabilistic picture of the rescatterings. Secondly, what we computed is a very specific discontinuity of the Mandelstam diagram, when two particles are close to the mass shell. This discontinuity is relevant only in the case when rescatterings take place on two distant targets. Only in this very case is the effective triple-Pomeron coupling G_{eff} large. The larger are the dimensions of the target the more important is the contribution of this kind of discontinuity to particle production. In particular, it will be dominant in inelastic collisions with heavy nuclei. Note that this kind of discontinuity of the Mandelstam diagram was never considered previously (to be more exact, we deal here with the unitarity Mandelstam diagram), since people were interested in the rescatterings in the interactions of non-extended objects. In this case, there are no near-mass-shell particles and the corresponding triple-Pomeron vertex is very small. Thirdly, there are no contributions which could cancel this contribution to the rescattering cross-section *). Fourthly, we are speaking about some triple-Pomeron coupling G_{eff} just to pay tribute to tradition. In fact, no information on the Reggeon structure of the amplitudes A_6 and A_4 is needed: one should just take physical amplitudes no matter what they are composed of. Fifthly, we do not neglect the usual Mandelstam diagrams altogether. In our opinion, they correspond to the double scatterings due to the true compositeness of the hadrons **). Rescatterings of the spectator quarks in the additive quark model correspond just to these Mandelstam diagrams.

*) Let us emphasize again that the RE we are interested in are the events with double inelastic interactions, so that in the final state there are two nucleons with large longitudinal momenta.

**) This point of view on the double scattering was expressed in the recent paper by Baker et al. ³³⁾ too. See also lectures by Gribov ³⁹⁾ and Anselm ⁴⁰⁾.

REFERENCES

- 1) O.V. Kancheli and S.G. Matinyan - Soviet J.Nucl.Phys. 11, 726 (1971).
- 2) O.V. Kancheli - JETP Letters 18, 274 (1973).
- 3) K. Gottfried - Phys.Rev.Letters 32, 957 (1974).
- 4) G. Winbow - "Multiparticle Production on Nuclei at Very High Energies"
Eds. G. Bellini, L. Bertocchi and P.G. Rancoita, ICTP Trieste
(1977), p. 133.
- 5) K. Zalewski - Ibid., p. 145.
- 6) N.N. Nikolaev - Ibid., p. 159.
- 7) Meng Ta-Chung - Ibid., p. 435.
- 8) Y. Afek, G. Berlad, A. Dar and G. Eilam - Ibid., p. 591.
- 9) E.L. Feinberg - Physics Reports 50, No 5 (1972).
- 10) V.I. Zakharov and N.N. Nikolaev - Soviet J.Nucl.Phys. 21, 227 (1975).
- 11) G.V. Davidenko and N.N. Nikolaev - Soviet J.Nucl.Phys. 24, 402 (1976).
- 12) N.N. Nikolaev - Phys.Letters B60, 363 (1976).
- 13) N.N. Nikolaev and V.R. Zoller - Phys.Letters B70, 99 (1977).
- 14) See, for example, A. Schwimmer - Nuclear Phys. B94, 445 (1976).
- 15) Yu.M. Shabelsky - Nuclear Phys. B132, 491 (1978).
- 16) A. Capella and A. Kaidalov - Nuclear Phys. B111, 477 (1976).
- 17) A. Capella and A. Krzywicki - Phys.Letters B67, 84 (1977).
- 18) E. Lehman - Nuclear Phys. B127, 331 (1977).
- 19) N.N. Nikolaev, A.Ya. Ostapchuck and V.R. Zoller - Lectures given at
the 6th ITEP Annual Winter School on Particle and Nuclear
Physics, Belye Stolby near Moscow (25 February - 5 March 1978),
to be published in Elementary Particles, Ed. I.Yu. Kobzarev,
by Atomizdat, Moscow.
- 20) E.M. Levin and M.G. Ryskin - Soviet J.Nucl.Phys. 17, 199 (1973) ; 18,
223 (1974).
- 21) V.V. Anisovich and V.M. Shekhter - Nuclear Phys. B55, 388 (1973).
- 22) G. Jancso et al. - Nuclear Phys. B124, 1 (1977), and references
therein.
- 23) J.E.A. Lys et al. - Phys.Rev. D16, 3137 (1977).
- 24) A. Sheng et al. - Phys.Rev. D12, 1219 (1975).
- 25) S. Dado et al. - Phys.Letters B60, 397 (1976).

- 26) Y. Eisenberg et al. - Phys.Letters B60, 305 (1976).
- 27) T. Dombeck et al. - Preprint ANL-HEP-PR-76-62 (1976).
- 28) S. Csorna et al. - Preprint VTL-PUB-35 (1977).
- 29) K. Moriyasu et al. - Preprint VTL-PUB-49 (1976).
- 30) A. Zieminski - Invited talk given at the 1st International Symposium on Hadron Structure and Multiparticle Production, Kazimierz, Poland (22-28 May 1977), Warsaw University Preprint.
- 31) V.A. Abramovsky, V.N. Gribov and O.V. Kancheli - Soviet J.Nucl.Phys. 18, 308 (1974).
- 32) N.N. Nikolaev and A.Ya. Ostapchuck - "Glauber Multiple Scattering Theory and Multiparticle Production Off Nuclei", submitted to Nuovo Cimento Letters.
- 33) M. Baker, H.J. Lubatti, E.O. Rogers and J.H. Weis - Phys.Rev. D17, 826 (1978).
- 34) N.N. Nikolaev - Phys.Letters B70, 95 (1977).
- 35) T. Kafka et al. - Phys.Rev. D16, 1261 (1977).
- 36) E.M. Levin and M.G. Ryskin - Soviet J.Nucl.Phys. 20, 280 (1975).
- 37) N.N. Nikolaev - JETP Letters 24, 348 (1976).
- 38) V.N. Gribov - Soviet Phys. JETP 29, 483 (1969).
- 39) V.N. Gribov - Lectures given at the 1st ITEP Annual Winter School on Particle and Nuclear Physics (January 1973), in Elementary Particles, Vol. 1, by Atomizdat, Moscow (1973).
- 40) A.A. Anselm - ibid, Vol. 3.

FIGURE CAPTIONS

- Figure 1 Diagram for the rescattering process.
- Figure 2 The structure of the inclusive spectrum in the rescattering events.
- Figure 3 The correspondence between the rescattering cross-section and the fan diagram of the Reggeon field theory.
- Figure 4 The y distribution of the rescatterings for the plateau-like spectra.
- Figure 5 The normalized spectra of γ quanta in the rescattering events [Ref. 28] are compared with the predictions of the fan diagram dominance model (broken curve), additive quark model (solid curve) and the eikonal model (dotted-dashed curve).
- Figure 6 The predicted excess multiplicities in the rescattering events are compared with the experimental data compiled in Ref. 30).
- Figure 7 A definition of the triple-Pomeron coupling G_{PPP} through the diffraction cross-section.
- Figure 8 Glauber's multiple scattering expansion for the hd elastic amplitude.
- Figure 9 The two-ladder production process which corresponds to the two-Pomeron exchange.
- Figure 10 The inelastic final states given by Glauber's double scattering amplitude. Note that there is no two-ladder production process.
- Figure 11 The Mandelstam non-planar two-Pomeron two-particle coupling which has a desired two-ladder production discontinuity.
- Figure 12 The integration contour in the Pomeron-particle energy s' plane. After the deformation of the integration contour, as shown by the arrow, one arrives at the dispersion integral of the discontinuity of M_2 .

- Figure 13 The normalized spectra of the secondary particles in the rescattering events predicted by the eikonal model. The curves labelled by 0 , ∇ , Δ correspond to the spectra of the leading (protons), secondary (pions) and positive (protons + positive pions) particles. The curve marked by $\square$ demonstrates the influence of the inelastic intermediate states on the spectrum of pions in the case of $\sigma_D = \sigma_{el}$. See Fig. 5 for comparison with the data.
- Figure 14 The normalized spectra of the charged particles in the rescattering events predicted by the additive quark model. See Fig. 5 for comparison with the experimental data.
- Figure 15 The contribution of the rescatterings to the two-particle correlation function, calculated in the fan diagram dominance approximation. The shaded area corresponds to the kinematically forbidden region.
- Figure 16 The contribution of the rescatterings to the two-particle correlation function in the eikonal model.
- Figure 17 The contribution of the rescatterings to the two-particle correlation function in the additive quark model.
- Figure 18 The contribution to the two-particle correlation function connected with the correlations in the input NN interactions.
- Figure 19 The additive quark model predictions for the KNO scaling functions in the rescattering events. The experimental multiplicity distributions shown were obtained in Ref. 23) by the analysis of the two-proton events.
- Figure 20 The estimated influence of the rescatterings on the mean transverse momenta of the secondary particles.
- Figure 21 Diagram for the rescattering cross-section. The crossed legs correspond to the on-mass-shell particles. The double-crossed lines correspond to the propagators which should be taken with the "wrong sign" prescription.
- Figure 22 A box diagram model for the six-point amplitude.

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