Ref.TH.2745-CERN

NON-EQUILIBRIUM IN THE VERY EARLY UNIVERSE

John Ellis
CERN -- Geneva

and

Gary Steigman
Bartol Research Foundation of the
Franklin Institute
Newark, DE 19711, U.S.A.

ABSTRACT

It is argued that the small horizon when the universe was close to the Planck temperature restricted the possible energies of elementary particles, preventing them from being in full thermal equilibrium. Grand unified particle interactions seems unlikely to have thermalized the universe much before the temperature fell to the mass scale of the baryon number violating bosons.

Ref.TH.2745-CERN
28 September 1979

Modern gauge theories of the different elementary particle interactions -- QCD¹⁾ for the strong interactions and the Weinberg-Salam²⁾ model for the weak and electromagnetic interactions -- are sufficiently convincing that one is led to contemplate^{3),4)} their grand unification into a theory of all elementary particle interactions. The energy scale at which this grand unification should take place is⁵⁾⁻⁷⁾ $0(10^{15})$ GeV, close to the Planck mass $m_P \approx 1.2 \times 10^{19}$ GeV. Simple grand unification models have some aesthetic appeal and phenomenological successes^{5),6)}. It is therefore natural to consider their implications for the evolution of the Universe at very early times when the temperature was very high: $T > 10^{15}$ GeV. During this period the Universe would have contained many heavy bosons (here called generically X bosons) whose interactions violate baryon number conservation. It has been suggested⁸⁾ that a C and CP violating component in the interactions and decays of these bosons may have generated an asymmetry between the quark and antiquark densities, which later became the observed net baryon asymmetry: $(\Delta B/N_Y)_0 = 0(10^{-9} \pm 1)$. Our purpose in this paper is to discuss more general aspects of the early evolution of the Universe in the context of grand unified theories and assess their impact on the generation of baryon number.

We first emphasize⁹⁾ that grand unified interactions and plausible patterns of quantum gravitational effects would not have been in equilibrium at temperatures between 10^{19} and 10^{16} GeV. We then point out that causality (if valid at this epoch) would impose a lower cut-off on the possible energy spectrum of particles when the temperature $\sim 10^{19}$ GeV: $E \geq E_{\min} = 0(5) M_P$. This makes it difficult to understand how the full Boltzmann energy distribution could have been filled. We then estimate the rate at which grand unified elementary particle interactions could have generated new particles, and find it questionable whether the Boltzmann distribution could have been filled out before the temperature dropped to $\sim 10^{16}$ GeV. This complicates estimation of the net baryon asymmetry⁸⁾, which conventionally involves the decays of heavy bosons as they drop out of an assumed thermal distribution.

We first address the question whether interactions in the Universe would have been in equilibrium at temperatures just below the Planck temperature T_P . The "known" grand unified elementary particle interactions would have had a $2 \leftrightarrow 2$ interaction rate

$$\Gamma = \left\langle \sum_{i,j,k,l} n_i n_j \sigma_{ij \rightarrow kl} v \right\rangle = n \left[\alpha^2 c \left(\frac{\hbar c}{kT} \right)^2 \right] \phi \quad (1)$$

where n_j is the number density of particles of species j and n is the total number density. In the cut-off distributions we will be considering fermions and bosons contribute equally:

$$n = g_T A \left(\frac{kT}{\hbar c} \right)^3 \quad (2)$$

In formula (2) g_T is the number of particle helicities [$g_T = 172$ in the minimal SU(5) model^{4),6)} with 3 generations of $\bar{5} + 10$ fermions] and A is a numerical factor for which a plausible range is 0.12 (for a Boltzmann distribution) to 10^{-2} (a typical non-equilibrium value as we shall see later). In Eq. (1) ϕ is a rate parameter defined by

$$\sum_{i,j,k,l} n_i n_j \langle \sigma_{ij \rightarrow kl} v \rangle = n^2 \left[\alpha^2 c \left(\frac{\hbar c}{kT} \right)^2 \right] \phi \quad (3)$$

If we just consider fermions and gauge bosons which have two helicity states $g_i = 2$, and factor out the internal degrees of freedom, then we have

$$\left[\alpha^2 c \left(\frac{\hbar c}{kT} \right)^2 \right] \phi \approx \left(\frac{2}{g_T} \right)^2 \sum_{i,j,k,l} \langle \sigma_{ij \rightarrow kl} v \rangle \quad (4)$$

We will assume that the high-energy cross-section for each reaction $ij \rightarrow kl$ is ^{*)} $\left[\alpha^2 c \left(\frac{\hbar c}{kT} \right)^2 \right]$ multiplied by a combinatorial factor for the number of contributing diagrams. We can either count these explicitly in an SU(5) model, or estimate the combinatorial factor by using the $1/N_c$ expansion developed for large unitary groups SU(N_c)¹⁶⁾. The first method gives

$$\phi = \alpha(20) \times (24)^2 \times \left(\frac{2}{g_T} \right)^2 \quad (5)$$

^{*)} There should be a factor of π in the numerator, but this is often counteracted by a numerical factor in the denominator.

and the second method gives an essentially identical result with 24 replaced by N_c^2 . We therefore use the estimate (5) for ϕ , corresponding to

$$\phi = \left(\frac{48}{g_\tau}\right)^2 \times O(20) = O(27) \quad (6)$$

for the simplest SU(5) model⁴⁾.

We need to compare the interaction rate (1) with the expansion rate, which is given by

$$\frac{t(T)}{t_p} = \left(\frac{3}{32\pi g_\tau B}\right)^{\frac{1}{2}} \left(\frac{T_p}{T}\right)^2 \quad (7)$$

in a homogeneous and isotropic radiation -dominated Robertson-Walker-Friedman (R-W-F) cosmology¹¹⁾. In Eq. (7) $t_p = \hbar/kT_p$ is the Planck time and B is a numerical coefficient appearing in the energy density

$$\rho = g_\tau B \left(\frac{kT}{c^2}\right) \left(\frac{kT}{\hbar c}\right)^3 \quad (8)$$

Combining (1) and (7) we see that

$$\Gamma t = \left(\frac{3g_\tau}{32\pi B}\right)^{\frac{1}{2}} A \alpha^2 \phi \left(\frac{T_p}{T}\right) \quad (9)$$

We therefore see that $\Gamma t > 1$ so that there is equilibrium with respect to $2 \rightarrow 2$ grand unified interactions only for

$$\left(\frac{T}{T_p}\right) < \left(\frac{3g_\tau}{32\pi B}\right)^{\frac{1}{2}} A \alpha^2 \phi \quad (10)$$

For the type of particle distributions we will be considering $A/\sqrt{B} \lesssim 1/5$, the value it would obtain in full thermal equilibrium, so that grand unified $2 \leftrightarrow 2$ interactions are only in equilibrium (using $g_T = 172$, $\phi = 2$ and $\alpha = 1/50$) when⁹⁾

$$T < 3 \times 10^{15} \text{ GeV} \quad (11)$$

The grand unified decay and inverse decay ($1 \leftrightarrow 2$) processes would have been negligible when $T \gg m_X$, the mass of the heavy baryon number violating bosons $\approx 5 \times 10^{14} \text{ GeV}$ ⁷⁾, since they are proportional to $m_X^2 / \sqrt{T^2 + m_X^2}$ ⁸⁾. Furthermore, when the temperature is above some critical temperature T_c which is probably within an order of magnitude of m_X the grand unification symmetry is exact and the X bosons are massless. We conclude⁹⁾ that grand unification interactions were not in equilibrium in the very early Universe.

What about quantum gravitational effects? They would look non-renormalizable at low temperatures $T \lesssim T_P$, with a rate characterized by the Planck mass scale^{8),9)}:

$$\Gamma_G = n \phi_G \left(\frac{T}{T_P} \right)^{m \geq 2} \text{ when } T \leq T_P \quad (12)$$

where ϕ_G is defined analogously to its grand unification counterpart ϕ (3). Combining (7) and (12) we see that for gravitational interactions

$$\Gamma_G t = \left(\frac{3g_T}{32\pi B} \right)^{\frac{1}{2}} A \phi_G \left(\frac{T}{T_P} \right)^{m+1} : T \leq T_P \quad (13)$$

It is clear from the form of Eq. (13) that at subsequent times $\Gamma_G t$ is less than its value at $T = T_P$, at which temperature it is plausibly $O(1)$. We therefore conclude⁹⁾ that quantum gravitational effects were probably out of thermal equilibrium when $T < T_P$, but could have been in equilibrium at $T \gtrsim T_P$.

Does this mean that the elementary particles have had a Boltzmann distribution at this temperature? Probably not. In a radiation dominated homogeneous and isotropic universe¹¹⁾ the diameter of the horizon increases with the age

of the Universe as $D_H = 4ct$. In the absence of acausal effects^{*)}, it is difficult to see how coherence could have been established beyond the horizon, so that one might anticipate a possible upper limit to the wavelengths of well-defined elementary particles:

$$\lambda \leq \lambda_{\max} = D_H \quad (14)$$

Corresponding to this longest wavelength there would be a minimum energy for particles within the horizon:

$$E \geq E_{\min} = \frac{\hbar}{4t} \quad (15)$$

Let us assume that quantum gravitational effects were sufficiently strong to establish a truncated thermal distribution among the particles present (15), but that for the epochs considered here there were no particles with lower energies. For $E_{\min}(t) > kT(t)$ this cut-off is important^{**)}: we hope it is a reasonable order-of-magnitude representation of the deviation from equilibrium to be expected at this early epoch.

With such a cut-off, the number densities of fermions and bosons are essentially the same:

$$n_{\pm} \rightarrow n_i = g_i A \left(\frac{kT}{\hbar c} \right)^3$$

where

$$A = \frac{1}{2\pi^2} \int_x^\infty y^2 e^{-y} dy = \frac{e^{-x}}{2\pi^2} (x^2 + 2x + 2) \quad (16)$$

*) Quantum gravity may in fact generate acausal effects, and all parts of the Universe seem to have started expanding at the same time, which argues for some acausal effect. This is beyond our present understanding.

***) At the grand unification mass of a few $\times 10^{14}$ GeV, this condition is $E \geq 10^{11}$ GeV. At the present epoch this condition excludes particles with $E \leq 10^{-43}$ GeV corresponding to 10^{-30} °K, whereas the black body radiation has a temperature $\sim 3^\circ\text{K}$,

and $x \equiv E_{\min}/kT$. By way of comparison, $A = A_{\text{eq}} \approx \frac{\zeta(3)}{\pi^2} \approx 0.12$ for a full equilibrium distribution. Similarly the energy densities

$$\rho_{\pm} \rightarrow g_{\pm} B \frac{kT}{c^2} \left(\frac{kT}{\hbar c} \right)^3$$

where $B = \frac{1}{2\pi^2} \int_x^{\infty} y^3 e^{-y} dy = \frac{e^{-x}}{2\pi^2} (x^3 + 3x^2 + 6x + 6)$ (17)

and for a full equilibrium distribution $B_{\text{eq}} \approx 3A_{\text{eq}} \approx 0.36$.

Using the time-temperature relation (7) for a radiation dominated R-W-F cosmology¹¹⁾, we can re-express the cut-off x as

$$x(t) \equiv \frac{E_{\min}(t)}{kT(t)} = \frac{1}{4} \left(\frac{t_p}{t} \right) \left(\frac{T_p}{T} \right) = \left(\frac{2\pi g_{\pm} B}{3} \right)^{\frac{1}{2}} \left(\frac{T}{T_p} \right) \quad (18)$$

If a full distribution were assumed so that $B = B_{\text{eq}}$, Eq. (18) tells us that we would have

$$x_{\text{eq}} \approx 11 \left(\frac{T}{T_p} \right) \quad (19)$$

which could only be $\ll 1$ (justifying the full equilibrium assumption) if the temperature T at which equilibrium was established were much less than T_p . We excluded this possibility in our previous analysis. If we assume an initial cut-off equilibrium at $T_i \approx T_p$ as our previous analysis suggests, then we can use Eq. (18) to solve for x_i , A_i and B_i , finding

$$x_i = x_p \approx 5 ; B_i = B_p \approx 0.07 ; A_i = A_p \approx 0.01 \quad (20)$$

We therefore conclude that the universe emerges from the epoch of quantum gravity at $T_i \simeq T_p$ with a non-equilibrium distribution of elementary particles strongly cut off at energies $E \lesssim 0(5) m_p$.

We now ask at what rate elementary particle interactions can generate new particles to fill out the Boltzmann distribution, and at what temperature full thermal equilibrium can be attained. We argued earlier that $1 \nleftrightarrow 2$ grand unified interactions are suppressed at temperatures $\gtrsim$ a few $\times 10^{15}$ GeV, and $2 \nleftrightarrow 2$ interactions clearly do not generate new particles! How about $2 \nleftrightarrow 3$ processes? We introduce analogously to the ϕ of Eq. (3) a rate factor ψ defined by

$$\sum_{i,j,k,l,m} n_i n_j \langle \sigma_{ij \rightarrow klm} v \rangle \equiv n^2 \left[\alpha^3 c \left(\frac{\hbar c}{kT} \right)^2 \right] \psi \quad (21)$$

The $1/N_c$ expansion¹⁰⁾ would suggest that $\psi = O(N_c) \phi$, and looking at explicit diagrams suggests to us that there may be an additional combinatorial factor of a few. We therefore guess that

$$\psi = O(25) \quad (22)$$

with a considerable uncertainty.

We now consider a specific arbitrary co-moving volume $V = V(t)$ which contains $N = N(t) = n(t)V(t)$ particles. N would be constant¹²⁾ in the case of full equilibrium, but in our case N increases (for $N < N_{eq}$) according to

$$\frac{1}{V} \frac{dN}{dt} = \beta n^2 \quad (23)$$

where

$$\beta = \psi \alpha^3 c \left(\frac{\hbar c}{kT} \right)^2 \quad (24)$$

Since $N = nV$ we can express Eq. (23) as

$$\frac{dN}{N^2} = \frac{\beta dt}{V^2} \quad (25)$$

Since $\beta \propto T^{-2} \propto t$, and the volume scales as the cube of the scale factor which is $\propto t^{\frac{1}{2}}$ for the radiation dominated early evolution considered here, we can integrate Eq. (25) to obtain

$$1 - \frac{N_i}{N} = 2 (\Gamma t)_i \left(\frac{T_i}{T} - 1 \right) \quad (26)$$

where $\Gamma = \beta n$ and

$$2 (\Gamma t)_i = \left(\frac{3g_{\tau} A_i^2}{8\pi B_i} \right)^{\frac{1}{2}} (\psi \alpha^3) \left(\frac{T_P}{T_i} \right) \quad (27)$$

In the interesting range of temperatures $T_i \approx T_P \gg T > m_X c^2/k$, equations (26) and (27) tell us that

$$1 - \frac{N_i}{N} \approx \left(\frac{3g_{\tau} A_i^2}{8\pi B_i} \right)^{\frac{1}{2}} (\psi \alpha^3) \left(\frac{T_P}{T} \right) \quad (28)$$

Putting in the usual numbers $g_{\tau} = 172$, $\alpha = 1/50$, A_i and B_i from Eq. (20), we finally get

$$1 - \frac{N_i}{N} \approx 1.6 \times 10^{-6} \psi \left(\frac{T_P}{T} \right) \quad (29)$$

We therefore see that if ψ is correctly estimated by Eq. (22), N is still $\approx N_i$ when T falls to a few $\times 10^{15}$ GeV, the temperature at which the X boson masses⁷⁾ can no longer be neglected. Thus the distribution of elementary particles does not seem to get fully thermalized before the temperatures at which the X bosons' masses start to distort their attempted Boltzmann distributions.

What are the implications of this conclusion for the baryon to photon ratio to be generated by B, C and CP violating processes when $T \lesssim m_X$? In an X boson decay model the baryon excess in each co-moving volume is related to the number of X bosons by

$$\Delta B = \sum N_X \quad (30)$$

The baryon excess can then be diluted by a factor up to A_i/A_{eq} due to the generation of other particles as their full Boltzmann distributions are built up during and after the decays of the X bosons. There might also be a significant Boltzmann suppression factor $(N_X/N_Y)_{eq}$ due to the finite X mass even if a thermal equilibrium distribution were established. Finally, there should be another, unrelated, dilution factor due the increase in the number of photons between the X boson decay and now, which has been estimated¹³⁾ to be

$$N_{\gamma 0}/N_{\gamma eq} \approx \left(\frac{11}{43} g_T \right) \quad (31)$$

We therefore find that the present ratio of baryons to photons could be as low as

$$\left(\frac{\Delta B}{N_{\gamma 0}} \right) \approx \sum \left(\frac{11}{43} g_T \right)^{-1} \left(\frac{A_i}{A_{eq}} \right) \left(\frac{N_X}{N_{\gamma}} \right)_{eq} \quad (32)$$

Specific grand unified models⁸⁾ calculate ζ , and we see from Eq. (32) that they should be shooting for a value 0(100 to 1000) times the observed value of $\left(\frac{\Delta B}{N_{\gamma}} \right) \approx 10^{-9 \pm 1}$. To help them on their way, we recall⁸⁾ that baryon number generation is only possible when thermal equilibrium is broken, and we find that disequilibrium may have existed at $T > m_X$, as well as for $T \lesssim m_X$ as has normally been assumed.

Clearly our analysis is subject to many uncertainties coming both from quantum gravity and from grand unified theories, and we hope that much refinement is possible. Nevertheless, it seems likely that the early universe passed through a non-equilibrium epoch. However this does not seem to require a qualitative change in the usual grand unified theory programme⁸⁾ for generating number.

ACKNOWLEDGEMENTS

One of us (J.E.) thanks A. Dolgov, M.K. Gaillard and S. Wolfram for interesting discussions, and Lawrence Berkeley Laboratory and the Bartol Research Foundation for hospitality while parts of this work were done. The other (G.S.) thanks the CERN Theory Division for hospitality during the initial stages of this work. The work at Bartol is supported by D.O.E. contract DE-A502-78ER05007.

REFERENCES

- 1) For reviews, see
H.D. Politzer - Phys. Reports 14C, 129 (1974);
W. Marciano and H. Pagels - Phys. Reports 36C, 137 (1978);
A. Peterman - Phys. Reports 53C, 157 (1979).
- 2) S. Weinberg - Phys. Rev. Lett. 19, 1264 (1967);
A. Salam - Proc. 8th Nobel Symposium, ed. N. Svartholm (Almqvist and Wiksells, Stockholm, 1968) p. 367.
- 3) J.C. Pati and A. Salam - Phys. Rev. Lett. 31, 661 (1973).
- 4) H. Georgi and S.L. Glashow - Phys. Rev. Lett. 32, 438 (1974).
- 5) H. Georgi, H.R. Quinn and S. Weinberg - Phys. Rev. Lett. 33, 451 (1974).
- 6) M.S. Chanowitz, J. Ellis and M.K. Gaillard - Nucl. Phys. B128, 506 (1977);
A.J. Buras, J. Ellis, M.K. Gaillard and D.V. Nanopoulos - Nucl. Phys. B135, 66 (1978).
- 7) T. Goldman and D.A. Ross - Phys. Lett. 84B, 208 (1979);
W. Marciano - Phys. Rev. D20, 274 (1979).
- 8) A. Yu. Ignatiev, N.V. Krosnikov, V.A. Kuzmin and A.N. Tavkhelidze, Phys. Lett. 76B, 436 (1978);
M. Yoshimura, Phys. Rev. Lett. 41, 381 (1978);
S. Dimopoulos and L. Susskind, Phys. Rev. D18, 4500 (1978);
A.D. Sakharov, Zh. Eksp. Teor. Fiz. 76, 1172 (1979);
B. Toussaint, S.B. Treiman, F. Wilczek and A. Zee, Phys. Rev. D19, 1036 (1979);
J. Ellis, M.K. Gaillard and D.V. Nanopoulos, Phys. Lett. 80B, 360 (1979);
S. Weinberg, Phys. Rev. Lett. 42, 850 (1979);
S. Dimopoulos and L. Susskind, Phys. Lett. 81B, 416 (1979).
- 9) J. Ellis, M.K. Gaillard and D.V. Nanopoulos, Ref. 8) and to be published.
- 10) G. 't Hooft - Nucl. Phys. B72, 461 (1974).
- 11) S. Weinberg, Gravitation and Cosmology, New York: Wiley (1972).
- 12) G. Steigman, Ann. Rev. Nucl. and Part. Sci. 29, 313 (1979);
A.D. Dolgov and Ya.B. Zeldovich, Cosmology and Elementary Particles, ITEP/Acad. of Sciences Moscow Preprint (1979).
- 13) G. Steigman, K.A. Olive and D.N. Schramm, Phys. Rev. Lett. 43, 239 (1979).

14.