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THE PION MASS DIFFERENCE IN THRESHOLD PION REACTIONS
ON LIGHT NUCLEI

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A B S T R A C T

We investigate the effect of the pion mass difference in scattering and photoproduction of pions on light nuclei near threshold. The effect is found to be most easily described in a nuclear K-matrix formalism where the nuclear K-matrix elements are shown to be independent of the mass difference.

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1. INTRODUCTION

During the last few years there has been a number of investigations of threshold pion reactions on light nuclei. Experimentally both scattering and photo-production of pions have been studied^{1),2)}. The theoretical studies employed the Faddeev equations or the Watson multiple scattering series. Both methods have their advantages. The Faddeev approach can be applied to pion scattering by the deuteron and allows an exact solution of the scattering problem. The Watson approach, on the other hand, is physically more transparent and is easily extended to other light nuclei. For the deuteron the two methods are identical. In two recent papers³⁾ we have shown how the static approximation is recovered for reactions on the deuteron. We have explicitly demonstrated that binding corrections are small. This result gives us confidence in the applicability of the static approximation.

In the experiments we are discussing the energy is only a few MeV above threshold. Hence, we expect large isospin breaking effects due to the pion mass difference, $\Delta m = m(\pi^\pm) - m(\pi^0) = 4.61$ MeV. They are commonly accounted for by working in the K-matrix formalism where the expression for the nucleon amplitudes reads.

$$f(k) = \frac{1}{1 - a(k^2) i k} a(k^2) \quad (1)$$

The matrix $a(k^2)$ is assumed isospin conserving and the momentum matrix k is diagonal with channel dependent matrix elements. The isospin breaking is important in channels with small matrix elements such as $\pi^0 N \rightarrow \pi^0 N$ and $\gamma N \rightarrow \pi^0 N$. In the second reaction the coupling between the channels $\gamma p \rightarrow \pi^0 p$ and $\gamma p \rightarrow \pi^+ n \rightarrow \pi^0 p$ through Eq. (1) changes the threshold value of $f(\gamma p \rightarrow \pi^0 p)$ by 40%.

These considerations apply also for nuclei provided $a(k^2)$ is interpreted as a nuclear amplitude. However, it is not clear whether $a(k^2)$ depends on the pion mass difference since it is $f(k)$ itself which is calculated through the multiple scattering series. A proper calculation of $f(k)$ does of course include the pion mass difference. On the other hand we shall argue that a simpler approach is to calculate $a(0)$, i.e., $f(0)$ in the absence of mass difference, and afterwards correct for the mass difference through Eq. (1).

That a careful treatment of the mass difference is necessary can be seen in pion deuteron scattering. At threshold the elastic amplitude is dominated by double scattering which in turn is dominated by double charge exchange, e.g., $\pi^0 p \rightarrow \pi^+ n$ followed by $\pi^+ n \rightarrow \pi^0 p$. Neglecting the mass difference double scattering is proportional to

$$\left\langle \frac{1}{r} \right\rangle = 0.45 \text{ fm}^{-1} \quad (2)$$

whereas inclusion of the mass difference gives

$$\left\langle \frac{1}{r} e^{-\kappa r} \right\rangle = 0.31 \text{ fm}^{-1} \quad (3)$$

with $\kappa = \sqrt{2m\Delta m} = 0.18 \text{ fm}^{-1}$. However, we must not forget that according to Eq. (1) also the single scattering term does give a term of the type $a(\pi^0 p \rightarrow \pi^+ n)a(\pi^+ n \rightarrow \pi^0 p)$ with a proportionality factor $ik = \kappa = 0.18 \text{ fm}^{-1}$. When this contribution is added to that of Eq. (3), terms linear in κ cancel exactly. If we want to go beyond the linear approximation it is not sufficient to use the complete expression (3) but also possible κ^2 -terms in $f(0)$ must be retained. The upshot is that in pion deuteron scattering the pion mass difference can be ignored. It is therefore natural to ask whether this result is true for all nuclei. This question will be answered in the subsequent sections.

2. UNITARITY FOR FIXED SCATTERERS

We shall first illustrate the concept of unitarity for an ideal case with no mass difference or isospin. We consider pion nucleus scattering near threshold. Our basic assumption is that the nuclear amplitude is obtained by first calculating the scattering amplitude $F(k)$ for fixed positions of the nucleons and afterwards taking the average in the proper nuclear state. The assumption of fixed nucleons implies that the recoil of the nucleons are neglected. This approximation is a good one since $m_\pi/M_N \ll 1$. Furthermore, we assume that the scattering by an individual nucleon is of the S-wave type. We write

$$f_l(k) = \frac{1}{1 - a_l ik} a_l = a_l + a_l ik \cdot a_l \quad (4)$$

neglecting powers of k^2 and higher.

The scattering by a system of fixed scatterers allows simple solutions only in certain circumstances. For the moment we assume a situation with non-overlapping potentials. In this case the solution to the multiple scattering problem is given by

$$F(k) = \sum_i e^{-i\vec{k} \cdot \vec{x}_i} f_i(k) e^{i\vec{k} \cdot \vec{x}_i} + \sum_{i \neq j} e^{-i\vec{k} \cdot \vec{x}_i} f_i(k) G_{ij}(k) f_j(k) e^{i\vec{k} \cdot \vec{x}_j} + \dots \quad (5)$$

with

$$G_{ij}(k) = \frac{e^{ikr_{ij}}}{r_{ij}} = \frac{1}{r_{ij}} + ik + \dots \quad (6)$$

and $r_{ij} = |\bar{x}_i - \bar{x}_j|$. This series can be summed in closed form but for the moment we are not interested in this expression. As remarked above $F(k)$ is considered an operator acting between nuclear states. In particular, the elastic scattering amplitude is given by $\langle \psi_0 | F(k) | \psi_0 \rangle$ where $|\psi_0\rangle$ is the nucleus ground state.

Our assertion is that $F(k)$ satisfies a unitarity condition identical to that of $f(k)$, Eq. (4). This must of course be the case but the actual demonstration helps us understand how to incorporate the mass difference.

We are only interested in terms linear in k . Therefore, we expand the amplitudes $f_\ell(k)$ as in Eq. (4) and the propagator as in Eq. (6). We introduce $g_{ij} = 1/r_{ij}$. The phase factors in Eq. (5) can be ignored since they give corrections of order k^2 . Now, consider the expansion of a particular f_j in a term of order N in Eq. (5). Leaving out index k we get for the linear contribution

$$\sum_{\dots i \neq j \neq l \dots} (a_i g_{i2} \dots a_i g_{ij}) a_j \cdot ik \cdot a_j (g_{jl} a_l \dots a_{N+1}) \quad (7)$$

Then we look at the corresponding term of order $N+1$ where instead the Green function is expanded (do not confuse the momentum k with the index k)

$$\sum_{\dots i \neq j \neq k \neq l \dots} (a_i g_{i2} \dots a_i g_{ij}) a_j \cdot ik \cdot a_k (g_{kl} a_l \dots a_{N+1}) \quad (8)$$

When expressions (7) and (8) are added together the sums over j and k become independent yielding

$$\left\{ \sum_{\dots i \neq j \dots} a_i g_{i2} \dots a_i g_{ij} a_j \right\} ik \left\{ \sum_{k \neq l \dots} a_k g_{kl} \dots a_{N+1} \right\} \quad (9)$$

Clearly, summing over all possible choices of initial f_j gives

$$F(k) = A(0) + A(0) ik A(0) + \dots \quad (10)$$

a result which is completely analogous to Eq. (4). On the right hand side of Eq. (10) we have used the notation $A(0)$ rather than $F(0)$. This was done in order to remember that $A(k^2)$ is a function of k^2 only, and not of k as $F(k)$. This again is in complete analogy with Eq. (4).

The unitarity condition, Eq. (10), is an operator equation. The condition on the nuclear amplitude becomes

$$\langle \psi_0 | F(k) | \psi_0 \rangle = \langle \psi_0 | A(0) | \psi_0 \rangle + \sum_n \langle \psi_0 | A(0) | \psi_n \rangle i k_n \langle \psi_n | A(0) | \psi_0 \rangle \quad (11)$$

where the sum runs over all nuclear states. In our simplified treatment the energy loss to the pion was neglected, i.e., $k_n = k$. This can only be correct if the nuclear states ψ_n are degenerate in energy. However, our derivation is correct also in the general case provided we interpret k as an operator. We then have $k_n^2 = k^2 + 2m_\pi (E_0 - E_n)$.

3. INTRODUCING THE MASS DIFFERENCE

In order to account for the pion mass difference the analysis must be slightly changed. We simplify the argument by neglecting the mass difference between the proton and the neutron. The first change is that we must now interpret Eq. (4) as an operator equation with a and k being represented by matrices. The k matrix is diagonal but with channel dependent matrix elements. For the diagonal elements we get

$$\langle 0 | f(k) | 0 \rangle = \langle 0 | a | 0 \rangle + \sum_n \langle 0 | a | n \rangle i k_n \langle n | a | 0 \rangle \quad (12)$$

where the second term gives rise to the isospin breaking. Analogously we interpret the momentum k in the Green function of Eq. (5) as an operator.

With these new interpretations the analysis of the previous section can be taken over step by step. We must only be careful and retain the correct ordering of all the operators. But by foresight that was already done in our derivation. Consequently, the final Eqs (10) and (11) are still correct provided k is considered as an operator in Eq. (10) and the momentum k_n in Eq. (11) correctly accounts for the possible mass difference between the intermediate and final states.

The result of our analysis is that there are two possible ways of calculating the threshold amplitude. The first is a direct evaluation of $\langle 0 | F(k) | 0 \rangle$ with due attention to the type of pion in each scattering step. The second and more practical way is to calculate $\langle 0 | F(k) | 0 \rangle$ through Eq. (11). From the derivation it follows that calculating matrix elements of $A(0)$ is done by neglecting the mass difference at all levels. They only enter through the intermediate state momentum k_n .

We illustrate the above methods for π^0 scattering. In the case of $\pi^\pm$ scattering or charge exchange the corrections are negligible. The pion nucleon amplitude is written as

$$a_{\pi N} = b^{(+)} - b^{(-)} \bar{\tau} \cdot \bar{\tau} = b^{(+)} \frac{1}{2} \{ \tau_p, \tau_n \} + b^{(-)} \frac{1}{2} [\tau_p, \tau_n] , \quad (13)$$

where α and β denote the isospin indices of the pion in the initial and final states, respectively. We use the values⁴⁾

$$b^{(+)} = -0.018 \text{ fm} , \quad (14a)$$

$$b^{(-)} = 0.130 \text{ fm} . \quad (14b)$$

For π^0 proton scattering $k_n = i\kappa_n$ with $\kappa_n = 0.21 \text{ fm}^{-1}$ yielding the threshold amplitude $f(\pi^0 p) = -0.025 \text{ fm}$, a substantial increase compared to $b^{(+)}$.

We then turn to π^0 nucleus scattering limiting ourselves to single and double scattering terms. In the unitarity correction to Eq. (11) it is then sufficient to calculate $A(0) = Ab^{(+)} + b^{(-)} \bar{\tau} \cdot \bar{\tau}$, $\bar{\tau}$ being the Pauli isospin operator the nucleus. It follows that the intermediate nuclear state must have the same isospin as the initial nucleus. Our conclusions concerning the mass difference correction to the threshold amplitude $\langle \psi_0 | F(0) | \psi_0 \rangle$ are

i) no correction for d and ${}^4\text{He}$

ii) non-vanishing correction for ${}^3\text{He}$ and ${}^3\text{H}$.

For the $({}^3\text{He}, {}^3\text{H})$ -system the complete formula is

$$\langle \pi^0 {}^3\text{N} | F(0) | \pi^0 {}^3\text{N} \rangle = 3b^{(+)} + (6b^{(+)^2} - 4b^{(-)^2}) \langle \frac{1}{r} \rangle - 2b^{(-)^2} \chi_n . \quad (15)$$

Since $\kappa_n \approx \frac{1}{2} \langle 1/r \rangle$ the mass difference correction is about 25% of the double scattering term but the net effect is substantially smaller than for the proton.

We conclude that the pion mass difference is unimportant when calculating the multiple scattering part of threshold pion nucleus amplitudes. We shall see later that the situation is radically different in photoproduction.

4. INTRODUCING FORM FACTORS

The multiple scattering series (5) applies in particular to scattering by the deuteron. Since the sum runs to infinity the pion can bounce back and forth between the nucleus any number of times. The propagator is always the same. Hence, we will get factors $1/r^N$ with arbitrarily high powers N . The corresponding expectation value diverges when N becomes sufficiently large rendering the multiple scattering series meaningless. The error in this argument is that Eqs (5) and (6) are incorrect for small relative separation between the nucleons. The reason is that

$$e^{i\vec{k}\cdot\vec{x}} + f(k) \frac{e^{ikr}}{r} \quad (16)$$

is not the exact solution to the Schrödinger equation.

We illustrate how this dilemma is solved by considering a problem which can be solved exactly. We assume that the scattering by a nucleon is described by a separable potential

$$V(\vec{x}, \vec{x}') = \text{const} \frac{e^{-\beta r}}{r} \cdot \frac{e^{-\beta r'}}{r'} \quad (17)$$

The exact wave function is then

$$\psi(\vec{x}) = e^{i\vec{k}\cdot\vec{x}} + f(k) \frac{1}{r} (e^{ikr} - e^{-\beta r}) \quad (18)$$

where $f(k)$ is the on-shell scattering amplitude. The scattered wave is now perfectly regular and approaches (16) for large separations.

The exact solution to the multiple scattering problem is however, not obtained by making the replacement

$$\frac{e^{ikr}}{r} \rightarrow \frac{1}{r} (e^{ikr} - e^{-\beta r}) \quad (19)$$

since we have one form factor at each vertex. The multiple scattering problem can be solved exactly just as for a single potential. The Schrödinger equation for a sum of potentials $V_i(\vec{x}, \vec{x}') = \lambda_i v_i(\vec{x}) v_i(\vec{x}')$ is

$$\psi(\vec{x}) = e^{i\vec{k}\cdot\vec{x}} + \int d^3x' d^3x'' G(\vec{x}, \vec{x}') \sum_i V_i(\vec{x}' - \vec{x}_i, \vec{x}'' - \vec{x}_i) \psi(\vec{x}'') \quad (20)$$

The scattering amplitude $F(k)$ is easily expressed as a multiple scattering series through

$$F(k) = \sum_i e^{-i\vec{k} \cdot \vec{x}_{fi}} F_i(k) e^{i\vec{k} \cdot \vec{x}_{in}} \quad (21a)$$

$$F_i(k) = f_i(k) + \sum_{j \neq i} f_i(k) G_{ij}(k) F_j(k) \quad (21b)$$

$$\begin{aligned} G_{ij}(k) &= \frac{1}{v_i(k)} \int d^3x d^3x' v_i(\vec{x}-\vec{x}_i) G(\vec{x}, \vec{x}') v_j(\vec{x}'-\vec{x}_j) \frac{1}{v_j(k)} \\ &= \frac{1}{r} \left\{ e^{i\vec{k} \cdot \vec{r}} - e^{-\beta r} \left(1 + \frac{r}{\beta} (\beta^2 + k^2) \right) \right\}_{r=r_{ij}} \end{aligned} \quad (21c)$$

where $f_j(k)$ is the on-shell scattering amplitude for a single scattering centre. In Eq. (21a) we have denoted the co-ordinate for the first hit nucleon by $\vec{x}_{in}$ and for the last hit by $\vec{x}_{fi}$. The explicit expression in Eq. (21c) is valid only when $\beta_i = \beta_j = \beta$.

The most important remark is that the unitarity condition Eq. (11) is still valid when the form factors are present even though the amplitude $A(0)$ itself will be different. To see this we observe that the phase factors in Eq. (21a) give corrections of the form $1 + O(k^2)$. For the propagator (21c) we have independently of the form factors

$$G_{ij}(k) = G_{ij}(0) + ik + \dots \quad (22)$$

Thus, our previous demonstration remains unchanged.

An additional bonus is that the propagator is regular at the origin since according to (21c)

$$\frac{1}{r} \rightarrow \frac{1}{r} \left\{ 1 - (1 + \beta r) e^{-\beta r} \right\} \quad (23)$$

No singularity of the type discussed above will develop. The model with a one term Yamaguchi potential works very well for pion nucleon scattering in the S_{11} channel. In the S_{31} channel two terms are necessary to get a good description.

5. π^0 PHOTOPRODUCTION

The interesting application of our formalism is to π^0 photoproduction. The nucleon threshold amplitude is

$$a_{\pi\gamma} = i\vec{\sigma} \cdot \vec{\epsilon} \left\{ E^{(+)} \delta_{\beta 3} + E^{(-)} \frac{1}{2} [\tau_{\beta}, \tau_3] + E^{(0)} \tau_{\beta} \right\} \quad (24)$$

where β is the isospin index of the pion. Close to threshold the k dependence is described by Watson's theorem

$$f_{\pi\gamma}(k_{\gamma}) = \frac{1}{1 - a_{\pi N} i k_{\pi N}} a_{\pi\gamma} \quad (25)$$

where $a_{\pi N}$ is the matrix (13) and where the diagonal matrix $k_{\pi N}$ has channel dependent matrix elements. It is obvious that all our previous remarks about nuclei go through unchanged provided we always keep the photoproduction matrix to the right. In Eq. (21a) we have, e.g., $\bar{k} = \bar{k}_{\gamma}$ and $\bar{k}' = \bar{k}'_{\pi}$. The unitarity relation reads

$$F_{\pi\gamma}(k_0) = A_{\pi\gamma}(k_0) + A_{\pi N}(0) i k_{\pi N} A_{\pi\gamma}(k_0) \quad (26)$$

where k_0 is the threshold momentum. The only new aspect is the momentum transfer in the production step which must not be expanded but kept at the threshold value.

We then turn to the applications. Our formulas are of interest only in those cases where the direct amplitude is small, i.e., for π^0 photoproduction. For the nucleon we get from Eqs (24) and (25) at threshold

$$f(\pi^0 N) = i\vec{\sigma} \cdot \vec{\epsilon} \left\{ E^{(+)} \pm E^{(0)} - 2\chi G^{(-)} (E^{(-)} \pm E^{(0)}) \right\} \quad (27)$$

where the upper sign is for the proton and the lower one is for the neutron. The values of the momentum κ are $\kappa(\pi^0 p) = 0.21 \text{ fm}^{-1}$ and $\kappa(\pi^0 n) = 0.15 \text{ fm}^{-1}$. Inserting numerical values for the multipoles $E^{(-)} = G = 0.021/m(\pi^+)$, $E^{(+)} = -0.048G$ and $E^{(0)} = -0.070G$ we get

$$f(\pi^0 p) = i\vec{\sigma} \cdot \vec{\epsilon} \left\{ -0.118 - 0.050 \right\} G, \quad (28a)$$

$$f(\pi^0 n) = i\vec{\sigma} \cdot \vec{\epsilon} \left\{ 0.022 - 0.043 \right\} G. \quad (28b)$$

Consequently both amplitudes are strongly affected by the unitarity correction coming from the mass difference.

The next application is the deuteron. We are only interested in single and double scattering terms. Since the deuteron has isospin $T = 0$ we get

$$\langle \pi^0 d | A_{\pi\gamma}(k_0) | \gamma d \rangle = i \vec{\sigma}_p \cdot \vec{\epsilon} \left\{ E^{(+)} S(k_0) - \left\langle \frac{e^{i \vec{k}_0 \cdot \vec{r}/2}}{r} \right\rangle (2b^{(-)} E^{(-)} - b^{(+)} E^{(+)} \right\} \quad (29)$$

where $S(k_0)$ is the deuteron form factor. There is no contribution to $\langle \pi^0 d | F(0) | \gamma d \rangle$ from the unitarity term since the intermediate two nucleon state must have isospin zero. Therefore only π^0 contributes in the intermediate state and it has $k_{\pi N} = 0$. Our conclusion is that there are no corrections from the mass difference. However, it is important to realize that the single scattering term is not the sum of the physical $\pi^0 p$ and $\pi^0 n$ photoproduction amplitudes of Eq. (28) since the latter are strongly affected by the mass difference.

Finally, we consider π^0 photoproduction on ${}^3\text{He}$ and ${}^3\text{H}$. We assume the nuclear co-ordinate space wave function to be symmetric and of the s-wave type. The spin-isospin part of the wave function is properly anti-symmetrized. For the detailed evaluations it is useful to know that

$$\sum_{k=1}^3 \vec{\sigma}^{(k)} \cdot \vec{\epsilon} |{}^3N\rangle = + \vec{\sigma}_h \cdot \vec{\epsilon} |{}^3N\rangle \quad (30a)$$

$$\sum_{k=1}^3 \vec{\sigma}^{(k)} \cdot \vec{\tau}^{(k)} |{}^3N\rangle = - \vec{\sigma}_h \cdot \vec{\tau}_h |{}^3N\rangle \quad (30b)$$

where $|{}^3N\rangle$ denotes a ${}^3\text{He}$ or ${}^3\text{H}$ state with arbitrary spin. $\vec{\sigma}_h$ and $\vec{\tau}_h$ are the total Pauli spin and isospin operators for the 3N system. After some calculations we get

$$\begin{aligned} \langle \pi^0 {}^3N | A_{\pi\gamma}(k_0) | \gamma {}^3N \rangle = i \vec{\sigma}_h \cdot \vec{\epsilon} \left\{ (E^{(+)} \mp E^{(0)}) S(k_0) \right. \\ \left. - \left\langle \frac{e^{i \vec{k}_0 \cdot \vec{r}_1}}{r_{12}} \right\rangle (4b^{(-)} E^{(-)} - 2b^{(+)} (E^{(+)} \mp E^{(0)})) \right\} \quad (31) \end{aligned}$$

where the upper (lower) sign corresponds to ${}^3\text{He}({}^3\text{H})$. Because of the extra minus sign in Eq. (30b) the single scattering term corresponds to the neutron combination for ${}^3\text{He}$ and the proton combination for ${}^3\text{H}$.

The (${}^3\text{He}, {}^3\text{H}$) system is analogous to the (p,n) system and we will consequently get a contribution from the unitarity term in Eq. (26). A trivial calculation gives

$$\begin{aligned} \langle \pi^0 {}^3\text{N} | F_{\pi\gamma}(k_0) | \gamma {}^3\text{N} \rangle &= \langle \pi^0 {}^3\text{N} | A_{\pi\gamma}(k_0) | \gamma {}^3\text{N} \rangle \\ &+ i\vec{\sigma}_k \cdot \vec{E} \, 2\pi S(k_0) k^{(\gamma)} (E^{(\gamma)} \mp E^{(\pi)}) \end{aligned} \quad (32)$$

where the upper (lower) sign is for ${}^3\text{He}({}^3\text{H})$. The values for κ are $\kappa(\pi^0 {}^3\text{He}) = 0.19 \text{ fm}^{-1}$ and $\kappa(\pi^0 {}^3\text{H}) = 0.17 \text{ fm}^{-1}$. The single scattering term of Eq. (31) and the mass corrections of Eq. (32) together give

$${}^3\text{He} : \quad \{ +0.022 + 0.046 \} G S(k_0) \quad (33a)$$

$${}^3\text{H} : \quad \{ -0.118 + 0.048 \} G S(k_0) \quad (33b)$$

The correction is of the same size as for the (p,n) system, but has the opposite sign. This derives from Eq. (30b) through which the sign of the charged pion photoproduction amplitudes becomes opposite to the nucleon case. Again we conclude that the single scattering contributions are very different from the physical threshold amplitudes on the nucleon.

The formulas above are valid for static nucleons. In the real world extra kinematical factors must be affixed to the single and double scattering terms. The double scattering term is multiplied by a factor $1 + m_\pi/M_N = 1.14$ relative to the single scattering term. In addition there is an extra over-all kinematical factor. In the experimental analysis²⁾ this factor as well as the nuclear form factor $S(k_0)$ are explicitly extracted. The remaining part of the matrix element defines the nuclear electric dipole amplitude $E(\pi^0 A)$. For the deuteron a factor $\frac{1}{2}$ is also extracted. We have calculated $E(\pi^0 A)$ using the Reid wave function⁵⁾ for the deuteron and the correlated Fearing wave function⁶⁾ for the (${}^3\text{He}, {}^3\text{H}$) system. Our numerical results are

$$E(\pi^0 p) = -0.168 G \quad (34a)$$

$$E(\pi^0 d) = -0.394 G \quad (34b)$$

$$E(\pi^0 {}^3\text{He}) = -0.248 \text{ G} , \quad (34c)$$

$$E(\pi^0 {}^3\text{H}) = -0.386 \text{ G} . \quad (34d)$$

The ratios $E(\pi^0 d)/E(\pi^0 p)$ and $E(\pi^0 {}^3\text{He})/E(\pi^0 p)$ are about 20% smaller than the experimental ratios. This is probably due to our neglect of P-wave contributions, form factors and pion absorption. The detailed analysis for the deuteron is given in Ref. 3).

SUMMARY

The importance of the pion mass difference in pion reactions on light nuclei near threshold has been investigated. We have found it particularly marked for π^0 photoproduction, essentially due to the extreme smallness of the direct π^0 photoproduction amplitude. For nucleons this phenomenon is treated in the K-matrix formalism. The same approach is applicable to nuclei. The important question is here how to calculate the nuclear K-matrix elements at threshold. We have explicitly shown that in this framework the K-matrix should be calculated for vanishing mass difference. Hence, the nuclear K-matrix approach allows us to rapidly assess the importance of the mass difference. In π^0 photoproduction it can be neglected for the deuteron but it is important for the (p,n) and (${}^3\text{He}$, ${}^3\text{H}$) systems. As a result, the connections between the single scattering terms and the physical nucleon amplitudes are completely lost.

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