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BINDING EFFECTS IN COHERENT π^0 PHOTOPRODUCTION
ON DEUTERIUM

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ABSTRACT

We have calculated the threshold amplitude for π^0 photoproduction on deuterium including binding corrections and the pion mass difference. Due to large cancellations among the binding corrections our result is remarkably well described by the static approximation. The main corrections come from the P wave contribution to the single scattering term.

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Threshold π^0 photoproduction by protons and deuterons has recently been measured at Saclay¹⁾. The aim of the experiment was to determine the threshold amplitude for π^0 photoproduction on neutrons. The single scattering contribution to the deuteron amplitude is very small. This fact follows from current algebra and PCAC. Due to this circumstance, the double scattering contributions to the deuteron amplitude become more important than the single scattering contributions. Consequently, they must be calculated with great care. Previous theoretical estimates²⁾ were in disagreement with experiment. In the present investigation we retain certain double scattering terms which were neglected by those authors and we get good agreement with experiment. Our method is identical to the one used in pion-deuteron scattering³⁾. Further details of our calculation are given in Ref. 4).

Our calculation is a multiple scattering calculation. As input we need the elementary nucleon amplitudes. At threshold the nuclear photoproduction amplitude is given by

$$F = i \vec{\sigma} \cdot \vec{\epsilon} E \quad (1)$$

where $\vec{\epsilon}$ is the polarization vector of the photon. We take for E the values obtained with the pseudovector pion-nucleon coupling in the Born approximation⁵⁾. The three isospin combinations are in this model: $E^{(-)} = G = 0.021/m(\pi^+)$; $E^{(0)} = -0.070G$ and $E^{(+)} = -0.048G$. Unfortunately, it is not sufficient to know the photoproduction amplitudes at threshold. The energy dependence close to threshold is important. We assume it is given by Watson's theorem. Then, the energy dependence in terms of the pion-nucleon momentum q is given by

$$E(q) = \frac{1}{1 - i a q} E \quad (2)$$

where a is the pion-nucleon scattering length. In this expression, both E and a are matrices in isospin space. We shall only retain terms which are, at most, linear in a . The higher order terms are not retained in the multiple scattering equations either. For the numerical illustration we have taken Höhler's⁶⁾ scattering lengths, i.e., $a^{(+)} = (-0.018 \pm 0.005)\text{fm}$ and $a^{(-)} = (0.130 \pm 0.004)\text{fm}$.

We define the deuteron photoproduction amplitude F_D by extracting certain trivial kinematical factors. The spin averaged cross-section is given by

$$\frac{d\sigma_D}{d\Omega} = \frac{q}{k} \frac{8}{3} \left| \frac{1 + m(\pi^0)/M_N}{1 + m(\pi^0)/M_D} \right|^2 |F_D|^2 \quad (3)$$

where k is the photon momentum in the deuteron c.m. system. At threshold, the pion momentum q is zero. In the static approximation the single scattering contribution to F_D is

$$F(SIA) = \frac{1}{2} \{ E(\pi^+ \rightarrow \pi^0 p) + E(\pi^+ \rightarrow \pi^0 n) \} S(k/2) \quad (4)$$

$$= E^{(+)} S(k/2) = -0.038 G,$$

where $S(k/2)$ is the deuteron form factor. At threshold we have $k = 0.661 \text{ fm}^{-1}$. Using the Mc Gee⁷⁾ wave function for the deuteron (neglecting the D state) we get $S(k/2) = 0.777$, so that $F(SIA) = -0.038G$.

The binding or non-static corrections to $F(SIA)$ are obtained by observing that the nucleon on which the photoproduction takes place is not at rest. Thus, the relative pion-nucleon momentum q in Eq. (2) is non-zero. The relative kinetic energy becomes

$$T_{kin} = -\epsilon_D - \frac{\bar{p}^2}{2M_N} - \frac{\bar{p}^2}{2(M_N + m(\pi^0))} \quad (5)$$

where ϵ_D is the deuteron binding energy and $\bar{p}$ the momentum of the spectator nucleon in the over-all c.m. system. The relative pion-nucleon momentum is imaginary $q = ik$. Neglecting smaller terms of order $m(\pi^0)/M_N$ we have

$$\mathcal{H} = \alpha \sqrt{\frac{2m(\pi^0)}{M_N}} \sqrt{1 + \bar{p}^2/\alpha^2} \quad (6)$$

with $\alpha^2 = M_N \epsilon_D$. Since the energy dependence is described by Eq. (2), we get a binding correction to the single scattering term

$$F(SBC) = -\alpha \sqrt{\frac{2m(\pi^0)}{M_N}} \{ a^{(+)} E^{(+)} + 2 a^{(-)} E^{(-)} \} I_M \quad (7a)$$

$$I_M = \int d\bar{p} \varphi(\bar{p}) \varphi(\bar{p} - k/2) \sqrt{1 + \bar{p}^2/\alpha^2} = 1.81. \quad (7b)$$

Numerically, $F(SBC) = -0.059G$ and more important than the single scattering term is the static approximation. This is a subtle effect and shows that great care is needed.

The evaluation of the double scattering term is slightly simpler. Here it is not necessary to account for the energy dependences in the individual amplitudes since this term is already of second order. The inverse propagator is

$$D(\vec{p}, \vec{p}') = \epsilon_0 + \frac{1}{2m(\pi^0)} (\vec{p} - \vec{p}')^2 + \frac{1}{2M_N} (\vec{p}^2 + \vec{p}'^2) \quad (8)$$

and the complete double scattering contribution

$$F(DS) = \left(1 + \frac{m(\pi^0)}{M_N}\right) \{a^{(+)} E^{(+)} - 2 a^{(-)} E^{(-)}\} R \quad (9a)$$

$$R = \frac{1}{2\pi^2} \int d^3\vec{p} d^3\vec{p}' \varphi(\vec{p}') \frac{1}{D(\vec{p}, \vec{p}')} \varphi(\vec{p} - \vec{k}/2) \quad (9b)$$

The kinematical factor in expression (9a) originates from the reduced mass factor in the elementary amplitude. Numerically, we have $F(DS) = -0.066G$. This is not the true value of the double scattering term since also $F(SBC)$ is of the double scattering type. We shall clarify the picture by following the approach of Ref. 3). To this end we write R as a sum of two terms, namely the static approximation and the binding correction

$$R = \left\langle \frac{e^{i\vec{k} \cdot \vec{r}/2}}{r} \right\rangle - \alpha \sqrt{\frac{2(m(\pi^0))}{M_N}} I_D = 0.224 f_m^{-1} \quad (10)$$

For the Mc Gee deuteron, the first term is $0.398 fm^{-1}$ and $I_D = 1.39$. Corresponding to the subdivision of Eq. (10) we have a corresponding subdivision of the double scattering term $F(DS) = F(DIA) + F(DBC)$. The numerical value for the binding correction is $F(DBC) = +0.052G$. Comparing this with the binding correction to the single scattering term $F(SBC) = -0.059G$ we observe a remarkable cancellation. It is not a coincidence. Calculating I_D to leading order in $m(\pi^0)/M_N$ gives exactly I_M . The only difference between $F(SBC)$ and $F(DBC)$ is in the isospin combinations of the nucleon amplitudes. The cancellation phenomenon therefore only applies to the $a^{(-)} E^{(-)}$ combination. There is a corresponding result for the small $a^{(+)} E^{(+)}$ combination but to get it we must calculate also a triple scattering term which has a nucleon-nucleon collision in between the photoproduction and pion-nucleon steps. Since this term is insignificant in the present application we do not discuss it further but refer to Refs. 3) and 4) for the details.

Summing up we have

$$F_D = F(SIA) + F(DIA) + \{F(SBC) + F(DBC)\} \quad (11)$$

with

$$F(SIA) = -0.038 G \quad (12a)$$

$$F(DIA) = -0.118 G \quad (12b)$$

$$F(SBC) + F(DBC) = -0.007 G. \quad (12c)$$

Clearly the residual binding correction is quite small, being reduced by a factor $m(\pi^0)/2M_N$ due to the cancellation.

Previous investigations of threshold photoproduction²⁾ used $F(SIA)$ for the single scattering term and $F(DS)$ for the double scattering term. The error in this approach is substantial. Physically, it means retaining binding corrections in the double scattering term but not in the single scattering term.

There are several small corrections to the above result. We shall in turn discuss the modifications due to: i) the P wave term in the photoproduction amplitude; ii) the off-shell form factor in the pion-nucleon amplitude; and iii) the mass difference between the charged and neutral pions.

The origin of the P wave term is the fact that the deuteron c.m. system differs from the nucleon c.m. system. In our calculation we have neglected the off-shell dependence in the energy and have interpreted the momenta in the photoproduction amplitude as the appropriate relative momenta (non-relativistically). In the nucleon c.m. system the momentum in the initial state is $\bar{k}_N = \bar{k} - \zeta \bar{p}$ and in the final state $\bar{q}_N = -\zeta \bar{p}$ where $\zeta = m(\pi^0)/(M_N + m(\pi^0))$ and $\bar{p}$ the momentum of the spectator nucleon. Thus, we replace Eq. (1) by the effective single scattering term

$$F = i\vec{\sigma} \cdot \vec{\epsilon} \left\{ E(q) - \zeta \bar{p} \cdot \vec{k} B \right\} \quad (13)$$

The only isospin combination which enters is $B^{(+)}$ and we have $B^{(+)} = 0.495G/m(\pi^+)^2$. Integration over the Fermi momentum $\bar{p}$ gives a P wave contribution to the single scattering term

$$F(PS) = -\frac{1}{4} \zeta k^2 B^{(+)} S(k/2) = -0.011 G, \quad (14)$$

i.e., a 10% correction to the over-all amplitude. In principle, there is also a correction coming from the frame dependence of the polarization vector of the photon. We have chosen ϵ_μ to be $(0, \vec{\epsilon})$ in the deuteron c.m. system. In the nucleon c.m. system there will be an additional time component. The correction from this term is, however, negligible.

Up to now we neglected the off-shell form factors in the pion-nucleon and photoproduction amplitudes. We have investigated their effects. The form factors were fixed by assuming a separable pion-nucleon interaction fitted to the pion-nucleon phase shifts, separately for the S_{11} and S_{31} channels. The form factors change $F(\text{SBC})$ and $F(\text{DS})$ by small amounts. The new results are

$$F(\text{SBC}) = -0.057 G \quad (15a)$$

$$F(\text{DS}) = -0.070 G. \quad (15b)$$

Including also the P wave, our revised prediction for the deuteron amplitude becomes: with form factors $F_D = -0.176G$; without form factors $F_D = -0.174G$.

The over-all influence of the form factors is small. In the individual terms that is not so. If, e.g., we had used the form factor for the S_{11} channel the final result would have been smaller by about 10%.

In the Saclay experiment one is not measuring the absolute value of the deuteron amplitude but rather the ratio to the proton amplitude $\gamma p \rightarrow \pi^0 p$. As pointed out by Laget⁸⁾ this amplitude is strongly affected by the pion mass difference $\Delta m_\pi = 4.61 \text{ MeV}$. The reason is that the direct amplitude for $\gamma p \rightarrow \pi^0 p$ is very small. As a result one must also consider the channel $\gamma p \rightarrow \pi^+ n \rightarrow \pi^0 p$ via unitarity, Eq. (2). The detailed expression is obtained by interpreting q as the pion-nucleon momentum in the appropriate channel. Since $M(\pi^+ n) > M(\pi^0 p)$ the momentum in the intermediate $\pi^+ n$ channel is imaginary, $q = i\kappa$. Neglecting insignificant contributions, the physical $M(\gamma p \rightarrow \pi^0 p)$ amplitude becomes

$$\begin{aligned} M(\gamma p \rightarrow \pi^0 p) &= E(\gamma p \rightarrow \pi^0 p) - \kappa A(\pi^+ n \rightarrow \pi^0 p) E(\gamma p \rightarrow \pi^+ n) \\ &= -0.118 G - 0.050 G = -0.168 G, \end{aligned} \quad (16)$$

using $\kappa = 0.21 \text{ fm}^{-1}$. Hence, we conclude that the mass difference is extremely important.

In the deuteron case the mass difference can be neglected. It enters the main terms proportional to $a^{(-)} E^{(-)}$ which have a charged pion in the intermediate state. There are two contributions, one from the double scattering term and one from the single scattering term. In the single scattering term we must change the κ of Eq. (6) by making the replacement $\alpha^2 \rightarrow \alpha^2 + M_N \Delta m_\pi$. The new value for the binding correction is $F(\text{SBC}, \Delta m_\pi \neq 0) = -0.071G$. In the double scattering term we must change the propagator in Eq. (8) by making the replacement $\epsilon_D \rightarrow \epsilon_D + \Delta m_\pi$. The transformation is the same as for the single scattering term. The new binding correction becomes $F(\text{DBC}, \Delta m_\pi \neq 0) = +0.058G$. The sum of the new binding corrections differs from our previous result by $-0.006G$ and can be

neglected. Mathematically, the cancellation is still complete to leading order in $m(\pi^0)/M_N$ since the substitutions in the two terms are the same.

To summarize, we find that there are large cancellations between the various binding corrections. Our final result is remarkably simple. The static approximation with single and double scattering terms gives an excellent description of the deuteron photoproduction amplitude. This is still true when the pion mass difference is considered.

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