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THE QUARK RECOMBINATION MECHANISM IN MULTI-HADRON PRODUCTION

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ABSTRACT

The lectures discuss the role of quarks and gluons in high energy hadron collisions, in particular in the dominant class of such collisions which are those of non-diffractive low- p_T type. The main progress made recently in this field concerns the secondaries produced in the fragmentation regions of the incident hadrons. The observed similarity in the x -distributions of incident valence quarks and secondary mesons at medium and large x is well accounted for by the quark recombination model. Studying the case of pp collisions, we explicitly exhibit how this similarity comes about and find on the basis of recent ISR data that, contrary to earlier claims, only a modest enhancement of the parton sea is needed to explain the absolute magnitude of the observed cross section. We also find that inclusive meson production at large x is best accounted for by additive contributions of recombination and triple-Regge mechanism. Starting from the meson production data, we then show that by using the hyperon production rate as additional input, one can estimate the probabilities for the various ways in which the three valence quarks of an incident proton recombine with each other and/or with sea partons to produce secondaries in the fragmentation region.

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1. INTRODUCTION

The aim of these lectures is to discuss certain aspects of low p_T reactions of hadrons in terms of quarks and gluons. The general problem of what happens to quarks and gluons in such collisions is under intensive investigation along several lines. I shall talk about a possible picture on which I have worked with S. Pokorski since 1974 and which underwent new developments since 1977 due to the work of Ochs, Das and Hwa, Teper, and others. The work reported here was done in collaboration with Pokorski and Kalinowski.

The picture says the following. At very high energy, non-diffractive collisions of low p_T type are characterized by excitation of the set of gluonic constituents of the incident hadrons, the valence quarks tending to fly through with retention of their original fraction x of incident momentum. These valence quarks hadronize by recombining with each other and/or with additional low- x partons from the parton sea (q , $\bar{q}$ and gluons); they produce in this way the outgoing hadrons in the fragmentation regions (regions of medium and high x of either sign). As to the hadrons produced in the central region (low- x region), they mainly result from hadronization of the excited gluon sea.

It should be noted that the quark behaviour postulated in the recombination model is distinct from the assumptions made for quasi-free quark behaviour in deep inelastic processes. The latter processes are characterized by short times, whereas quark recombination is a soft process involving longer time intervals. Hence, we cannot say a priori that the quark recombination picture is bound to be right, just as one was entitled to question the assumption of quasi-free behaviour of quarks in deep inelastic processes before the experimental evidence imposed itself. Experiment has to decide on the validity of the quark recombination picture and, if qualitative success is encountered, it will reveal the quantitative features of recombination. As to Quantum Chromodynamics, it may be quite compatible with the quark recombination model, but this compatibility may be hard to establish since in QCD soft processes require a non-perturbative approach which is still lacking.

2. QUARK RECOMBINATION (GENERALITIES)

In the picture here discussed, the basic mechanism for production of hadrons in the fragmentation regions is a recombination mechanism involving the valence quarks and sea partons of the same incident hadron ^{*)}. In the case of an incident proton, it can operate in three ways :

- i) Recombination of a single valence quark : Consider the inclusive reaction

$$p + \text{hadron} \rightarrow M + \dots \quad (1)$$

producing a meson M in the proton fragmentation region. M is assumed to originate through recombination (some authors prefer the word fusion) of a valence quark q of p , which retains its original x -value, with a sea antiquark $\bar{q}$ of low x , the x -value of M being approximately

$$x_M \approx x_q + x_{\bar{q}} \approx x_q \quad (2)$$

This relates the x -distribution of the meson M to the x -distribution of the valence quark q in the incident proton. A single valence quark could also recombine with two sea quarks to give a baryon.

- ii) Recombination of two valence quarks : Consider, for example, the reaction

$$p + \text{hadron} \rightarrow Y + \dots \quad (3)$$

producing a hyperon Y in the proton fragmentation region. Y is assumed to originate :

- (a) either through recombination of two valence quarks q, q' of p with a strange quark s of the sea, in which case

$$x_Y \approx x_q + x_{q'} + x_s \approx x_q + x_{q'} \quad , \quad (4)$$

^{*)} Recombination of sea partons from different incident hadrons gives hadronic systems of a mass growing with collision energy (except when all the recombining partons have $x \rightarrow 0$).

- (b) or as in i) through recombination of one valence quark q of p with two sea quarks s, q'' , in which case

$$x_Y \simeq x_q + x_s + x_{q''} \simeq x_q .$$

- iii) Recombination of all three valence quarks : The reaction

$$p + \text{hadron} \rightarrow p + \dots \quad (5)$$

in the proton fragmentation region can proceed as in i) or ii), or through recombination of the three valence quarks q, q', q'' of the incident proton giving for the outgoing proton the x -value

$$x_p \simeq x_q + x_{q'} + x_{q''} .$$

The recombination mechanisms ii) and iii) involve the joint probability distribution $g(x_1, x_2, x_3)$ of the valence quarks in the proton. The recombination mechanism iii) was proposed in 1974 for the "leading particle" reaction (5) by S. Pokorski and the present author, on the basis of the observation that the same average value ~ 0.5 is found experimentally for $\langle x_p \rangle$ and $\langle x_q + x_{q'} + x_{q''} \rangle$, the former from leading particle measurements in pp collisions, the latter from deep inelastic lepton-nucleon collisions ¹⁾. Possible forms for the function $g(x_1, x_2, x_3)$ were discussed in this context ²⁾.

The recent upsurge of interest in the recombination model started with an observation by Ochs ³⁾ to the effect that the x -distribution of pions in the fragmentation region of an incident proton is similar to that of the valence quarks which they share with this proton. The first model calculation for recombination was published by Das and Hwa ⁴⁾. Refs. 3 and 4 concern the recombination mechanism i) for meson production. The most extensive proton-proton collision data available for this type of analysis are the inclusive meson distributions measured at the CERN Intersecting Storage Rings, especially by the CERN-Holland-Lancaster-Manchester Collaboration. Figs. 1-3 are taken from the latest publication of these data by Singh et al. ⁵⁾. The centre-of-mass energy is $s^{\frac{1}{2}} = 45 \text{ GeV}$ and the x region covered is $x \gtrsim 0.4$. Figs. 1 and 2 show that,

up to normalization, the x -distributions of π^+ and K^+ mesons (which can be formed by recombination of an up quark of the proton with a sea antiquark) are strikingly similar to the up quark distribution $u(x)$ derived from the SLAC data on electron-nucleon deep inelastic scattering ⁶⁾, the agreement being particularly good for π^+ . The π^- distribution and its comparison with the down quark distribution $d(x)$ are shown on Fig. 3, where the agreement is good up to $x \approx 0.7$, the π^- spectrum being slightly above $d(x)$ for larger x . Note that the $u(x)$ and $d(x)$ used for the comparison refer to the region of relatively low momentum transfers (low Q^2) studied in the SLAC experiments, where Bjorken scaling was nevertheless found to hold approximately. We come back to the question of scaling violation in section 7.

The K^- production spectrum is very different from the $\pi^\pm$ and K^+ spectra. The invariant cross section $E d^3\sigma/dp^3$ drops rapidly, about as $(1-x)^6$, and is only 2.5% of the K^+ cross section at $x = 0.7$. Note that the exponent is slightly lower than the one predicted by the dimensional-counting rule, which is 7 or 9 depending on whether one assumes single quark or no quark interchange in the collision. The recombination model here discussed makes no specific prediction for K^- production because this meson has no valence quark in common with the incident proton. This holds also for the K^{*0} or φ resonances of which the K^- could be a decay product. One might consider K^- , K^{*0} or φ production as a recombination of two sea quarks, and various authors have done this. We believe that such a picture is doubtful for sizable x of the meson produced, and that sea gluons must play an important role by contributing a considerable amount of additional momentum.

As shown by Singh et al., none of the $\pi^\pm$ and K^+ spectra follows a simple $(1-x)^N$ law, in contradistinction with the indications given by cruder data. This means that applications of the dimensional-counting rule of Brodsky, Farrar, and Matveev et al. ⁷⁾ can only have indicative value. Another interesting result of Ref. 5 is that for π^+ and K^+ a triple-Regge fit works, but only for $x \gtrsim 0.7$, and in this interval it gives the expected Regge trajectories with the correct slope (the N_α trajectory for π^+ and a superposition of the Λ and Σ_δ trajectories for K^+). For π^- a triple-Regge fit again works for $x \gtrsim 0.7$, but in this case it gives the N_α trajectory instead of the expected Δ_δ one. We note that there is no theoretical reason against quark recombination and triple-Regge mechanism being both at work, and it seems that the contribution of the latter becomes apparent at large x only. This matter is studied further in section 5.

3. RECOMBINATION OF A VALENCE QUARK WITH A SEA ANTIQUARK (EQUATIONS)

The most striking fact mentioned above is the similarity between the invariant meson cross section of Figs. 1-3 and the valence quark probability distributions $u(x)$, $d(x)$. How this similarity can be understood in the recombination model will now be discussed in some detail for process (1)

$$p + \text{hadron} \rightarrow M + \dots$$

For the inclusive cross section integrated over transverse momenta the basic formula is

$$\frac{1}{\sigma} \frac{d\sigma}{dk_M} = \int f(k_p, k_v, k_s) r(k_v, k_s) \delta(k_M - k_v - k_s) dk_v dk_s \quad (6)$$

where momenta are taken in the centre-of-mass frame and only their longitudinal components are written down, k_p for the incident proton, k_v and k_s for two of its constituents (a valence quark q_v and a sea antiquark $\bar{q}_s$), and k_M for the meson M produced by the recombination of q_v and $\bar{q}_s$. All these longitudinal momenta are taken to be positive.

The function f is the joint distribution function of k_v and k_s for given k_p . The recombination probability is denoted by the function r multiplied by the delta function for momentum conservation. If the recombination would involve additional partons, for example, gluons, their momenta would have to be included in f , r and the delta function. σ is the relevant integrated cross section (total inelastic cross section excluding diffraction dissociation, $\sigma \approx 28$ mb for pp collisions).

We now go over to the x-variables

$$k_M = x_M k_p, \quad k_v = x_v k_p, \quad k_s = x_s k_p. \quad (7)$$

For large k_p , the combination $k_p^2 f$ is expected to become a function of x_v and x_s with only a slow dependence on k_p reflecting the increasing excitation of the parton sea as k_p increases. We shall write

$$k_p^2 f(k_p, k_v, k_s) = F(x_v, x_s) \quad (8)$$

As to $r(k_v, k_s)$, it is not expected to depend on k_p . This makes it invariant for a longitudinal Lorentz boost, so that for large momenta it depends only on the ratio of k_v and k_s . We therefore write

$$r(k_v, k_s) = R(x_s/x_v) \quad (9)$$

With (8) and (9) the cross section formula becomes

$$\frac{1}{\sigma} \frac{d\sigma}{dx_M} = \int_0^1 \int_0^1 F(x_v, x_s) R(x_s/x_v) \delta(x_M - x_v - x_s) dx_v dx_s \quad (10)$$

The single valence quark distribution $G_v(x)$ is related to F by

$$G_v(x) = n_v \int_0^1 F(x, x_s) dx_s \int_0^1 F(x', x_s) dx' dx_s \quad (11)$$

with n_v the number of valence quarks of type q_v in the proton ($n_v = 2$ for u quarks, $n_v = 1$ for d). Depending on whether q_v is up or down, $G_v(x)$ is the function $u(x)$ or $d(x)$ compared with the data in Figs. 1-3. We recall that the structure functions νW_2 of deep inelastic lepton-nucleon scattering are linear combinations of the $x G_v(x)$ with the squares of the quark charges as coefficients.

The distribution of sea partons is believed to be concentrated at small x and to become singular as x^{-1} for $x \rightarrow 0$, this singularity being needed to

reproduce the diffractive low- x limit of deep inelastic lepton-nucleon scattering. We therefore expect our function $F(x_v, x_s)$ to be concentrated at small x_s , say $x_s \lesssim \epsilon \approx 0.1$, and to grow as x_s^{-1} for $x_s \rightarrow \infty$. For eq.(11), this means that the ratio must be calculated first for the integrals taken from a positive x_0 to 1, the limit $x_0 \rightarrow 0$ being taken afterwards. The situation is different for eq. (10), which would give an infinite cross section unless the recombination function has a zero at $x_s = 0$. By symmetry it should also have a zero at $x_v = 0$. We shall write

$$R(x_s/x_v) = x_s x_v (x_s + x_v)^{-2} \rho(x_s/x_v) \quad (12)$$

and assume $\rho(0)$ finite. Das and Hwa⁴⁾ adopt this form with constant ρ , justifying it by applying the dimensional-counting rule to R :

$$R \propto (1 - x_v) \quad \text{for } x_v + x_s = 1, \quad x_v \rightarrow 1.$$

With (12) the cross section is given by

$$x_M \frac{d\sigma}{dx_M} = \sigma \int x_s F(x_M - x_s, x_s) \rho\left(\frac{x_s}{x_M - x_s}\right) \frac{x_M - x_s}{x_M} dx_s \quad (13)$$

For small ϵ and not too small x_M , we approximate $(x_M - x_s)/x_M$ by one and $\rho(x_s(x_M - x_s)^{-1})$ by $\rho(0)$ over the interval $0 < x_s \lesssim \epsilon$ where the sea contributes. The invariant cross section formula then reduces to the simple form

$$x_M \frac{d\sigma}{dx_M} = \sigma \rho(0) \tilde{G}_v(x_M) \quad (14)$$

with

$$\tilde{G}_v(x) = \int_0^x F(x - x_s, x_s) x_s dx_s \quad (15)$$

Comparison of eqs. (11) and (15) suggests that $\tilde{G}_v(x)$ is likely to be similar but not identical in shape to the valence quark distribution $G_v(x)$, as found experimentally (Figs. 1-3). The extent of the similarity can be discussed

as follows. For $x_v \lesssim 1 - \varepsilon$, one can assume factorization of F , i.e.

$$F(x_v, x_s) = G_v(x_v) G_s(x_s) \quad (16)$$

$\tilde{G}_v$ is then given by the convolution

$$\tilde{G}_v(x) = \int_0^x G_v(x-x') G_s(x') x' dx' \quad (17)$$

For $x > \varepsilon$ and small ε one gets the simple approximation

$$\tilde{G}_v(x) \approx G_v(x) \int_0^1 G_s(x') x' dx' \quad (18)$$

i.e. proportionality of $\tilde{G}$ and G for $\varepsilon < x < 1-\varepsilon$. The situation is different for $x \lesssim \varepsilon$, where one must know the low- x behaviour of $G_v(x)$, which is believed to be

$$G_v(x) \propto x^{-\alpha} \quad \text{with } \alpha \sim 0.5. \quad (19)$$

Eqs. (17) and (14) then give the small- x behaviour

$$\tilde{G}_v(x) \propto x^{1-\alpha}, \quad \frac{d\sigma}{dx_M} \propto x_M^{-\alpha} \quad (20)$$

No meaningful comparison with data can be made here. For $x_v \gtrsim 1-\varepsilon$ the constraint $x_v + x_s \leq 1$ prevents the validity of the factorization (16) and again $\tilde{G}_v$ is no longer expected to follow the shape of G_v , as seems to be indicated by the data.

Eq. (18) can also be written

$$\tilde{G}_v(x) = \bar{x}_s G_v(x) \quad (21)$$

where

$$\bar{x}_s = \int_0^1 G_s(x') x' dx' \quad (22)$$

is the average momentum fraction carried by the sea antiquark $\bar{q}_s$ involved in the recombination. Under the assumptions for which (18) holds, in particular,

$$\varepsilon < x < 1-\varepsilon, \quad \varepsilon \ll 1 \quad (23)$$

the cross section formula (14) becomes

$$x_M \frac{d\sigma}{dx_M} = \sigma \bar{x}_s \rho(0) G_V(x_M). \quad (24)$$

We shall use it in this approximate form for our further discussion.

4. RECOMBINATION OF A VALENCE QUARK WITH A SEA ANTIQUARK (COMPARISON WITH DATA)

To compare with data, one needs the invariant cross section $E d^3\sigma/dp^3$ integrated over the transverse momentum p_T . The ISR measurements of Refs. 5 and 8 cover the range $0.35 < p_T < 1.45$ GeV. For π^+ and K^+ the data are fitted in Ref. 5 to expressions of the form :

$$\text{i) } \exp(-B p_T), \quad \text{ii) } \exp(-C p_T^2). \quad (25)$$

Within the experimental errors, the variation of B or C with x is found to be negligible for $x > 0.5$. Using those fits, the value 28 mb for σ and the $G_V(x) = u(x)$ function from deep inelastic electron-nucleon scattering ⁶⁾ one can estimate $\bar{x}_s \rho(0)$. We have worked at $x = 0.7$ and we have adopted an estimate of 0.16 for $u(0.7)$, a rather high value which takes into account that low Q^2 are relevant to our case (see section 6). We then find

$$P_{u\bar{d} \rightarrow \pi^+} \equiv \bar{x}_s \rho(0) \approx 0.06 - 0.09 \quad (26)$$

for π^+ production ($u\bar{d}$ recombination), the lower (upper) value corresponding to the form ii) (i) of p_T dependence. While we have taken a rather high $u(x)$, the cross section value used contains some π^+ produced by diffraction dissociation or resulting from resonance decay, so that (26) is probably a reasonable estimate.

For K^+ production ($u\bar{s}$ recombination), the value of $\bar{x}_s \rho(0)$ is found from table 9 of Ref. 5 to be $\sim 25\%$ of the value for π^+ production. Note that in the model the quantity (26) has a simple physical meaning. It is the probability $P_{ud \rightarrow \pi^+}^-$ that a valence u quark recombines with a sea $\bar{d}$ antiquark to produce a π^+ .

We can now discuss the size of (26), and in particular whether it requires a strong enhancement of the sea, as was found to be necessary by Das and Hwa ⁴⁾, whose approach differs from ours by the fact that they adopted for F the following factorized ansatz

$$F(x_v, x_s) = \beta G_v(x_v) G_s(x_s) (1-x_v-x_s) \quad (27)$$

β being a constant, G_v and G_s the valence and sea quark distributions, and $(1-x_v-x_s)$ a phase space factor introduced ad hoc. This enhancement question is discussed in detail in a recent paper by Teper ⁹⁾.

The value of $\rho(0)$ is basically unknown. If the function $\rho(x_s/x_v)$ in eq. (12) is taken to be a constant ρ , one must have $\rho \leq 4$ to satisfy the requirement that the recombination probability cannot exceed one, $R \leq 1$, a requirement noted by Teper. Das and Hwa adopted $\rho = 6$ on the basis of an alleged sum rule, eq. (6) of Ref. 4, which is not compelling as far as we can see; in view of $R \leq 1$ it could only hold if $R \equiv 1$. Of course, $\rho(x_s/x_v)$ need not be a constant, and therefore $\rho(0)$ can be larger than 4, but probably not by a big factor. Adopting $\rho(0) \simeq 4$ we deduce for the average momentum fraction of the sea $\bar{d}$ antiquark involved in the recombination

$$\bar{x}_s \simeq 0.015 - 0.02 \quad (28)$$

The total momentum fraction carried by all sea quarks and antiquarks in the nucleon is estimated from the SLAC deep inelastic scattering data to be $\bar{x}_{\text{sea}} \simeq 0.05$. It is natural to assume that it is shared equally between $u, \bar{u}, d, \bar{d}$, with a small additional fraction for $s, \bar{s}$ (this fraction is estimated to be 11% if we use our above determination that K^+ production is $\sim 25\%$ of π^+ production, assuming the same $\rho(0)$ for both cases). The share of $\bar{d}$ in

$\bar{x}_{\text{sea}}$ is therefore $x_{\bar{d}} \approx 0.01$, and we conclude from (28) that the sea enhancement required is only a factor 1.5 - 2, instead of the factor 4 resulting from Das and Hwa's analysis (see Teper's discussion, Ref. 9).

We believe that our result is reasonable and that the high value of Das and Hwa resulted from their use of eq. (27), which has the serious defect of not satisfying the consistency condition (11). Following Teper, it must be pointed out, however, that other suppression factors are ignored in going from $\bar{x}_{\text{sea}} \approx 0.05$ to $\bar{x}_{\bar{d}} \approx 0.01$, namely a factor 9 for colour and a factor 4 for spin. Teper proposes that the necessary matching of colour and spin of u and $\bar{d}$ to recombine into π^+ should be achieved thanks to the sea gluons (presumably through capture or emission).

The comparisons of Figs. 1-3 between meson spectra and valence quark distributions, which represent a nice success for the recombination picture, assume that the pions and kaons are directly produced particles and are not decay products of resonances. As shown by Teper, this may be true to a reasonable approximation in the x range considered in Figs. 1-3 ($x \gtrsim 0.5$), but direct measurements of meson resonance production are needed, not only to verify this point, but also because such resonances will themselves be produced by the same quark recombination mechanism. They will provide further tests of the model and allow it to be used at lower x values. The importance of this question will also become apparent in the next section.

5. TRIPLE-REGGE MECHANISM

We now expand on the remarks at the end of section 2. Figs. 4 and 5 from Ref. 5 give the triple-Regge fits at large x for $\pi^\pm$, K^+ production and the comparison with the relevant Regge trajectories. The variables used are the familiar ones, $M^2 = s(1-x)$ being the square of the missing mass and t the relativistic square of the energy-momentum transfer. We recall that the triple-Regge formula is

$$E \frac{d^3\sigma}{dp^3} = \frac{s}{\pi} \frac{d^2\sigma}{dt dM^2} = \beta(t) (1-x)^{1-2\alpha(t)}$$

The data for $pp \rightarrow \pi^- X$ are particularly interesting. Fig. 4 shows a clear break at $x \approx 0.7$ (already visible in Fig. 3), and Fig. 5 shows poor agreement with the Δ trajectory. The explanation we propose is that valence quark recombination and triple-Regge mechanism contribute additively :

$$E \frac{d^3 \sigma}{dp^3} = \left(E \frac{d^3 \sigma}{dp^3} \right)_{\text{Recomb.}} + \beta(t) (1-x)^{1-2\alpha(t)}$$

the latter becoming important only at large x . Fig. 6 shows that the π^- data are easily accounted for by this additive model. The Regge trajectory used is the Δ one,

$$\alpha_{\Delta}(t) = 0.13 + 0.90 t$$

and the residue function is found to be described by the simple form

$$\beta_{\Delta}(t) = 27 e^t \quad \mu\text{b GeV}^{-2}$$

in the range $-0.55 \lesssim t \lesssim -0.2 \text{ GeV}^2$. At smaller t ($t \lesssim -0.6$) $\beta_{\Delta}(t)$ is found to flatten off. For the recombination term we have taken

$$\left(E \frac{d^3 \sigma}{dp^3} \right)_{\text{Recomb.}} = \gamma(p_T) d(x)$$

with the coefficient $\gamma(p_T)$ reduced by 10% compared with the fits of Fig. 3. One has $\gamma(0.55 \text{ GeV}) \approx 2\gamma(0.75 \text{ GeV})$.

Fig. 5 shows that, contrary to the case of π^- production, the π^+ and K^+ production data are in good agreement with the corresponding Regge trajectories. The reason is that, in these cases, both triple-Regge and recombination mechanisms imply similar x dependences for large x . Thus recombination leads to a dependence as $u(x) \propto (1-x)^3$, while with the trajectories given in Fig. 5 the Regge exponents become at $t = -0.55 \text{ GeV}^2$

$$1 - 2\alpha(-0.55) = 2.8 \text{ for } \pi^+, \quad 2.5 \text{ and } 3.4 \text{ for } K^+.$$

In contrast we have for π^- production $d(x) \propto (1-x)^{4.5}$ and

$$1 - 2\alpha_{\Delta}(-0.55) = 1.7,$$

i.e. a much steeper decrease for the recombination than for the Regge term. The large- x dominance of the Regge term will even be stronger at $p_T = 0$, where t becomes positive.

Another case of interest is $\pi^- p \rightarrow \pi^0 X$, experimentally studied in Ref. 10 at large x and interpreted in terms of the triple-Regge mechanism. The authors of Ref. 10 find quite good agreement, except for the fact that the trajectory $\alpha(t)$ derived from the triple-Regge fit for $0.71 \leq x \leq 0.98$ falls somewhat below the expected ρ -trajectory, whereas there is agreement for $0.81 \leq x \leq 0.98$. This can again be explained by addition of Regge and recombination terms. The latter is expected to decrease as $1-x$, while the former has the Regge exponent

$$1 - 2\alpha_{\rho}(t) \approx 0 \quad \text{for} \quad t \approx 0.$$

We end this section by pointing out that additivity of recombination and triple-Regge mechanisms is physically reasonable, because the mechanisms are different when expressed in terms of partons. The triple-Regge case amounts to the substitution of an incident valence (anti)quark of low x by another one (in $\pi^- p \rightarrow \pi^0 X$ for example), or of two incident valence quarks of low x by a low- x antiquark (in $pp \rightarrow \pi X$ for example), but the incident valence quark flying through need not have its x -value close to the one of the outgoing hadron, as is the case in the recombination mechanism.

6. PROBABILITIES FOR SINGLE, DOUBLE AND TRIPLE VALENCE QUARK RECOMBINATION

Returning to the recombination model, we now make use of the probability (26) for $u\bar{d} \rightarrow \pi^+$ recombination in order to estimate the probability $P_{u \rightarrow M}$ that a given valence u quark recombines with any sea antiquark to give any meson including the meson resonances. Such an estimate was made by Teper⁹⁾, with the result

$$P_{u \rightarrow M} \approx (3.4 - 4.5) P_{u\bar{d} \rightarrow \pi^+} \quad (29)$$

We take the same range of values for $P_{d \rightarrow M}$. We have checked that at $x = 0.7$ this is compatible with the π^- production data of Ref. 5 and the value of the valence d quark distribution function. Combining (26) and (28) we find

$$\begin{aligned} P_1 = P_{u \rightarrow M} = P_{d \rightarrow M} &\approx (3.4 - 4.5) \cdot (0.06 - 0.09) \\ &\approx 0.2 - 0.4 \end{aligned} \quad (30)$$

P_1 is the probability that a given valence quark recombines with any sea anti-quark. In the framework of the model (sections 1 and 2), $1-P_1$ is then the probability for a given valence quark to recombine with quarks and form a baryon. The above estimates hold for $x \gtrsim 0.5$ and for recombination of one valence quark. We do not know what happens at small x , nor what correlations may exist between the behaviour of the three valence quarks. But if we adopt the estimate (29) for all x we can attempt to guess the probabilities for the various types of recombinations listed in section 2. This will now be done.

The probability $1-P_1$ is composed of three contributions

$$1 - P_1 = P_1' + P_2 + P_3 \quad (31)$$

corresponding to the three ways in which a given quark can recombine to form a baryon, either with two sea quarks (probability P_1'), or with one sea and one valence quark (probability P_2), or with the two remaining valence quarks (probability P_3). These contributions can be related to the probabilities for the overall fate of the set of three valence quarks of an incident nucleon, namely the probabilities A_1 that they all recombine separately with sea partons, A_2 that two of them recombine together with a sea quark to form a baryon, and A_3 that all three recombine together to form a nucleon. The relations are

$$\begin{aligned} P_1 + P_1' &= A_1 + \frac{1}{3} A_2 \\ P_2 &= \frac{2}{3} A_2 \\ P_3 &= A_3 \end{aligned} \quad (32)$$

and (31) is equivalent to $A_1 + A_2 + A_3 = 1$. All probabilities A_i , P_i and P_1' refer to non-diffractive collisions.

In addition to our above estimate of P_1 , one can make use of the production rate of hyperons to obtain information on the size of the A_i . Let $2B$ be the mean number of hyperons produced in the fragmentation regions of a non-diffractive pp collision. The recombination model gives for the hyperon production per proton

$$\begin{aligned}
 B = & A_2 [\xi\eta + (1-\xi)\eta' \xi' + \xi\eta'(1-\xi') + 2\xi\eta'\xi'] \\
 & + A_1 [3\eta^2\eta'\xi' + 3\eta\eta'^2 (2\xi'(1-\xi') + 2\xi'^2) \\
 & + \eta'^3(3\xi'(1-\xi')^2 + 6\xi'^2(1-\xi') + 3\xi'^3)] \\
 & + A_1 \eta^3 [1-(1-\xi)^3] \\
 = & 3\eta'\xi'A_1 + (\xi + \xi'\eta')A_2 + A_1\eta^3 [1-(1-\xi)^3]
 \end{aligned} \tag{33}$$

ξ is the probability for a sea quark to be strange and $1-\xi' = (1-\xi)^2$ the probability for two sea quarks to be both non-strange. We have also defined

$$\eta = 1 - \eta' = P_1 / (P_1 + P_1') \tag{34}$$

A valence quark which recombines with sea partons only has probability η to do so with an antiquark, and η' to do so with two sea quarks. We assume η to be the same irrespective of what happens to the other valence quarks. The various terms of eq. (33) were first written so as to correspond to the various ways in which 1, 2 or 3 hyperons can be formed by recombination of the valence quarks with 1 to 6 sea quarks, or by recombination of the 3 surplus sea quarks available when all 3 valence quarks have recombined with sea antiquarks into mesons (term in $A_1\eta^3$, which was not considered before but should be included since it is likely to populate the fragmentation region). We have assumed these various recombinations to be uncorrelated.

Using (32) and (34) we rewrite (33) as

$$B = \xi A_2 + 3\xi P_1' + A_1 \eta^3 [1 - (1-\xi)^3] \quad (35)$$

To first order in ξ this simplifies to

$$b = B/\xi = A_2 + 6P_1' + 3A_1 \eta^3 \quad (36)$$

a formula which becomes exact if in counting hyperons one gives weight 2 and 3 to hyperons of strangeness - 2 and - 3 respectively. These equations combined with the fact that P_1, P_1', P_2 and P_3 are all positive, impose interesting constraints on b . We shall use the simpler form (36). For given P_1, P_1' one finds that the minimum of b is obtained for $A_2 = 0$ and has the value

$$b_{\min} = 3(P_1 + P_1') (\eta^3 - 2\eta + 2) = 3P_1 \eta^{-1} (\eta^3 - 2\eta + 2) \quad (37)$$

The maximum of b at fixed P_1, P_1' is obtained for $A_1 = 0$ if $P_1 + P_1' \leq \frac{1}{3}$, and for $A_1 + A_2 = 1$ if $\frac{1}{3} \leq P_1 + P_1' \leq 1$. Its value is

$$b_{\max} = \left\{ \begin{array}{ll} 3(P_1 + P_1')(3 - 2\eta) = 3P_1 + 9P_1' & \text{for } P_1 + P_1' \leq \frac{1}{3} \\ \frac{3}{2}(1 - \eta^3) + \frac{3}{2}(P_1 + P_1')(3\eta^3 - 4\eta + 3) & \text{for } \frac{1}{3} \leq P_1 + P_1' \leq 1 \end{array} \right\} \quad (38)$$

Notice that $b = b_{\min} = b_{\max}$ for $P_1' = 0$ as well as for $P_1 + P_1' = 1$. Fig. 7 illustrates the shape of the allowed domain of b versus P_1, P_1' .

We now discuss the values of our parameters for pp collisions. From the K^+/π^+ production ratio of 0.25 we deduce

$$\xi = 0.25/(2 + 0.25) = 0.11.$$

The best experimental data to estimate B are those from Blobel et al.¹¹⁾, who give for pp collisions at 24 GeV

$$2B_{\Lambda} = 0.057, \quad 2B_{\Sigma^+} = 0.028, \quad 2B_{\Sigma^-} = 0.0009.$$

The Λ production rate includes Λ from Σ^0 decays. These data are at sufficiently low energy to assume that most hyperons result from beam and target particle fragmentation, but still high enough not to expect a large change of the fragmentation cross sections as one goes to higher energies. The data of Ref. 11 include diffraction dissociation, but this is approximately corrected for by the fact that the values (39) are obtained by dividing the production cross section by the full inelastic cross section. The Ξ production is not given in Ref. 11, but for pp collisions at 19 GeV, Alpgard et al.¹²⁾ give $2B_{\Xi^-} \approx 0.001$ with a large error. Adding up we obtain $B = 0.048$. Hyperon production data at Fermilab and ISR energies are too incomplete to determine B reliably, but they suggest a somewhat larger multiplicity. Values in the range

$$b = B/\xi \approx 0.6 - 0.75 \quad (39)$$

may therefore be expected and shall be adopted for our discussion. Since P_1 is in the range $0.2 - 0.4$ and the function $b_{\min}/3P_1$ of eq. (37) has the shape shown in Fig. 8, we find that our constraints limit P_1 to low values, $P_1 \approx 0.2 - 0.25$, and η to values close to one, $\eta \gtrsim 0.8$ or $P_1' \lesssim 0.04$.

We can now estimate ranges of values for the probabilities A_1, A_2, A_3 . If $P_1' = 0$, i.e. $\eta = 1$, we have

$$b = 3P_1 = 3A_1 + A_2 \quad (P_1' = 0) \quad (40)$$

as the only constraints beyond positivity. Hence

$$A_1 \leq P_1, \quad A_2 \leq 3P_1, \quad 1 - 3P_1 \leq A_3 \leq 1 - P_1 \quad (41)$$

Taking $P_1 \approx 0.2 - 0.25$ this gives

$$A_1 \lesssim 0.25, \quad A_2 \lesssim 0.75, \quad 0.25 \lesssim A_3 \lesssim 0.8 \quad (42)$$

If $P_1' > 0$, i.e. $\eta < 1$, the A_i are determined in terms of P_1, P_1' and b :

$$\begin{aligned}
 A_1 &= (P_1 + 3P_1' - \frac{1}{3}b)/(1 - \eta^3) \\
 A_2 &= 3(P_1 + P_1' - A_1) \\
 A_3 &= 1 - 3(P_1 + P_1') + 2A_1
 \end{aligned} \tag{43}$$

They can be rewritten as

$$\begin{aligned}
 A_1 &= P_1 + P_1' - \frac{1}{3}(b - b_{\min})/(1 - \eta^3) \\
 A_2 &= (b - b_{\min})/(1 - \eta^3) \\
 A_3 &= 1 - P_1 - P_1' - \frac{2}{3}(b - b_{\min})/(1 - \eta^3)
 \end{aligned}$$

As long as $P_1 + P_1' \leq \frac{1}{3}$, $b_{\max}$ is given by the first equation (38) and corresponds to

$$A_1 = 0, \quad A_2 = 3(P_1 + P_1'), \quad A_3 = 1 - 3(P_1 + P_1').$$

As we have seen in connection with eq. (39), unless P_1 is close to the lower limit of its range ($P_1 \approx 0.2$) and P_1' close to zero, b is expected to be close to $b_{\min}$, which implies

$$A_1 \approx P_1 + P_1', \quad A_2 \approx 0, \quad A_3 \approx 1 - P_1 - P_1'.$$

Closer estimates can be obtained if we make more special but still plausible probabilistic assumptions on the relative sizes of P_1 , P_1' , P_2 and P_3 . Consider a given valence quark of the proton and call p_v its probability to recombine with another given valence quark, and p_s its probability to recombine with a given sea quark or antiquark. We neglect the dependences of these probabilities on momenta and flavours, as well as possible correlations. We then have

$$P_1 = n_s p_s$$

$$P_1' = \frac{1}{2} n_s (n_s - 1) p_s^2 \quad (44)$$

$$P_2 = 2 n_s p_s p_v$$

$$P_3 = p_v^2$$

where n_s is the effective number of sea quarks (or antiquarks) available for recombination. Hence

$$P_2^2 = 4P_1^2 P_3 \quad (45)$$

$$P_1' = c P_1^2, \quad c = (n_s - 1)/(2n_s) \quad (46)$$

Taking $1 \leq n_s < \infty$ as range for n_s , we have for c

$$0 \leq c \leq 0.5 \quad (47)$$

Using $P_3 = 1 - P_1 - P_1' - P_2$, we solve eq. (45) for P_2

$$P_2 = 2P_1(1 - P_1 - P_1' + P_1^2)^{\frac{1}{2}} - 2P_1^2 = 2P_1(1 - P_1 + (1 - c)P_1^2)^{\frac{1}{2}} - 2P_1^2 \quad (48)$$

For given P_1 , P_1' the values of the A_i and of b are then all fixed. We give a few examples for $c = 0$ and $c = 0.5$ (the values for $c = 0.5$ being given between brackets).

P_1	b	A_1	A_2	A_3
0.2	0.6 (0.72)	0.06 (0.08)	0.43 (0.42)	0.51 (0.50)
0.3	0.9 (1.12)	0.12 (0.17)	0.53 (0.51)	0.35 (0.32)
0.4	1.2 (1.53)	0.21 (0.31)	0.57 (0.51)	0.22 (0.18)

Keeping in mind eq. (39) we conclude that under our more specific assumptions we are again led to a low value for P_1 , $P_1 \approx 0.2$, and to $P_1' = c P_1^2 \ll P_1$. They imply small A_1 , and A_2 of the same order of magnitude as A_3 (whereas the early considerations of Refs. 1 and 2 assumed dominance of A_3). Since P_1' is small, A_2 -type processes are mostly those where, out of the three valence quarks of the incident proton, two recombine with a sea quark q_s to give a baryon b and the third with a sea antiquark $\bar{q}_s'$ to give a meson m , where b and m may be resonances but the b - m system is not. Hence the usefulness of meson-baryon correlation studies in the proton fragmentation region of non-diffractive collisions. The recombination model relates them to correlations among the proton valence quarks.

Under our assumptions, we find A_3 to be considerable, probably $\gtrsim 0.4$. This is in contradiction with the two-quark-chain model of inelastic collisions (as obtained by cutting the cylindrical Pomeron structure of the dual topological unitarization scheme), which implies $A_3 = 0$. Our findings are in favour of schemes with an important gluon exchange contribution to the Pomeron. They are compatible with complete dominance of gluon exchange in hadronic collisions at very high energy.

7. DISCUSSION

Our analysis suggests that lower values of P_1 are more likely than higher ones. As already mentioned in section 4, P_1 and $\bar{x}_s \rho(0)$ may be somewhat smaller than the estimates in eqs. (30) and (26), due to diffraction dissociation, resonance decay, and the fact that the value of $G_V(x) = u(x)$ to be used when applying eq. (24) to π^+ production may have to be taken somewhat larger than the one given at the same x by the SLAC electron-nucleon scattering data. The reason for this latter expectation is that the recombination process here considered involves low momentum transfers, and the violation of Bjorken scaling in lepton-nucleon scattering is such that $u(x)$ at large x increases for decreasing momentum transfer (decreasing Q^2). Physically, the valence quark takes along more of its gluon cloud in the recombination process than in high Q^2 lepton-nucleon scattering. At $x \gtrsim 0.5$, the effects of diffraction dissociation, resonance decay and Bjorken scaling violation should not be very large, however,

if the similarity in shape of $(x^{-1} d\sigma/dx)_{\pi^+}$ and $u(x)$ is to be explained by the recombination model. In particular, we doubt that they could completely eliminate the need for an enhancement of the parton sea, although at ISR energies the enhancement may be somewhat less than the factor 1.5 - 2 mentioned in section 4, and much less than the factor required by Das and Hwa⁴⁾.

Our whole discussion has been limited to the region $x \gtrsim 0.5$. To extend it to lower x one has to face the problem of resonances, mainly mesonic^{*)}, but also baryonic. Accurate work will become possible when more data will be available on resonance production in the fragmentation region. Upper limits for the resonance multiplicities can be estimated in terms of the probabilities P_1 , P_1' , A_2 and A_3 . They are rather small, which is encouraging for the experimental work. It would be advisable to study the resonance contents of the fragmentation region by imposing a cut-off of perhaps $x \gtrsim 0.2$ on the particles considered, also when they are resonance decay products, so as to avoid contamination from decays of the resonances produced in the central region. The study of strange particle production should be very useful, as illustrated already by the information we were able to extract above from very limited data. Also the study of non-resonant systems of two or three particles is instructive, as shown by Lockman et al. for the production of multi-pion systems in the fragmentation region at the ISR¹⁴⁾.

In these lectures, the equations of the quark recombination model have only been given for single valence quark recombination with a sea antiquark and for cross sections integrated over transverse momenta. Extension to the various other recombination processes can be carried out to test the model and exploit all the information to be extracted from the experimental study of the fragmentation regions. Also the extension to incident mesons is likely to become very interesting, all the more so that it should be, with the Drell-Yan process, one of the very few ways of getting information on the quark-gluon structure of mesons^{**) .}

*) For a study of meson resonance production in the recombination model see R.G. Roberts et al.¹³⁾.

**) For an application to incident pions, see R.C. Hwa and R.G. Roberts¹⁵⁾.

8. OTHER PRODUCTION PROCESSES AND OTHER MODELS

We add a few brief remarks on other cases of hadron production. The quark recombination model proposes an answer to the question of what happens in low- p_T non-diffractive collisions to the valence quarks of the incident hadrons. It extends naturally to processes involving leptons and to high- p_T hadronic collisions; it can then be applied to those incident valence quarks which undergo no high- p_T process in the reaction. In deep inelastic lepton-nucleon scattering, for example, the model can be applied to the target fragmentation region and will permit detailed comparisons with nucleon fragmentation in low- p_T hadronic reactions. Similar applications can be found in Drell-Yan processes. Much work is being done in this direction ¹⁶⁾.

Coming back to the purely hadronic reactions, we note that the quark recombination model covers only the fragmentation region. In the spirit of the model, most of the centrally produced particles, i.e. at very high energy the majority of the secondaries, must originate entirely from the sea. They may be produced through hadronization of the excited gluon sea, as would be expected if the Pomeron is gluon-exchange dominated.

As mentioned above, the two-quark-chain model of central particle production deduced from dual topological schemes is of another nature and, if dominant, would require $A_3 = 0$. A comparison of this model with central particle production data has recently been worked out by Capella et al. ¹⁷⁾. For the fragmentation regions, they reject the recombination model and are led to adopt a model where a diquark carrying about 95% of the incident proton momentum fragments into the observed secondaries. This type of parton fragmentation model had been proposed and studied by B. Andersson et al. ¹⁸⁾. While it can account for the data, it does not explain the similarities shown by Figs. 1-3 between inclusive meson spectra and valence quark distributions, similarities which give at present the main justification for the recombination model ^{*)}.

*) Another approach to hadron fragmentation in the dual topological scheme has recently been proposed by G. Cohen-Tannoudji et al. ¹⁹⁾. It leads to results which are rather close to those of the recombination model.

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FIGURE CAPTIONS

- Figure 1 Inclusive π^+ production cross section at the CERN ISR, for centre-of-mass energy 45 GeV. Data from J. Singh et al.⁵⁾.
- Figure 2 Same as Fig. 1 for K^+ production.
- Figure 3 Same as Fig. 1 for π^- production.
- Figure 4 Comparison of the data of Figs. 1,2,3 at $t = 0.55 \text{ GeV}^2$ with triple-Regge fits, from Ref. 5.
- Figure 5 Comparison of triple-Regge fits with expected Regge trajectories, from Ref. 5.

Figure 6 Fit of inclusive π^- production at $p_T = 0.55$ and $0.75 \text{ GeV}/c$ with a sum (dotted line) of a triple-Regge term (full line) and a recombination term (dashed line). Data from Ref. 5.

Figure 7 Allowed domain of b as function of P_1, P_1' . The domain is bounded by the surfaces $OAB, AA''B''B$ and $OA''B''$. The straight lines OL, LL'' and OL'' show how these surfaces are constructed. The curve $A''B''$ has the equations

$$P_1 + P_1' = 1, \quad b = 3(P_1^3 - 2P_1 + 2)$$

Its minimum is in the point N with $P_1 = (2/3)^{1/2} \approx 0.82$. The table gives the co-ordinates of various points of the figure.

Figure 8 The shape of the function $b_{\min} / 3P_1$ of eq.(37).

FIG. 1

FIG. 2

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FIG. 3

FIG.4

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FIG.5

FIG.6

	P_1	P'_1	b
A	0	$1/3$	3
A'	0	$1/3$	2
A''	0	1	6
B	$1/3$	0	1
B''	1	0	3
M	.62	.38	3
N	.82	.18	2.73

FIG.7

FIG.8