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UNIFIED GAUGE MODEL WITH A NON-LINEAR CHIRAL
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A B S T R A C T

A chiral unified gauge model based upon a phenomenological chiral $SU(4) \times SU(4)$ meson baryon Lagrangian is constructed for the gauge group $SU(2)_L \times U(1)$. The obtained effective weak interaction Lagrangian has the usual current $\times$ current structure where the charged weak current is of the Cabibbo type and the neutral current satisfies the condition $\Delta S = 0$ of the GIM-scheme. A generalization of the $SU(3) \times SU(3)$ symmetry breaking scheme of Oakes yields mass formulas for the $SU(4)$ hadron multiplets.

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1. INTRODUCTION

The recent discovery of new particles carrying another quantum number, charm, naturally suggests extending the chiral group $SU(3) \times SU(3)$ ¹⁾ to $SU(4) \times SU(4)$ ²⁾. Indeed, several authors³⁾ have investigated this larger group already before the new experimental achievements, trying to shed light on existing ideas on $SU(3) \times SU(3)$ and $SU(2) \times SU(2)$ symmetries of strong interactions. Further support in favour of $SU(4) \times SU(4)$ has been supplied by the developments in unified gauge theories of leptons and the attempts to incorporate quarks in such theories without having sizeable strangeness-changing neutral currents⁴⁾.

This work is devoted to the study of unified gauge models based upon non-linear chiral $SU(4) \times SU(4)$ Lagrangians. Among our motivations for using the framework of non-linear Lagrangians⁵⁾⁻⁷⁾ it is worth mentioning the following: first, it has been shown that non-linear chiral $SU(3) \times SU(3)$ Lagrangians give in the tree and one-loop approximation a satisfactory description of the leptonic, semi-leptonic and radiative decays of the $SU(3)$ meson octet (π, K, η)⁸⁾. Secondly, the unified models which are derived on the basis of them contain explicitly the fields of the observable hadron multiplets.

In a recent paper⁹⁾ we have investigated a unified gauge model for the weak and electromagnetic interactions of mesons based upon a non-linear chiral $SU(4) \times SU(4)$ meson Lagrangian. There extensive use has been made of the technique of non-linear realizations of symmetry groups⁶⁾. In the present paper we shall continue this kind of investigation by considering a non-linear meson-baryon Lagrangian which is a straightforward extension of usual $SU(3) \times SU(3)$ Lagrangians^{6),8),10)}. Weak and electromagnetic interactions are then included by defining gauge-covariant chiral derivatives with respect to the local gauge group $SU(2)_L \times U(1)$ of the Weinberg-Salam model¹¹⁾. The final result is an effective weak Lagrangian of the current $\times$ current type, with charged weak currents having a generalized Cabibbo structure and with a neutral current which conserves strangeness.

Obviously, the enlarged chiral symmetry must be broken much more strongly than ordinary $SU(3) \times SU(3)$ because of the appearance of heavy charmed hadrons. To reproduce (with an uncertainty of, say 10%) the masses of the 15-plet of 0^- -mesons and 20-plet of $\frac{1}{2}^+$ -baryons with a minimum number of arbitrary parameters, we finally used a suitable extension of the symmetry breaking mechanism proposed some years ago by Oakes¹²⁾. This scheme provides us with an interesting relationship between the strength of the $SU(2) \times SU(2)$ breaking (the finiteness of the pion mass), the non-conservation of strangeness in weak interactions and the Cabibbo angle.

The paper is organized as follows. In Section 2 we define the chiral $SU(4) \times SU(4)$ meson-baryon Lagrangian. Sections 3 and 4 contain the gauge covariant derivatives for the group $SU(2)_L \times U(1)$ and the chiral unified Lagrangian, respectively. In Section 5 we discuss the $SU(4) \times SU(4)$ breaking and the role of the Cabibbo angle, and in Section 6 we quote the effective weak Lagrangian. Finally, Section 7 contains a summary and a brief discussion of the results.

2. THE STRONG INTERACTION LAGRANGIAN

Let us consider the 0^- -mesons and $\frac{1}{2}^+$ -baryons belonging to the 15- or 20-dimensional representations of the group $SU(4)$. Their corresponding fields are denoted by ϕ_i ($i = 1, 2, \dots, 15$) and $B_c^{[ab]}$ ($a, b, c, = 1 \dots 4$)¹³⁾, respectively (for explicit expressions, cf., Appendix A). It is further convenient to introduce the dimensionless meson fields $\xi_i = \phi_i/f$, where f is a parameter with the dimension of a mass to be recognized later on as an averaged pseudoscalar meson decay constant.

In the following we shall deal with the non-linear field transformations $(\xi, B) \xrightarrow{g} (\xi', B')$ or, explicitly⁶⁾,

$$g e^{i\xi \cdot A} = e^{i\xi' \cdot A} e^{iu'(\xi, g) \cdot V}$$

$$B_\ell^{[mn]}, = (U^+)_i^m (U^+)_j^n U_\ell^k B_k^{[ij]}, \quad (1)$$

$$U = e^{iu'(\xi, g) \cdot V},$$

where

$$\xi \cdot A = \sum_{i=1}^{15} \xi_i A_i \quad \text{etc.}$$

Here g is an arbitrary element of the chiral group $SU(4) \times SU(4)$ which may be parametrized in the vicinity of unity by $g = e^{i\alpha \cdot A} e^{iu \cdot V}$; $V_i = (\lambda_i/2)\mathbf{1}$, $A_i = (\lambda_i/2)\gamma^5$ ($i = 1, 2, \dots, 15$) are the vector and axial generators of the Lie algebra of the chiral group, and λ_i are the $SU(4)$ Gell-Mann matrices^{*)}.

*) We use the normalization $\text{Tr } A_i A_k = \text{Tr } V_i V_k = 2\delta_{ik}$, $\text{Tr } A_i V_k = 0$, where the trace is taken over internal and Lorentz indices.

Furthermore, we need the Cartan forms $D_\mu \xi_i, \theta_\mu^i(\xi)$ ⁶⁾

$$\begin{pmatrix} D_\mu \xi_i \\ \theta_\mu^i(\xi) \end{pmatrix} = -\frac{i}{2} \text{Tr} \left\{ \begin{pmatrix} A_i \\ V_i \end{pmatrix} e^{-i\xi \cdot A} \partial_\mu e^{i\xi \cdot A} \right\} \quad (2)$$

The starting point of our analysis is the following phenomenological meson-baryon Lagrangian extended from $SU(3) \times SU(3)$ ^{6),8),10)} to $SU(4) \times SU(4)$

$$\mathcal{L}_{\text{Strong}} = \mathcal{L}_{\text{inv}}(D_\mu \xi; B, D_\mu B) + \Delta \mathcal{L} \quad (3)$$

$\mathcal{L}_{\text{inv}}$ being the $SU(4) \times SU(4)$ invariant part and $\Delta \mathcal{L}$ is a symmetry breaking term, the discussion of which will be postponed to Section 5. We have ^{*})

$$\begin{aligned} \mathcal{L}_{\text{inv}}(D_\mu \xi; B, D_\mu B) = & \frac{f^2}{2} D^\mu \xi_i D_\mu \xi_i + \bar{B}(i\gamma^\mu D_\mu - m_0)B - \\ & - g_A [\alpha (\bar{B} \gamma^\mu \gamma^5 V_i B)_d + (1-\alpha) (\bar{B} \gamma^\mu \gamma^5 V_i B)_f] D_\mu \xi_i \end{aligned} \quad (4)$$

(sum over repeated indices is understood). Here $g_A \approx 1.25$ is the axial vector renormalization constant, m_0 is an averaged baryon mass. The subscripts d, f in expressions like $(\bar{B} V_i B)_{d(f)}$ denote d or f couplings defined by

$$(\bar{B} V_i B)_{d(f)} = \frac{1}{2} \bar{B}_{[mn]}^a (V_i)_a^b B_b^{[mn]} + (-) \bar{B}_{[bn]}^m (V_i)_a^b B_m^{[an]} \quad (5)$$

^{*}) From Eq. (4) one can obtain, in the lowest order of ξ , the usual pseudo-scalar meson-baryon coupling by performing a field redefinition

$$B = \exp \{ -2i\gamma^5 g_A [\alpha (V_i)_d + (1-\alpha) (V_i)_f] \xi_i \} \psi$$

and taking into account the Goldberger-Treiman relation for the coupling constants.

and α is the corresponding mixing parameter. Finally, the covariant derivative of the baryon tensor is given in terms of the Cartan form $\theta_\mu^i(\xi)$ by

$$\bar{B} i \gamma^\mu D_\mu B = \frac{1}{2} \bar{B}_{[mn]}^a i \gamma^\mu \partial_\mu B_a^{[mn]} - (\bar{B} \gamma^\mu V_i B)_f \theta_\mu^i(\xi). \quad (6)$$

The invariance of (4) under the field transformations (1) can easily be shown by using the transformation laws for the Cartan forms (2) ⁶⁾.

3. GAUGE-COVARIANT DERIVATIVES

Let us now introduce the electro-weak interactions into the strong interaction Lagrangian (3). As gauge group we choose the group $G_W = SU(2)_L \times U(1)$ of the standard Weinberg-Salam model ¹¹⁾. In order to obtain the gauge-covariant derivatives $\bar{D}_\mu \xi, \bar{D}_\mu B$ of the hadron fields we have first to find the (non-linear) transformation laws of the fields (ξ, B) with respect to G_W . This can easily be achieved by embedding G_W into the global chiral group. We introduce the following 4×4 matrix representation of the generators of G_W ^{*)}

$$\begin{aligned} SU(2)_L : \hat{C}_i &= \frac{1+\gamma^5}{2} \frac{C_i}{2}, \quad C_i = \begin{pmatrix} \sigma_i & 0 \\ 0 & \sigma_1 \sigma_i \sigma_1^{-1} \end{pmatrix} \\ U(1) : \hat{Y} &= \frac{Y_W}{2} + \frac{1-\gamma^5}{2} \frac{C_3}{2} \end{aligned} \quad (i=1, \dots, 3) \quad (7)$$

where σ_i are usual Pauli matrices and y_W denotes the weak hypercharge. The matrix σ_1 appearing in C_i is required by our choice of $SU(4)$ λ matrices which correspond to the quark spinor

$$q = \begin{pmatrix} u \\ d \\ s \\ c \end{pmatrix}$$

*) Strictly speaking, the group G_W must be embedded into the group $U(4) \times U(4)$ since the generator $\hat{Y}/2$ contains the unit matrix. The latter one leads, however, to an irrelevant phase factor only which drops out of the transformation law (8). The problem of embedding more general gauge groups G_W into the global group $U(N) \times U(N)$ has been discussed by Weinberg ¹⁴⁾.

but not

$$q = \begin{pmatrix} u \\ d \\ c \\ s \end{pmatrix}$$

[cf., also Eq. (23)]. The operator of the electromagnetic charge reads

$$Q = \hat{C}_3 + \frac{\hat{Y}}{2} = \frac{1}{2} (C_3 + Y_W) .$$

We may then postulate the following field transformations

$$(\xi, B) \xrightarrow{h} (\xi', B') , \quad h \in G_W$$

[compare Eq. (1)]

$$h e^{i\xi \cdot A} = e^{i\eta \frac{\hat{Y}}{2}} [e^{i\xi' \cdot A} e^{i u'(\xi, h) \cdot V}]$$

$$B_{\ell}^{[mn]}, = (U^+)_{\ell}^m (U^+)_{\ell}^n U_{\ell}^k B_k^{[ij]} \quad (8)$$

where

$$h = e^{i\eta \frac{\hat{Y}}{2}} e^{i\varepsilon \cdot \hat{C}} , \quad \varepsilon \cdot \hat{C} = \sum_{i=1}^3 \varepsilon_i \hat{C}_i$$

$$U = e^{i u'(\xi, h) \cdot V}$$

As we now consider co-ordinate-dependent gauge transformations (8), we need further a set of gauge bosons W_{μ}^i, X_{μ} associated to the generators $\hat{C}_i, \hat{Y}/2$ of G_W . They obey the transformation laws

$$i\kappa W_{\mu} \cdot \hat{C} = e^{i\varepsilon(x) \cdot \hat{C}} (\partial_{\mu} + i\kappa W_{\mu} \cdot \hat{C}) e^{-i\varepsilon(x) \cdot \hat{C}} ,$$

$$i\rho X_{\mu}' = i\rho X_{\mu} - i\partial_{\mu} \eta(x) \quad (9)$$

where κ, ρ are coupling constants.

The gauge-covariant derivatives $\bar{D}_\mu \xi$, $\bar{D}_\mu B$ associated to the "electro-weak" group G_W will now be defined by the equation

$$\begin{pmatrix} \bar{D}_\mu \xi_i \\ \bar{\theta}_\mu^i(\xi) \end{pmatrix} = -\frac{i}{2} \text{Tr} \left\{ \begin{pmatrix} A_i \\ V_i \end{pmatrix} e^{-i\xi \cdot A} \left(\partial_\mu + i\kappa W_\mu \cdot \hat{C} + i g X_\mu \frac{\hat{Y}}{2} \right) e^{i\xi \cdot A} \right\} \quad (10)$$

and Eq. (6). It can easily be shown that under the gauge transformations (8) and (9), the transformation laws of $\bar{D}_\mu \xi$, $\bar{D}_\mu B$ take the same form as the transformation laws of $D_\mu \xi$, $D_\mu B$ in the case of the transformations (1).

4. CHIRAL UNIFIED LAGRANGIAN

In Ref. 15) a minimal substitution rule has been proposed for introducing electromagnetism into a non-linear chiral $SU(3) \times SU(3)$ Lagrangian. In a similar way electro-weak interactions can be introduced into our non-linear $SU(4) \times SU(4)$ theory by applying the generalized substitution rule $(D_\mu \xi_i, D_\mu B) \rightarrow (\bar{D}_\mu \xi_i, \bar{D}_\mu B)$ to the Lagrangian (3). The chiral unified Lagrangian thus obtained reads

$$\mathcal{L}_{unif} = \mathcal{L}_{inv}(\bar{D}_\mu \xi; B, \bar{D}_\mu B) + \Delta \mathcal{L} \quad (11)$$

where

$$\begin{aligned} \mathcal{L}_{inv}(\bar{D}_\mu \xi; B, \bar{D}_\mu B) = & \mathcal{L}_{inv}(D_\mu \xi; B, D_\mu B) - \\ & - \frac{\kappa}{2\sqrt{2}} (W^{+\mu} j_\mu^W + h.c.) - \frac{\kappa}{2\cos\theta_w} Z^\mu j_\mu^Z - \\ & - e A^\mu j_\mu^A + \mathcal{L}_{bil}(A_\mu, W_\mu^\pm, Z_\mu) . \end{aligned} \quad (12)$$

Here $W_\mu^\pm$, Z_μ and A_μ are the fields of the charged and neutral vector bosons and of the photon, respectively,

$$W_\mu^\pm = \frac{1}{\sqrt{2}} (W_\mu^1 \mp i W_\mu^2), \quad Z_\mu = \cos \theta_w W_\mu^3 - \sin \theta_w X_\mu \quad (13)$$

$$A_\mu = \sin \theta_w W_\mu^3 + \cos \theta_w X_\mu,$$

θ_w is the Weinberg angle defined by $\tan \theta_w = \rho/\kappa$, and e is the electromagnetic charge, $e = \kappa \sin \theta_w$ (11). The term $\mathcal{L}_{bil}$ in Eq. (12) contains expressions bilinear in the vector fields [see Ref. (9)]. Finally, the weak charged hadronic and neutral currents j_μ^W , j_μ^Z and the electromagnetic current j_μ^A are given by

$$j_\mu = -f^2 \bar{D}_\mu \xi_i \langle P A_i(\xi) \rangle + (\bar{B} \gamma_\mu V_i B)_f \langle P V_i(\xi) \rangle +$$

$$+ g_A [\alpha (\bar{B} \gamma_\mu \gamma^5 V_i B)_d + (1-\alpha) (\bar{B} \gamma_\mu \gamma^5 V_i B)_f] \langle P A_i(\xi) \rangle, \quad (14)$$

where P is a projection operator defined by $P = \{2\hat{C}_+, 2\hat{C}_3 - 2 \sin^2 \theta_w Q, Q\}$ for $\{j_\mu^W, j_\mu^Z, j_\mu^A\}$, respectively, and $\hat{C}_\pm = \hat{C}_1 \pm i\hat{C}_2$. Henceforth, we use the short-hand notations

$$\langle \dots \rangle = \frac{1}{2} \text{Tr} (\dots)$$

$$\begin{pmatrix} A_i(\xi) \\ V_i(\xi) \end{pmatrix} = e^{i\xi \cdot A} \begin{pmatrix} A_i \\ V_i \end{pmatrix} e^{-i\xi \cdot A}$$

5. SU(4) $\times$ SU(4) BREAKING, CABIBBO ANGLE AND PCAC

Note that the chiral-invariant part of Eq. (3) describes massless (Goldstone) mesons and equal-mass baryons. In order to get a physical hadron mass spectrum we have yet to choose a suitable $SU(4) \times SU(4)$ breaking term $\Delta \mathcal{L}$. To this end, let us extend the $SU(3) \times SU(3)$ symmetry-breaking scheme of Oakes¹²⁾ to $SU(4) \times SU(4)$. The main idea behind this approach is to break $SU(4) \times SU(4)$ [or $SU(3) \times SU(3)$] first down to $SU(2) \times SU(2)$. The breaking of the latter group, and thus a finite pion mass, is then achieved by a field rotation around the seventh axis in $SU(4)$ space which mixes the strange and non-strange hadrons. The mixing angle turns out to be the Cabibbo angle.

Let us illustrate these ideas by discussing first the symmetry breaking mechanism generating the meson masses. The respective $SU(4) \times SU(4)$ symmetry breaking term $\Delta\mathcal{L}_m$ is usually assumed to transform predominantly as a sum of terms from the representation $(4, 4^*) + (4^*, 4)$. We have $(V_0 = (1/2\sqrt{2})\mathbf{1})$

$$\Delta\mathcal{L}_m = \frac{\mu^2}{\sqrt{8}} f^2 \langle (V_0 + a V_8 + b V_{15}) e^{i2\xi \cdot A} \rangle. \quad (15)$$

To preserve $SU(2) \times SU(2)$ symmetry we express the parameters μ^2 , a , b by the masses of the neutral K and D meson as follows

$$\begin{aligned} \mu^2 &= m_{K^0}^2 + m_{D^0}^2, \quad a = -\sqrt{\frac{8}{3}} m_{K^0}^2 / \mu^2 \\ b &= -(3m_{D^0}^2 - m_{K^0}^2) / \sqrt{3} \mu^2. \end{aligned}$$

Then, $SU(2) \times SU(2)$ will be broken, too, by rotating the "bare" fields ξ_i around the seventh axis in $SU(4)$ space. This yields, finally,

$$\Delta\mathcal{L}_m = \frac{f^2}{2} \langle U_c^+ \begin{pmatrix} 0 & 0 & m_{K^0}^2 \\ & m_{K^0}^2 & 0 \\ & 0 & m_{D^0}^2 \end{pmatrix} U_c e^{i2\tilde{\xi} \cdot A} \rangle_{\Delta S=0} \quad (16)$$

where the new "physical" meson fields $\tilde{\xi}$ are defined by

$$e^{i2\tilde{\xi} \cdot A} = U_c^+ e^{i2\xi \cdot A} U_c; \quad U_c = e^{i2\theta V_7},$$

and we have retained only the strangeness-conserving parts in Eq. (16). We then get the following mass formulas for the 15-plet of 0^- -mesons

$$\begin{aligned} m_{\pi^+}^2 &= m_{K^0}^2 \sin^2 \theta, \quad m_{K^+}^2 = m_{K^0}^2 \cos^2 \theta \\ m_{D^+}^2 &= m_{D^0}^2 + m_{\pi^+}^2, \quad m_{F^+}^2 = m_{D^0}^2 + m_{K^+}^2 \\ m_{\eta_8}^2 &= m_{K^0}^2 \left(\frac{1}{3} + \cos^2 \theta \right), \quad m_{\eta_{15}}^2 = \frac{1}{6} (m_{K^0}^2 + 9 m_{D^0}^2). \end{aligned} \quad (17)$$

With $\theta \approx 0.26$, $m_{K^0} = 498$ MeV, $m_{D^0} = 1863$ MeV we obtain $m_{\pi^+} = 128$ MeV, $m_{K^+} = 481$ MeV, $m_{D^+} = 1867$ MeV, $m_{F^+} = 1924$ MeV which is in rough agreement with the experimental mass values. Equation (16) contains further a term describing π^0 - η_8 - η_{15} mixing. A complete discussion of the π^0 - η - η' - η_c mass matrix would require the inclusion of the SU(4) singlet η_0 which is, however, outside the scope of this paper. The baryon mass splitting may be described in an analogous way by choosing the following symmetry breaking term, close in spirit to Eq. (16),

$$\Delta \mathcal{L}_b = \frac{1}{2} \bar{B}_{[mn]}^i \begin{pmatrix} 0 & a \sin^2 \theta \\ & a \cos^2 \theta \\ & & b \end{pmatrix}_j \tilde{B}_j^{[mn]} +$$

$$+ \bar{B}_{[in]}^m \begin{pmatrix} 0 & c \sin^2 \theta \\ & c \cos^2 \theta \\ & & d \end{pmatrix}_j \tilde{B}_m^{[jn]}, \quad (18)$$

where $\tilde{B}_{\lambda}^{[mn]}$ are rotated "physical" baryon fields, and we have taken into account the two possible ways of coupling the baryon tensors.

With the parameter choice^{16)*} $a = -127$ MeV, $b = -1221$ MeV, $c = 264$ MeV, $d = 1494$ MeV and $m_0 = 2690$ MeV we get the predictions for the masses of the baryon 20-plet collected in Table 1.

We must next rewrite the unified Lagrangian (11) in terms of the physical fields $(\tilde{\xi}, \tilde{B})$. Clearly, we have $\mathcal{L}_{\text{inv}}(D_\mu \xi; B, D_\mu B) = \mathcal{L}_{\text{inv}}(D_\mu \tilde{\xi}; \tilde{B}, D_\mu \tilde{B})$. In terms of the physical fields the currents are again given by an expression like Eq. (14). However, the projection operator P is now replaced by $\tilde{P} = \{2\hat{C}_+(\theta), 2\hat{C}_3 - 2 \sin^2 \theta_W Q, Q\}$, $\hat{C}_+(\theta) = U_C^\dagger \hat{C}_+ U_C$, and $\bar{D}_\mu \tilde{\xi}_i$ is defined by an equation analogous to (10) where ξ_i and $\hat{C}_i$ are replaced by $\tilde{\xi}_i$, $\hat{C}_i(\theta)$.

Let us decompose the charged projection operator $\tilde{P}^{\text{ch}} = 2\hat{C}_+(\theta)$ as follows:

$$\tilde{P}^{\text{ch}} = \cos \theta \left[(V^{1+i2} + A^{1+i2}) + (V^{13-i14} + A^{13-i14}) \right] +$$

$$+ \sin \theta \left[(V^{4+i5} + A^{4+i5}) - (V^{11-i12} + A^{11-i12}) \right], \quad (19)$$

$$V^{k \pm i\ell} = V^k \pm i V^\ell \quad \text{etc.}$$

*) Ref. 16) contains also a discussion of the additional electromagnetic mass corrections.

Inserting Eq. (19) into the expression for j_μ^W we obtain for the charged weak current the generalized Cabibbo structure of the GIM scheme⁴⁾

$$j_\mu^W = \cos\theta [j_\mu^{1+i2} + j_\mu^{13-i14}] + \sin\theta [j_\mu^{4+i5} - j_\mu^{11-i12}] \quad (20)$$

From Eq. (20) follows the important result that the same $SU(4)$ rotation which was originally introduced in the mass terms in order to break chiral $SU(2) \times SU(2)$ also relates the different weak hadronic currents. Indeed, the correct form of the $SU(3)$ weak hadronic current will be reproduced when we identify the mixing angle θ with the Cabibbo angle. Finally, the neutral current can be rewritten in the well-known form

$$j_\mu^Z = j_\mu^{(2\hat{C}_3)} - 2 \sin^2\theta_w j_\mu^A \quad (21)$$

where $j_\mu^{(2\hat{C}_3)}$ is the current belonging to the third component of the weak isospin.

It is not difficult to prove the following PCAC relations for the (total) axial vector currents^{*)}

$$\begin{aligned} \partial^\mu j_{5\mu}^{W, tot} = \sqrt{2} f \{ & \cos\theta (m_{\pi^-}^2 \pi^- + m_F^2 F^-) + \\ & + \sin\theta (m_{K^-}^2 K^- - m_{D^-}^2 D^-) \} + O(\tilde{\xi}^3) \end{aligned} \quad (22)$$

*) Eqs (22) have been derived by using standard techniques¹⁷⁾. We have, e.g.,

$$\partial^\mu j_\mu^{W, tot} = - \frac{\delta \mathcal{L}}{\delta(\varepsilon_+/2\sqrt{2})} = - \frac{\delta \Delta \mathcal{L}}{\delta(\varepsilon_+/2\sqrt{2})}.$$

The variation $\delta \Delta \mathcal{L}_m$ then follows by using the field variation $(\varepsilon_+ = \frac{1}{\sqrt{2}}(\varepsilon_1 - i\varepsilon_2))$

$$\delta_+ e^{i2\tilde{\xi} \cdot A} = i \frac{\varepsilon_+}{2\sqrt{2}} ([C_+(\theta), e^{i2\tilde{\xi} A}]_- + \gamma^5 \{C_+(\theta), e^{i2\tilde{\xi} A}\}_+).$$

Other field variations do not contribute to (22).

$$\partial^\mu j_{5\mu}^{Z,tot} = f \left\{ m_{\pi^0}^2 \pi^0 + \frac{m_{\eta_8}^2}{\sqrt{3}} \eta_8 - \sqrt{\frac{2}{3}} m_{\eta_{15}}^2 \eta_{15} \right\} + O(\xi^3)^{(22 \text{ cont'd})}$$

whereas $\partial^\mu j_{\mu}^{A,tot} = 0$.

Let us note that the parameter f introduced in Eq. (4) by dimensional reasons is now recognized as a (averaged) meson decay constant. The case of equal decay constants, $f_\pi \simeq f_K \simeq f_D \simeq f_F \dots \simeq f$, considered here corresponds, in particular, to a $SU(4)$ invariant vacuum. If necessary, f splittings can, however, be taken into account in our model by replacing in the final expressions (e.g., currents)

$$\xi_i = \phi_i/f \rightarrow \hat{\xi}_i = \phi_i/f_i, \quad f \mathcal{D}_\mu \xi_i \rightarrow (\hat{f} \mathcal{D}_\mu \hat{\xi})_i$$

etc., where $\hat{f}$ is the diagonal matrix $\hat{f} = \text{diag}(f_1, \dots, f_{15})$.

6. THE EFFECTIVE LAGRANGIAN

Our final aim is to derive from the Lagrangian (11) (rewritten in terms of physical fields) an effective Lagrangian describing weak and weak/radiative decays of ordinary and charmed hadrons to first order in the weak interaction constant $G \sim 10^{-5}/M_P^2$. In order to get also leptonic and semileptonic decays of hadrons we must yet include the lepton sector of the standard Weinberg-Salam model¹¹⁾. The leptonic charged and neutral currents are

$$\begin{aligned} (j_\mu^W)_{\text{lept}} &= \bar{l} \gamma_\mu (2\hat{C}_+) l, \quad l = \begin{pmatrix} \nu \\ e \\ \mu \\ \nu' \end{pmatrix} \\ (j_\mu^Z)_{\text{lept}} &= \bar{l} \gamma_\mu (2\hat{C}_3 - 2\sin^2\theta_w Q) l. \end{aligned} \quad (23)$$

Using the standard approximations for the propagators of the W and Z bosons, we obtain the following effective Lagrangian describing processes in second order (κ^2, ρ^2) of weak perturbation theory

$$\mathcal{L}_{\text{eff}} = \mathcal{L}_{\text{weak}} + \mathcal{L}_{\text{weak/el.m.}} \quad (24)$$

where

$$\mathcal{L}_{weak} = -\frac{\bar{G}}{\sqrt{2}} \left(J^{\mu, Z} J_{\mu}^Z + J^{\mu, W} J_{\mu}^W \right), \quad (25)$$

$$\begin{aligned} \mathcal{L}_{weak/elm} = & -\frac{\bar{G}}{\sqrt{2}} e \frac{f^2}{8} A^{\mu} \left\{ H(C_+) (j_{\mu}^{0, W})_{mes}^+ + h.c. \right\} + \\ & + \left(H\left(\frac{C_3}{2}\right) - 4 \sin^2 \theta_w \langle [Q, e^{i2\tilde{\xi} \cdot A}] [Q, e^{-i2\tilde{\xi} \cdot A}] \rangle \right) (j_{\mu}^{0, Z})_{mes} \end{aligned} \quad (26)$$

The currents J_{μ} are the sum of the leptonic currents (23) and of the "free" hadronic currents $j_{\mu}^0 = j_{\mu}$ ($e = \kappa = 0$). $H(X)$ is a function describing the coupling of the vector bosons and pseudoscalar mesons⁹⁾

$$\begin{aligned} H(X) = & \langle [Q, e^{i2\tilde{\xi} \cdot A}] \left([X, e^{-i2\tilde{\xi} \cdot A}]_- - \gamma^5 \{X, e^{-i2\tilde{\xi} \cdot A}\}_+ \right) + \\ & + (\gamma^5, A_i) \rightarrow (-\gamma^5, -A_i) \rangle. \end{aligned} \quad (27)$$

Finally, $\bar{G} = \sqrt{2} \kappa^2 / 8\bar{M}_W^2$ is a corrected Fermi constant which coincides up to a negligible term $O(f^2(G/\sqrt{2})^2)$ with the usual expression $G = \sqrt{2} \kappa^2 / 8M_W^2$ *).

As we see, the effective weak Lagrangian (25) has the usual current $\times$ current structure of the conventional weak interaction theory. It involves $SU(3) \times SU(3)$ ^{8), 18)} and $SU(4) \times SU(4)$ ¹⁹⁾ substructures that have previously been derived only in a heuristic way. Using the mesonic part of the Lagrangian (25) we have calculated in the tree approximation some typical leptonic and semileptonic decays of charmed D and F mesons⁹⁾. The predictions agree in general with similar calculations²⁰⁾ based on the quark model, the PCAC hypothesis and certain

*) These corrections arise because the total masses $\bar{M}_{W, Z}$ of the vector bosons get contributions from both the Higgs mechanism in the lepton sector ($M_{W, Z}$) and the interaction with the hadron sector.

$$\bar{M}_W^2 = M_W^2 + \left(\frac{f\kappa}{\sqrt{2}} \right)^2, \quad M_Z^2 = M_Z^2 + \frac{1}{2} \left(\frac{f\kappa}{\cos \theta_w} \right)^2.$$

assumptions on the behaviour of form factors (cf., Table 2). Furthermore, it is worth mentioning that the weak neutral part of (25) does not contain strangeness changing terms like $-\bar{G}/\sqrt{2} (-f \partial^\mu K^0 + K^- \partial^\mu \pi^+ + \dots) (j_\mu^Z)_{\text{lept}}$. This way, the decays $K_L^0 \rightarrow \mu^+ \mu^-$, $K^- \rightarrow \pi^- e^+ e^-$, etc. are forbidden in agreement with the experimental data. The expression $\mathcal{L}_{\text{weak/elm}}$ (cf. Eq. (26)) describes "inner" weak/radiative processes with participating mesons.

7. SUMMARY AND DISCUSSION

The main purpose of this paper has been the construction of a non-linear Weinberg-Salam-GIM-type model based upon a phenomenological $SU(4) \times SU(4)$ meson-baryon Lagrangian. The electro-weak interactions have then been introduced into this strong interaction Lagrangian by a principle of minimal coupling. Our final result is an effective Lagrangian which describes weak and weak/radiative processes of hadrons and leptons. Its weak part (25) involves, in particular, $SU(3) \times SU(3)$ ^{8),18)} and $SU(4) \times SU(4)$ ¹⁹⁾ substructures that have previously been derived only in a heuristic way. Besides, the chiral unified model yields predictions for leptonic and semileptonic decays of charmed D and F mesons agreeing with quark model calculations ²⁰⁾. By extending the symmetry breaking mechanism of Oakes ¹²⁾ to chiral $SU(4) \times SU(4)$, we get, finally, a reasonable picture of the masses of the 15-plet of 0^- -mesons and 20-plet of $\frac{1}{2}^+$ -baryons, respectively.

To conclude, let us comment on the non-leptonic part of the weak interaction Lagrangian (25). As is well known, there is the old problem of the $|\Delta I| = \frac{1}{2}$ rule in non-leptonic decays according to which strangeness-changing transitions with $|\Delta I| = \frac{1}{2}$ are strongly enhanced in comparison with $|\Delta I| = 3/2$ transitions. Usually, such an enhancement is taken into account by multiplying the $|\Delta I| = \frac{1}{2}$ amplitudes by a factor $X \sim 1/\sin \theta \cos \theta$. The explanation of this enhancement factor requires, however, additional dynamical assumptions (e.g., octet or 20-plet dominance ^{20),21)}, inclusion of renormalization effects from the strong interaction ²²⁾, etc.) a discussion of which is outside the scope of this paper.

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APPENDIX A

Let us quote, for completeness, the explicit expressions for the 4×4 meson matrix

$$M = \sqrt{2} \sum_{i=1}^{15} V_i \phi_i$$

and the tensor representation of the baryon wave functions. We have

$$M = \begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta_8}{\sqrt{6}} + \frac{\eta_{15}}{\sqrt{12}} & \pi^+ & K^+ & \bar{D}^0 \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} + \frac{\eta_8}{\sqrt{6}} + \frac{\eta_{15}}{\sqrt{12}} & K^0 & D^- \\ K^- & \bar{K}^0 & -\frac{2}{\sqrt{6}}\eta_8 + \frac{\eta_{15}}{\sqrt{12}} & F^- \\ D^0 & D^+ & F^+ & -\frac{\sqrt{3}}{2}\eta_{15} \end{pmatrix}$$

where

$$\pi^\pm = \frac{1}{\sqrt{2}}(\phi_1 \mp i\phi_2), \quad D^\pm = \frac{1}{\sqrt{2}}(\phi_{11} \pm i\phi_{12}), \quad \bar{D}^0 = \frac{1}{\sqrt{2}}(\phi_9 - i\phi_{10}) \text{ etc.},$$

and¹³⁾

$$\begin{aligned} B_1^{[12]} &= \frac{A_2^+}{\sqrt{6}} - \frac{B_2^+}{\sqrt{2}} & B_1^{[13]} &= \frac{A_1^+}{\sqrt{6}} + \frac{B_3^+}{\sqrt{2}} & B_1^{[14]} &= \frac{\Lambda}{\sqrt{6}} + \frac{\Sigma^0}{\sqrt{2}} \\ B_1^{[23]} &= B_3^{++} & B_1^{[24]} &= \Sigma^+ & B_1^{[34]} &= p \\ B_2^{[12]} &= \frac{A_2^0}{\sqrt{6}} - \frac{B_2^0}{\sqrt{2}} & B_2^{[13]} &= B_3^0 & B_2^{[14]} &= \Sigma^- \\ B_2^{[23]} &= \frac{A_1^+}{\sqrt{6}} - \frac{B_3^+}{\sqrt{2}} & B_2^{[24]} &= \frac{\Lambda}{\sqrt{6}} - \frac{\Sigma^0}{\sqrt{2}} & B_2^{[34]} &= N \\ B_3^{[12]} &= -B_1^0 & B_3^{[13]} &= \frac{A_2^0}{\sqrt{6}} + \frac{B_2^0}{\sqrt{2}} & B_3^{[14]} &= -\Xi^- \\ B_3^{[23]} &= -\left(\frac{A_2^+}{\sqrt{6}} + \frac{B_2^+}{\sqrt{2}}\right) & B_3^{[24]} &= \Xi^0 & B_3^{[34]} &= -\sqrt{\frac{2}{3}}\Lambda \end{aligned}$$

$$B_4^{[12]} = -C_1^+ \quad B_4^{[13]} = C_2^+ \quad B_4^{[14]} = -\sqrt{\frac{2}{3}} A_2^0$$

$$B_4^{[23]} = -C_2^{++} \quad B_4^{[24]} = \sqrt{\frac{2}{3}} A_2^+ \quad B_4^{[34]} = \sqrt{\frac{2}{3}} A_1^+$$

$$B_i^{[ik]} = 0$$

notations		SU(3)	(I , I ₃)	S	C	mass(MeV)
P		8	(1/2 , 1/2)	0	0	949
N		8	(1/2 , -1/2)	0	0	958
Λ		8	(0 , 0)	-1	0	1109
Σ ⁺		8	(1 , 1)	-1	0	1179
Σ ⁰		8	(1 , 0)	-1	0	1191
Σ ⁻		8	(1 , -1)	-1	0	1204
Ξ ⁰		8	(1/2 , 1/2)	-2	0	1297
Ξ ⁻		8	(1/2 , -1/2)	-2	0	1315
A ₁ ⁺ ^[13]	C ₀ ⁺ ^[20]	3̄	(0 , 0)	0	1	2260
A ₂ ⁺	A ⁺	3̄	(1/2 , 1/2)	-1	1	2460
A ₂ ⁰	A ⁰	3̄	(1/2 , -1/2)	-1	1	2476
B ₃ ⁺⁺	C ₁ ⁺⁺	6	(1 , 1)	0	1	2426
B ₃ ⁺	C ₁ ⁺	6	(1 , 0)	0	1	2439
B ₃ ⁰	C ₁ ⁰	6	(1 , -1)	0	1	2452
B ₂ ⁺	S ⁺	6	(1/2 , 1/2)	-1	1	2618
B ₂ ⁰	S ⁰	6	(1/2 , -1/2)	-1	1	2629
B ₁ ⁰	T ⁰	6	(0 , 0)	-2	1	2791
C ₂ ⁺⁺	X _U ⁺⁺	3	(1/2 , 1/2)	0	2	3647
C ₂ ⁺	X _d ⁺	3	(1/2 , -1/2)	0	2	3664
C ₁ ⁺	X _S ⁺	3	(0 , 0)	-1	2	3894

Table 1

Table 2

Decay	Rate (chiral model) ⁹⁾	Rate (quark model) ²⁰⁾
$\bar{D}^- \rightarrow \mu \bar{\nu}$	$2 \times 10^8 \text{ s}^{-1}$	$1.9 \times 10^8 \text{ s}^{-1}$
$F^- \rightarrow \mu \bar{\nu}$	$3 \times 10^9 \text{ s}^{-1}$	$4.2 \times 10^9 \text{ s}^{-1}$
$\bar{D}^0 \rightarrow K^+ e \bar{\nu}$	$1 \times 10^{11} \text{ s}^{-1}$	$1.4 \times 10^{11} \text{ s}^{-1}$
$F^- \rightarrow \eta e \bar{\nu}$	$1 \times 10^{11} \text{ s}^{-1}$	$1.1 \times 10^{11} \text{ s}^{-1}$
$\bar{D}^0 \rightarrow \pi^+ e \bar{\nu}$	$1.2 \times 10^{10} \text{ s}^{-1}$	$1.4 \times 10^{10} \text{ s}^{-1}$
$\bar{D}^0 \rightarrow K^0 \pi^+ e \bar{\nu}$	$2.1 \times 10^6 \text{ s}^{-1}$	
$F^- \rightarrow K^+ K^- e \bar{\nu}$	$1.1 \times 10^9 \text{ s}^{-1}$	

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