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HOW TO REGULARIZE THE INFRA-RED AND MASS SINGULARITIES IN QCD

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E R R A T U M

Page 6 is changed into :

Equations (6), (7), (8) lead to the same result for $\Delta \frac{DY}{qq}$ with

$$a - b_q - b_{\bar{q}} = \left(1 + \frac{4\pi^2}{3}\right) \quad (9)$$

which was also obtained by using dimensional regularization ^{1),8)}.

We now are able to explain why the incorrect off-shell assignment ^{1), 6a), 6b)} lead to discrepancies if compared with the on-shell results : it is inconsistent with the double-cut-rule. All external legs of the DY diagrams (Fig. 2) were here given an off-shell mass $\sqrt{m^2 - \mu^2}$, different from the internal propagator mass m , in order to prevent anomalous cuts.

This off-shell assignment leads to the values :

$$a = \left(-4 + \frac{5\pi^2}{3}\right) \quad b_q = \left(-\frac{5}{2} - \frac{\pi^2}{3}\right) = b_{\bar{q}} \quad (10)$$

$$a - b_q - b_{\bar{q}} = \left(1 + \frac{7\pi^2}{3}\right)$$

The same value for $a - b_q - b_{\bar{q}}$ is also obtained by choosing $m^2 = 0$ ¹⁾. However, in the latter case a is changed into $(-1 + \pi^2)$ and $b_q, b_{\bar{q}}$ are changed into $(-1 - (2\pi^2/3))$. As already mentioned above, this choice is inconsistent with the cutting rule. Taking as an example Fig. 2a with $m_1^2 = m_3^2 = m^2 - \mu^2$ and $m_2^2 = m_4^2 = m^2$ the double cut rule automatically determines the mass assignment in all other graphs which, as one immediately recognizes, disagrees with the old off-shell choice. Notice that in some diagrams the external quark lines are on-shell. As stated above, this introduces anomalous cuts which are responsible for the needed $-\pi^2$ factor of a in Eq. (10) in order to arrive at the correct result of Eq. (6).

Comparison with Eq. (6) therefore reveals that the δ coefficients in Eq. (2) remain unchanged. The explanation is clear : since the DY graphs represent the absorptive part of the virtual Compton amplitude, all internal divergences are cancelled by adding the vertex graphs to real gluon production (e.g., Fig. 3a₁ + Fig. 3a₅, with $m_3^2 = m_4^2 = m^2 - \mu^2$, $m_1^2 = m_2^2 = m^2$) ; the sum reveals only initial state M divergences (but not infra-red divergences). Having set $\mu^2 \rightarrow 0$, the M divergences of the virtual Compton amplitude are then seen to agree with those obtained when using on-shell regularization.

HOW TO REGULARIZE THE INFRARED AND MASS SINGULARITIES IN QCD

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A B S T R A C T

The parton cross-sections in perturbative-QCD are mass(M) and infrared(IR) divergent. The singularities are prevented by on/off-shell parton masses or by dimensional regularization. All mass assignments must consistently be chosen in agreement with Kinoshita's double-cut-rule. The "off-shell" assignment, often used in the past, is incorrect. We demonstrate that Drell-Yan mass factorization in order- α_s is independent of all regularizations; the coefficient function is unique.

Several articles¹⁾⁻⁴⁾ recently aimed at determination of the deep-inelastic (DI) and Drell-Yan (DY) coefficient function in the asymptotic freedom (AF) region of QCD. They are calculated from the perturbative parton cross-sections through cancellation of the IR-singularities and subsequent M-factorization. The individual contributions to the parton cross sections can be M- and IR-singular (we leave aside the UV-problem). All M- and IR-divergences are prevented by appropriate M-regularization, the most familiar one being "on-and off-shell", or by dimensional regularization. Each of them renders a different perturbative cross-section but M-factorization is expected to leave a unique regularization independent form. In DI-scattering all three methods indeed give identical results. In the DY-process on-shell calculation agrees with dimensional regularisation, but the off-shell result is different, and here is where the problem starts.

Several groups^{3a), 3b), 1)} have used the off-shell mass assignment each providing a new answer due to calculation mistakes, incorrect argumentation and/or assumptions. All of them depart from an inconsistent "off-shell" mass assignment. It is incorrect to assume that a selected external quark line has in all diagrams one and the same off-shell mass. The mass assignment cannot be chosen at will but must be conform with Kinoshita's⁵⁾ double-cut-rule! This insight will allow us to resolve the DY-discrepancy.

We first introduce the structure of the order- α_s parton cross-sections for DI-scattering and the DY-process. Straightforward calculation (in the Feynman gauge) gives¹⁾:

$$\hat{\sigma}_{q\bar{q}}^{DY} = \frac{4}{3} \frac{\alpha_s}{2\pi} \left[\sum_{j=1}^2 \left\{ \frac{1+z^2}{(1-z)_+} \ln \frac{-(1-z)Q^2}{z^2 M_j^2(z)} - 1 + z + P_j(z) \right\} + a \delta(1-z) \right] \quad (1)$$

$$\hat{\sigma}_{q,j}^{DI} = \frac{4}{3} \frac{\alpha_s}{2\pi} \left[\left\{ \frac{1+z^2}{(1-z)_+} \ln \frac{-Q^2}{z^2 M_j^2(z)} + 2 + 3z - \frac{3/2}{(1-z)_+} + P_j(z) \right\} + b_j \delta(1-z) \right] \quad (2)$$

with

$$M_j^2(z) = \bar{z}^{-1} (1-z)^{-1} \left\{ z(1-z) p_j^2 - z p^2 - (1-z) m_0^2 \right\} \quad (3)$$

$$H_j(z) = \{ M_j^2(z) \}^{-1} \{ 4m_0 m_j - m_0^2 + p_j^2 - p^2 \} \quad (4)$$

Eq. (1) defines the DY parton cross-section for $q\bar{q} \rightarrow \mu^+ \mu^- \dots$ with $z \equiv \tau/x_1 x_2$, $\tau \equiv Q^2/s$ and the parton momentum fractions $x_1 x_2$. Similarly, the DI-parton cross-section (or structure function) in Eq. (2) describes the sub-process $q \gamma^* \rightarrow q + \dots$ with $z \equiv -q^2/2p_j q$. p_j stands for the incoming parton momentum corresponding to a quark (for $j = 1$) or anti-quark (for $j = 2$). We therefore use the short notation $d\sigma_q^{DI}$, b_q , $b_{\bar{q}}$ etc. The expressions between curly brackets are referred to as "hard-gluon" parts since they arise from the radiative gluon diagrams with an upper cut-off in the z -integration in order to prevent IR-and/or M-divergence. It is convenient to use the $1/(1-z)_+$ - procedure instead. a and b_i are referred to as "soft/virtual gluon" contributions. Both the hard part [via $M_j^2(z)$ and $A_j(z)$] as well as the δ -coefficient depend on the chosen mass assignment which was introduced in order to obtain finite cross-sections.

Careful evaluation of the M-factorisation⁶⁾ leaves the following DY-correction term:

$$\begin{aligned} \Delta_{q\bar{q}}^{DY} &\equiv \hat{\sigma}_{q\bar{q}}^{DY} - \hat{\sigma}_q^{DI} - \hat{\sigma}_{\bar{q}}^{DI} \\ &= \frac{4}{3} \frac{\alpha_s}{2\pi} \left[\left\{ 2 \frac{1+z^2}{(1-z)_+} \ln(1-z) - 4z - 6 + \frac{3}{(1-z)_+} \right\} \right. \\ &\quad \left. + (a - b_q - b_{\bar{q}}) \delta(1-z) \right] \end{aligned} \quad (5)$$

The Drell-Yan cross section with order α_s non-leading corrections is calculated with the simple DY-formula using scale dependent momentum distributions and the correction term (5) included in the integrand. The hard gluon part (in curly brackets) no longer depends on the M- or IR-regularizations. It, however, is more difficult to expose the uniqueness of the remaining soft/virtual-part. We shall confine ourselves in the rest of this paper to demonstrate how a and b are correctly evaluated in order to obtain the same $\Delta_{q\bar{q}}^{DY}$ for all regularizations methods.

We realize that DY M-factorization consists in the subtraction of M-singular terms from different reactions. It therefore is essential that M-regularization is carried out consistently, which means that one and the same mass assignment is used. For pedagogical reasons we briefly go back to the Feynman graphs contributing to DI-scattering. All final state M-singularities here can be generated from M-finite Feynman-diagrams, the virtual Compton amplitude, by imposing cuts (à la Cutkosky). The correct mass choice therefore is effected on this latter level before cutting. What is the more general procedure also valid for the Drell-Yan process? Kinoshita's double-cut-rule^{5),7)}, which resulted from analysis of the analyticity properties of Feynman graphs with respect to their internal and external masses, gives the clue. It starts from a vacuum-bubble (Fig. 1) and introduces cuts on two groups of lines, called A and B, generating in this way products of Feynman amplitudes. They correspond to transition probabilities of physical processes with A and B as their initial and final states. The same vacuum-bubble therefore can describe different physical processes. Appropriate cuttings generate from Fig. 1 all transition probabilities following from the lowest order Feynman diagrams in DY-, DI- and e^+e^- processes, i.e. $q + \bar{q} \rightarrow \gamma^* + g$ (Fig. 2), $\gamma^* + q \rightarrow q + g$ (Fig. 3), $e^+e^- \rightarrow \gamma^* \rightarrow q\bar{q} + g$ (Fig. 4). In A and B all partons are on-shell. Phase space integration over the group of final state particles brings us to the parton cross-sections. The vacuum-bubble stands representative for a vacuum-to-vacuum amplitude whose discontinuity is taken by cutting lines. If all particles are massless it cannot be cut since some of the contributions to the discontinuity will be divergent although the complete sum is finite. The divergences cancel since the external and internal lines become degenerate in some region of phase space which, according to the KLN-theorem, results in IR- and M-finite expressions. All IR- (M-) singularities were prevented by introducing auxiliary masses in the cut-diagrams (viz. cross sections) - a procedure which is very dangerous as we will see later on. The correct regularization starts from the vacuum-bubble giving each line between two vertices a number. Notice that a quark line between two photon vertices represents the same quark; in the vacuum-bubble of Fig. 1 we therefore have a quark-q line (left) and a quark- $\bar{q}$ line (right). Regularization is achieved by introducing on each line an auxiliary mass (which may vanish) such that all cut-graphs exist. Once the mass assignment on the vacuum-bubble is fixed it holds consistently for all cut-graphs. We now can immediately apply the KNL-theorem. Since summation over the final states (lines B) is carried out, all

IR-singularities cancel among the DI-graphs and the DY-graphs separately. If, furthermore, summation over the initial states (lines A) is formed, all mass singularities disappear and we are back at the doubly-cut vacuum-to-vacuum amplitude which, since singularity-free, permits the zero-mass limit.

Whilst determining $\Delta_{q\bar{q}}^{DY}$ we do not sum over A and conclude that the DI and DY M-singularities remain. Since in both processes the M-divergences appear in a universal way, the above correction term is unique and must be independent of the regularization procedure. We explain "universal". Consider Fig. 2 for the DY- and Fig. 3 for the corresponding DI-process which both can be seen to originate from the same vacuum-bubble Fig. 1a. If the DI-diagrams a_1 and a_5 are added and similarly a_2 to a_6 , all (IR- and M-) divergence associated with particles 3,2,5 is cancelled whereas the M-divergence due to particles 4,1,5 remains. The opposite happens if a_3 is added to a_7 and similarly a_4 to a_8 ; the M-divergence 3,2,5 survives. Whilst adding the DY-diagrams we cancel all IR- and internal M-divergences. The result is factorizable in two terms, one containing all M-divergence due to lines 4,1,5 (quark-q) and similarly the other due to lines 3,2,5 (quark-q). These divergences appear as logarithms with the same coefficient in the DI- and DY-process. This illustrates "universal". If the DY- and DI- mass divergences are mutually subtracted we can safely proceed to the zero-mass-limit. It is uniform in the sense that it does not depend on the auxiliary mass assignment. We emphasize that this statement is only correct if the corresponding DI- and DY-processes are derived from the same vacuum-bubble with the same consistent mass assignment concerning lines 1 to 5. If this constraint is not observed the DY-correction term will not be unique.

We introduce the correct mass assignment. The "on-shell" regularization uses one quark mass m (vanishing or not) with $p_i^2 = m^2$ for all external lines, and a gluon propagator mass λ . This choice is in agreement with the double-cut-rule. For $\lambda^2 \ll m^2 \neq 0$, λ can be seen to prevent the IR-divergences and m all M-singularities; for $m^2 = 0$ λ regularizes both. In the "off-shell" method the singularities are prevented by going off-shell with the external lines. In the past one incorrectly assigned to all diagrams the same external off-shell masses; $p_i^2 < m^2$ was chosen in order to prevent the anomalous cuts. Proceeding now correctly we go back to the vacuum-bubble and introduce an auxiliary mass $\sqrt{m^2 + \mu^2}$ on some appropriate lines instead of m .

After cutting one finds for a selected external line the off-shell mass $p^2 = m^2 + \mu^2$ in some of the diagrams but $p^2 = m^2$ in others. If $\mu^2 \ll m^2 \neq 0$, μ serves as IR- and m as M-regulator whereas for $m^2 = 0$ μ takes the role of both. Notice that the off-shell method always involves anomalous cuts (due to the difference in masses) which are completely cancelled in the correction term.

In the following we determine a and b of Eq. 5 assuming off-shell mass assignment with $\mu^2 \neq 0$; in the first calculation we set $m^2 \neq 0$, and use $m^2 \neq 0$ in the second. In both cases we compare with the analogous on-shell results. The auxiliary masses for off-shell calculation are defined by :

$$\text{Fig. 1.a} \quad m_1^2 = m_2^2 \equiv m^2 \neq 0, \quad m_3^2 = m_4^2 \equiv m^2 + \mu^2$$

$$\text{Fig. 1.b} \quad m_7^2 = m_9^2 = m_{10}^2 \equiv m^2, \quad m_8^2 \equiv m^2 + \mu^2$$

$$\text{Fig. 1.c} \quad m_{11}^2 = m_{13}^2 \equiv m^2 + \mu^2, \quad m_{12}^2 = m_{14}^2 \equiv m^2$$

$$m_5^2 = \lambda^2 = 0, \quad m_6^2 = q^2$$

giving

$$a = \left(-4 + \frac{2\pi^2}{3}\right), \quad b_{\bar{q}} = \left(-\frac{5}{2} - \frac{\pi^2}{3}\right) = b_{\bar{q}} \quad (6)$$

Had we chosen $m^2 = 0$ instead, the result would be changed to

$$a = \left(-\frac{11}{4} + \frac{\pi^2}{3} - \frac{3\pi i}{2}\right)$$

$$b_{\bar{q}} = \left(-\frac{15}{8} - \frac{\pi^2}{2} - \pi i\right), \quad b_{\bar{q}} = \left(-\frac{15}{8} - \frac{\pi^2}{2} - \frac{\pi i}{2}\right) \quad (7)$$

Due to the positiveness of μ^2 anomalous cuts appear in several of the diagrams.

Calculating those terms which have no anomalous cuts results in $b_1 = (-\frac{11}{4} - \frac{\pi^2}{3})$ whereas the anomalous cut pieces contribute $b_2 = (-1 - 2\frac{\pi^2}{3} - \frac{3\pi}{2}i)$. Changing μ^2 from positive to negative values would remove these cuts in some diagrams but automatically re-introduce them in others. If $m^2 \neq 0$, μ^2 replaces only the IR-singularities, and since these are separately cancelled in the DI- and DY-cross-sections, we may not expect anomalous cuts for this case [see Eq. (6)]. The analogous on-shell calculation²⁾ uses the mass-assignment

$$m_i^2 \equiv m^2 \quad (i=1, \dots, 14), \quad m_5^2 \equiv \lambda^2 \neq 0, \quad m_6^2 \equiv q^2$$

If $m^2 \neq 0$ we find the result of Eq.(6). Instead, for $m^2 = 0$ we have:

$$a = (-\frac{7}{2}), \quad b_q = (-\frac{9}{4} - \frac{4\pi^2}{3}) = b_{\bar{q}} \quad (8)$$

Eqs. (6), (7) and (8) lead to the same result for $\Delta_{q\bar{q}}^{DY}$ with:

$$a - b_q - b_{\bar{q}} = (1 + \frac{4\pi^2}{3}) \quad (9)$$

which was also obtained by using dimensionality regularization.

We now are able to explain why the incorrect off-shell assignment^{1)3.a)3.b)} leads to discrepancies if compared with the on-shell results: it is inconsistent with the double-cut-rule. The external legs of all DY-diagrams (Fig. 2) were here given an off-shell mass $\sqrt{m^2 - \mu^2}$, different from the internal propagator mass m , in order to prevent anomalous cuts; this choice is inconsistent with the cutting rule. Taking as an example Fig. 2.a₄ with $m_1^2 = m_3^2 = m^2 - \mu^2$ and $m_2^2 = m_4^2 = m^2$ the double-cut-rule automatically determines the mass assignment in all other graphs which as one immediately recognizes disagrees with the old off-shell choice. Notice that in some diagrams the external quark lines are on-shell (eg. Fig. 2.a₃); this, as stated above, introduces anomalous cuts which are responsible for the needed π^2 -factor in a in order to arrive at the correct result of Eq.(6). We analyse the changes whilst going from the incorrect to the correct off-shell assignment. The earlier (wrong) values of $a = (-4 + \frac{5\pi^2}{3})$ and $b_q, b_{\bar{q}} = (-\frac{5}{2} - \frac{\pi}{3})$, giving $a - b_q - b_{\bar{q}} = 1 + \frac{7\pi^2}{3}$, were found to be the same

for $m^2 = \text{finite or vanishing}$. Comparison with Eq. (6) therefore reveals that the δ -coefficients in Eq. (2) remain unchanged. The explanation is clear: since the DI-graphs represent the absorptive part of the virtual Compton amplitude, all internal divergences are cancelled by adding the vertex graphs to real gluon production (e.g., Fig. a₁ + Fig. 3.a₅, with $m_3^2 = m_4^2 = m^2 - \mu^2$, $m_1^2 = m_2^2 = m^2$); the sum reveals only initial state M-divergences (but no IR-divergences). Having set $\mu^2 \rightarrow 0$, the M-divergences of the virtual Compton amplitude then are seen to agree with those from using on-shell regularization.

Applying the old off-shell mass assignment on $e^+ e^- \rightarrow q \bar{q} + g$ reveals its total cross section in disagreement with the Jost-Luttinger result⁸⁾. Taking (incorrectly) the final state quarks in all contributing diagrams off-shell not only violates the double-cut-rule but also unitarity. All double-cut-graphs can be generated by applying the Cutkosky rules on the vacuum polarization diagram which is free of M- and IR-singularities; they appear as soon as cuts are introduced (Fig.4). We regularize the theory by introducing auxiliary masses in the vacuum polarization graph which renders the cut-graphs finite. In their sum all M- and IR-singularities then cancel as well as the anomalous cut terms. As a result the final answer is real and identical with the Jost-Luttinger expression also obtained via on-shell mass assignment with a λ -regulator. The old off-shell mass assignment with all external quarks off-shell ($p_i^2 < m^2$) can never be achieved by cutting vacuum polarization graphs and therefore its analytic expression will be different and incorrect.

We conclude:

- (i) The on-shell, off-shell and dimensional regularizations applied on a chosen parton process give different cross-sections. M-factorization, however, will establish a unique answer for any process (DI, DY, $e^+ e^-$) provided the mass assignment was chosen in agreement with the double-cut-rule.
- (ii) Correct off-shell regularization leads to anomalous cuts and technically involved calculations. The on-shell method with a gluon regulator mass runs into the same problems in higher orders (three gluon coupling). Dimensionality regularization^{1),4)} with all particles massless is doubtless the most attractive tool to deal with the IR- and M-singularities.
- (iii) Earlier phenomenological calculations using on-shell mass assignment have determined correctly the order- α_s DY coefficient functions; it was found that the non-log correction term in DY surmounts the leading-log result by a factor ~ 3 at lower Q^2 -values⁹⁾. All analogous off-shell results have to be taken with caution.

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FIGURE CAPTIONS

- Figure 1 : The vacuum-bubbles leading to the order- α_s Feynman diagrams in DI, DY and e^+e^- processes.
- Figure 2 : Order- α_s cut-diagrams of the DY-process with line-numbering derived from the vacuum-bubble in Fig. 1.
- Figure 3 : Order- α_s cut-diagrams of DI-scattering with line-numbering derived from the vacuum-bubble in Fig. 1.
- Figure 4 : Cut-diagrams derived from the vacuum-polarization graph with line-numbering derived from the vacuum-bubble in Fig. 1.

Fig. 1

Fig. 2

Fig. 3

Fig.4