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A REALISTIC DESCRIPTION OF DEEP INELASTIC STRUCTURE FUNCTIONS

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ABSTRACT

We show that a predictive and successful description of all observable deep inelastic processes can be achieved in a theoretical framework where quarks are confined.

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1. INTRODUCTION

Deep inelastic phenomena play a central role in contemporary subnuclear physics. The simple "partonic" behaviour that deep inelastic inclusive cross-sections do seem to exhibit has largely contributed to give quarks a prominent position in our understanding of hadronic matter. However, in the light of the striking elusiveness of the quark degrees of freedom, the question we must ask is: What is the real meaning of partons?

In the last few years a very useful theoretical toy like the parton model¹⁾ has, through QCD, developed into a very sophisticated approach to the physics of hadronic phenomena, ranging from deep inelastic inclusive cross-sections to jets and the structure of final states. Whatever the future developments of this theoretical approach may turn out to be, we consider its present formulation -- based on perturbation theory -- highly unrealistic, due to the unsolved problems of quark confinement and of the real physical link between partons and hadrons.

It is the desire of a realistic understanding of the partonic concept that prompted one of us (G.P.)²⁾ to propose a new way -- which was called the massive quark model (MQM) -- to look at the quark degrees of freedom, which is free from the very start from the difficulties of "unconfined quarks". It was shown³⁾ that a very general framework could be built on a few simple ideas -- to be summarized in Section 2 -- which is capable of describing deep inelastic scattering (DIS) structure functions in a realistic fashion and in great detail.

Bjorken scaling as well as its violations were found to be general and simple consequences of the approach and its structure was predicted with precision. In addition, several features of the final states were predicted, jets included, which have recently found very remarkable confirmation. In this paper we address ourselves to the MQM description of deep inelastic structure functions both in electron-nucleon and in neutrino-nucleon scattering.

The very successful description, which we are able to achieve, of all observable deep inelastic structure functions in terms of only six parameters, is, in our opinion, a counter-argument to the widespread belief that QCD is the only sensible theory of strong interactions which is available so far, especially in view of its present, very grave, unphysical elements.

2. THE BASIC MQM IDEAS AND THEIR APPLICATION TO DIS

The MQM approach has been described in detail in Ref. 3), where a full discussion and motivation of its basic ideas was given. Here we shall briefly summarize them:

- i) The quark degrees of freedom exist only in finite space-time domains (bags);
- ii) In the "bag" domains, quarks have the same behaviour as low (effective) mass hadrons, in particular high-energy quark Green's functions are Regge behaved;
- iii) Quarks, like in the quark-parton model, have a point-coupling to electromagnetic and weak currents.

Based on the previous three points we now proceed to describe the inclusive DIS cross-sections.

The process we wish to study appears in Fig. 1. The cross-section is obtained, through the optical theorem, by the imaginary part of the virtual forward Compton-like amplitude, whose MQM diagram is reported in Fig. 2.

In the Bjorken limit ($Q^2 \rightarrow \infty$, $\nu \rightarrow \infty$, with $Q^2/2\nu = x$ fixed), by the Regge ansatz [see (ii) above] the amplitude of Fig. 2 is dominated by the expansion of Fig. 3, where the sum runs over the leading Regge trajectories of hadronic physics (Pomeron, vector mesons, tensor mesons, etc.). Notice that the two terms appearing in the Mueller-Regge expansion correspond to the nucleon fragmentation into quarks and antiquarks. As a result we can write (neglecting for a moment the spin $\frac{1}{2}$ nature of quarks)

$$\text{Im } A(\nu, Q^2) \xrightarrow{\text{Bj-limit}} \sum_i \sum_F [f_{N,F}^i(x) + \bar{f}_{N,F}^i(x)] \left(\frac{2\nu}{M^2}\right)^{\alpha_i(0)}, \quad (2.1)$$

where F runs over the quark flavours ($u, d, s, c, \dots$), and $f_{N,F}^i$ [$\bar{f}_{N,F}^i$] are the fragmentation functions of the hadron N into a quark (antiquark) of flavour F . For deep inelastic electron-proton scattering we obtain, for instance (Q_F is the electric charge of the quark of flavour F)

$$\frac{\nu}{M^2} W_2^{ep}(\nu, Q^2) \xrightarrow{\text{Bj-limit}} \sum_i \sum_F Q_F^2 [f_{P,F}^i(x) + \bar{f}_{P,F}^i(x)] \left(\frac{2\nu}{M^2}\right)^{\alpha_i(0)}, \quad (2.2)$$

which implies that the leading trajectory -- the Pomeron -- having $\alpha_P(0) = 1$, gives rise asymptotically to Bjorken scaling, a completely non-trivial result. Thus we see that the famous Bjorken scaling behaviour is, at the quark level, nothing but the well-known Feynman scaling behaviour of high-energy hadronic physics.

In this way the MQM is able to reproduce most of the striking results of the quark-parton model, without the well-known difficulties related to the question: which kind of physical objects are the partons?⁴⁾ The very simple answer that the MQM gave, long before the advent of QCD²⁾, is that partons are just the quark degrees of freedom of hadronic matter, whose existence is confined inside the hadronic bags.

Equation (2.2) provides not only an explanation of asymptotic Bjorken scaling, but it describes also the approach to asymptotic scaling, through the contribution coming from non-leading Regge trajectories, with $\alpha_M(0) = \frac{1}{2}$. We shall show in Section 4 that this description is strongly supported by experimental data.

But our MQM analysis of structure functions can go much further by exploiting the full apparatus of the Mueller-Regge approach. In fact, we can study the limits $x \rightarrow 1$ and $x \rightarrow 0$, and obtain very useful information for building up the model of the DIS structure functions which shall be discussed in Section 3. For $x \rightarrow 1$, it is the triple Regge limit which determines the behaviours of the fragmentation functions, according to the diagrams in Fig. 4, which give us the results:

$$f_{N,F}^P(x) \xrightarrow{x \rightarrow 1} C_{N,F}^P (1-x)^{1-2\alpha_{2q}(0)} \quad (2.3)$$

$$\bar{f}_{N,F}^P(x) \xrightarrow{x \rightarrow 1} \bar{C}_{N,F}^P (1-x)^{1-2\alpha_{4q}(0)}, \quad (2.4)$$

where $\alpha_{2q}(0)$ and $\alpha_{4q}(0)$ are the intercepts of the $2q$ and $4q$ trajectories which dominate the triple Regge limit of the quark and antiquark fragmentation functions, respectively.

It is very easy to show²⁾ that between (2.3) and the asymptotic behaviour of the magnetic nucleon form factor, the well-known Drell-Yan-West connection holds⁵⁾:

$$G_M^N(Q^2) \xrightarrow{Q^2 \rightarrow \infty} (Q^2)^{\alpha_{2q}(0)-1} \quad (2.5)$$

When $x \rightarrow 0$, on the other hand, it is the double Regge limit which dominates. According to the diagrams in Fig. 5, we have

$$f_{N,F}^P(x) \xrightarrow{x \rightarrow 0} \sum_j C_{jN}^F x^{-\alpha_j(0)} = C_{PN}^F x^{-1} + C_{MN}^F x^{-1/2} + \dots \quad (2.6)$$

$$\bar{f}_{N,F}^P(x) \xrightarrow{x \rightarrow 0} \sum_j \bar{C}_{jN}^F x^{-\alpha_j(0)} = \bar{C}_{PN}^F x^{-1} + \bar{C}_{MN}^F x^{-1/2} + \dots \quad (2.7)$$

In the next Section we shall make use of these results in order to construct a simple and very predictive model for DIS scaling and its violations.

3. A MODEL FOR DIS STRUCTURE FUNCTIONS

We shall now use the MQM results described in the preceding Section to construct a simple model for deep inelastic scattering structure functions. We shall also take into account the non-scaling effects due to the exchange of non-leading meson trajectories with $\alpha_M(0) = \frac{1}{2}$. Besides these scaling-violating terms, which decrease like $(Q^2)^{-\frac{1}{2}}$, we have other scaling violations coming from kinematics (quark and target mass effects) which decrease like m^2/Q^2 ; these terms shall not be considered here, thus limiting our analysis to values of Q^2 above some minimum value, say, $Q_{\min}^2 = 2 \text{ GeV}^2$. There is, however, a region ($x \rightarrow 0$) where it was shown that one can take into account both the kinematical constraint

$$\nu W_2(Q^2, \nu) \xrightarrow{Q^2 \rightarrow 0} Q^2 \left. \frac{d}{dQ^2} \nu W_2(Q^2, \nu) \right|_{Q^2=0}, \quad (3.1)$$

and the dispersive contribution from nearby vector meson poles, to write (m_ρ^2 is the ρ mass-squared)

$$\nu W_2(Q^2, \nu) \xrightarrow{x \rightarrow 0} \frac{Q^2}{Q^2 + m_\rho^2} F_2(0), \quad (3.2)$$

which works accurately for all values of Q^2 (see Section 4). According to our previous discussion, for $x \rightarrow 0$ we can write for the generic quark (antiquark) fragmentation function $q(x, Q^2)$ [$\bar{q}(x, Q^2)$] the following expansions:

$$q(x, Q^2) \xrightarrow{x \rightarrow 0} \frac{A}{x} + \frac{B}{x^{1/2}} + \frac{A'}{x\sqrt{Q^2}} + \frac{B'}{x^{1/2}\sqrt{Q^2}} + O\left(\frac{1}{Q^2}\right) \quad (3.3)$$

$$\bar{q}(x, Q^2) \xrightarrow{x \rightarrow 0} \frac{A}{x} + \frac{\bar{B}}{x^{1/2}} + \frac{\bar{A}'}{x\sqrt{Q^2}} + \frac{\bar{B}'}{x^{1/2}\sqrt{Q^2}} + O\left(\frac{1}{Q^2}\right) \quad (3.4)$$

where use has been made of the Pomeron charge conjugation properties. When $x \rightarrow 1$, we write on the other hand:

$$q(x, Q^2) \xrightarrow{x \rightarrow 1} C(1-x)^2 + \frac{D}{\sqrt{Q^2}} x^{1/2} (1-x)^{5/2} + O\left(\frac{1}{Q^2}\right) \quad (3.5)$$

$$\bar{q}(x, Q^2) \xrightarrow{x \rightarrow 1} E(1-x)^7 + \frac{F}{\sqrt{Q^2}} x^{1/2} (1-x)^{13/2} + O\left(\frac{1}{Q^2}\right) \quad (3.6)$$

where it is worth noticing the powers $5/2$ and $13/2$ of the $(1-x)$ terms and the assumption that, according to dimensional arguments⁷⁾,

$$\alpha_{nq}(0) = 1 - n. \quad (3.7)$$

Our results for the DIS structure functions for the process E, can be written as

$$2x F_1^{(E)}(x, Q^2) = F_2^{(E)}(x, Q^2) = x [q^{(E)}(x, Q^2) + \bar{q}^{(E)}(x, Q^2)] \quad (3.8)$$

and

$$xF_3^{(E)}(x, Q^2) = x [q^{(E)}(x, Q^2) - \bar{q}^{(E)}(x, Q^2)], \quad (3.9)$$

$$q^{(E)}(x, Q^2) = \alpha^{(E)} (1-x)^3 + \frac{\theta_2^{(E)}}{\sqrt{x}} (1-x)^5 + \frac{\tilde{\theta}^{(E)}}{2} \frac{(1-x)^9}{x} \frac{Q^2}{Q^2 + m_p^2} \quad (3.10)$$

$$+ \frac{\delta^{(E)}}{\sqrt{Q^2}} x^{1/2} (1-x)^{5/2} + O\left(\frac{1}{Q^2}\right)$$

$$\bar{q}^{(E)}(x, Q^2) = \frac{\theta_1^{(E)}}{x^{1/2}} (1-x)^7 + \frac{\lambda}{\sqrt{Q^2}} x^{1/2} (1-x)^{13/2} + \frac{\tilde{\theta}^{(E)}}{2} \frac{(1-x)^9}{x} \frac{Q^2}{Q^2 + m_p^2} \quad (3.11)$$

$$+ O\left(\frac{1}{Q^2}\right).$$

A few remarks are in order. Due to the accuracy of the parametrization (3.2) we have neglected altogether the $O(1/\sqrt{Q^2})$ terms in (3.3) and (3.4), owing to their experimental smallness. Both (3.8) and (3.9) are given as sums of terms of the form $x^\alpha (1-x)^\beta$, the values of α and β being chosen in such a way as to yield the correct limits for $x \rightarrow 0$ and $x \rightarrow 1$, as well as a smooth and "minimal" extrapolation between these two limits. Finally, the Callan-Gross relation, which in the MQM receives only $O(1/Q^2)$ violations, has been implemented in (3.8).

Thus we are able to describe the three structure functions for any deep inelastic scattering process E in terms of the parameters:

$$\alpha^{(E)}, \theta_1^{(E)}, \theta_2^{(E)}, \tilde{\theta}^{(E)}, \delta^{(E)} \text{ and } \lambda^{(E)}.$$

Apart from the usual quark-parton model relationships³⁾ between the different observable deep-inelastic scattering processes -- electroproduction, charged and neutral currents scattering on protons and neutrons -- with the help of simple models for $x \rightarrow 0$ and $x \rightarrow 1$ behaviours we obtain all deep inelastic processes above $Q^2 \cong 2 \text{ GeV}^2$ in terms of only six parameters. Let us see how this can be done.

a) Double Regge limit ($x \rightarrow 0$)

The model we make for the double Regge amplitude is given by the diagrams of Fig. 6⁸⁾. Assuming for baryons $SU(6)$ symmetry, and 56 dominance in the intermediate state, and taking into account that P exchange is a singlet in the t channel, while M exchange is a singlet in the s channel⁹⁾, we compute the $SU(3)$ Clebsch-Gordan coefficients of Table 1.

b) Triple Regge limit ($x \rightarrow 1$)

The diagram in Fig. 7 describes the model for the triple Regge limit. Due to the $x \rightarrow 1$ limit, however, SU(6) wave functions are not necessarily accurate. There are strong reasons for believing¹⁰⁾ that for $x \rightarrow 1$ the configuration dominates in which the active quark spin coincides with the proton spin; this fact, together with the SU(3) structure of the P and M trajectories, produces the entries for α and δ of Table 2.

4. RESULTS AND COMPARISON WITH EXPERIMENT

By trial and error, without attempting any χ^2 optimization, we have determined the six free parameters as:

$$\alpha_0 = .85 ; \theta_{10} = .10 ; \theta_{20} = .41 ; \tilde{\theta}_0 = .13 \quad (4.1)$$

and

$$S_0 = 2.11 \text{ GeV} , \lambda_0 = .01 \text{ GeV} . \quad (4.2)$$

The resulting structure functions for different values of Q^2 and different DIS processes are calculated by making use of Table 2 and of the expressions of the type (3.8) and (3.9). In Fig. 8 we have reported $F_2(x, Q^2)$ for electroproduction on proton and neutron, and for the different Q^2 values 4, 40, 5000 GeV. We notice that because of the small value of δ_0 in en deep inelastic scattering, scaling violations on neutrons are much smaller than on protons. Figure 9 exhibits $F_2(x, Q^2)$ and $xF_3(x, Q^2)$ for the same values of Q^2 for neutrino scattering off proton and neutron targets; the striking fact here is that $xF_3^{VP}(x, Q^2)$ does not show any scaling violation, while νn scattering has substantial departures from a scaling behaviour. The predictions for neutral currents scattering for $Q^2 = 4$ and 40 GeV² appear in Fig. 10.

Let us now turn to a comparison with the data. Figure 11 compares the CDHS data on charged currents scattering off an isoscalar target with our prediction (3.2) of scaling violations for $\nu W_2(x, Q^2)$ when $x \rightarrow 0$. Electro- and muon-production experiments at SLAC and FNAL are compared with our predictions in Fig. 12, for three different Q^2 values (0, 12.5 and 22.5 GeV²) as compiled by H. Anderson¹²⁾. Figures 13 and 14 summarize the experimental comparison with the CDHS data¹¹⁾ on ν scattering off an isoscalar target. Finally, Figs 15 and 16 take up the issue of moments, and compare our results with BEBC¹³⁾ and FNAL data. The agreement is outstanding throughout.

5. CONCLUSIONS AND OUTLOOK

Figures 11 to 16 are our best answer to the question whether there exist any alternative to the QCD picture of DIS. We have shown that the MQM not only provides a realistic approach to deep inelastic scattering phenomena, but it does so in an extremely accurate fashion. The emerging picture is one of scaling violations playing a significant role only for moderate values of Q^2 (i.e., $Q^2 \lesssim 10 \text{ GeV}^2$) which is strongly supported by present data.

It remains to be shown that final states too are accurately described in this approach; perhaps this will provide an even more convincing argument in favour of our contention that the MQM is a valid alternative to the prevalent theoretical ideas based on QCD. We are presently working on this very interesting problem.

Table 1

The SU(3) Clebsch-Gordan coefficients for the amplitude of Fig. 6.
By definition $\langle B | J^\dagger J | B \rangle = \text{Tr}(J^\dagger J) \bar{K}(\bar{B}B) + \text{Tr}([B, \bar{B}] J^\dagger J)$.

(i,j)	q	$\bar{q}$
(P,P)	$3 \text{ Tr}(JJ^\dagger) \text{Tr}(\bar{B}B)$	$3 \text{ Tr}(JJ^\dagger) \text{Tr}(\bar{B}B)$
(P,M)	$3 \langle B J^\dagger J B \rangle$	$\text{Tr}(JJ^\dagger) \text{Tr}(\bar{B}B)$
(M,P)	$\text{Tr}(JJ^\dagger) \text{Tr}(\bar{B}B)$	$3 \langle B J^\dagger J B \rangle$
(M,M)	$\langle B J^\dagger J B \rangle$	$\langle B J^\dagger J B \rangle$

Table 2
The relative weights of the parameters describing all
observable deep inelastic structure functions ($y = \sin^2 \theta_W$)

Parameter	Electroproduction			Charged currents		Neutral currents			
	ep	en		vp	vn	F_2^{vp}	F_2^{vn}	F_3^{vp}	F_3^{vn}
$\tilde{\theta}/\tilde{\theta}_0$	2	2		6	6	$3(1 - 2y + \frac{20}{9}y^2)$	$3(1 - 2y + \frac{20}{9}y^2)$	0	0
θ_2/θ_0	3	2		6	12	$3(\frac{3}{2} - \frac{10}{3}y + 4y^2)$	$3(\frac{3}{2} - \frac{8}{3}y + \frac{8}{3}y^2)$	$3(\frac{3}{2} - \frac{10}{3}y)$	$3(\frac{3}{2} - \frac{8}{3}y)$
θ/θ_{10}	2/3	2/3		4	4	$1 - 2y + \frac{20}{9}y^2$	$1 - 2y + \frac{20}{9}y^2$	$1 - 2y$	$1 - 2y$
α/α_0	7/9	1/3		2/3	10/3	$1 - \frac{22}{9}y + \frac{28}{9}y^2$	$1 - \frac{14}{9}y + \frac{4}{3}y^2$	$1 - \frac{22}{9}y$	$1 - \frac{14}{9}y$
δ/δ_0	1	1/9		0	32/9	$\frac{8}{9} - \frac{8}{3}y + 4y^2$	$\frac{8}{9} - \frac{8}{9}y + \frac{4}{9}y^2$	$\frac{8}{9} - \frac{8}{3}y$	$\frac{8}{9}(1 - y)$
λ/λ_0	3	2		6	12	$3(\frac{3}{2} - \frac{10}{3}y + 4y^2)$	$3(\frac{3}{2} - \frac{8}{3}x + \frac{8}{3}y^2)$	$3(\frac{3}{2} - \frac{10}{3}y)$	$3(\frac{3}{2} - \frac{8}{3}y)$

REFERENCES AND FOOTNOTES

- 1) R.P. Feynman, Photon-Hadron interactions, Benjamin N.Y. (1972).
- 2) G. Preparata, Phys. Rev. D7 (1973) 2973.
- 3) A complete account of the MQM developments is in G. Preparata in Lepton Hadron Structure, A. Zichichi Ed., Academic Press (1975), p. 54.
- 4) Curiously enough Feynman in his famous book never makes the identification of quarks and partons. The quark-parton model was developed by many authors only later, as a simplified approach to Light-Cone Physics. The MQM identification of confined quarks and partons was, when it was proposed (1972), a fairly bold theoretical idea.
- 5) S.D. Drell and T.M. Yan, Phys. Rev. D1 (1970) 2402;
G.B. West, Phys. Rev. Letters 24 (1970) 1206.
- 6) G. Preparata, Phys. Letters 36B (1972) 56.
- 7) This follows from attributing a canonical short distance behaviour to the quark degrees of freedom. For form factors [see (2.5)] this relation works obviously out for mesons ($n = 1$) and for baryons ($n = 2$).
- 8) The diagram in Fig. 6 was shown in Ref. 3) to give a good description of high energy hadronic cross-sections.
- 9) This is a simple consequence of SU(3) symmetry.
- 10) G. Preparata and J. Soffer (in preparation).
- 11) CDHS Collaboration, "Inclusive interaction on High-energy neutrinos and antineutrinos in iron" (submitted to Zeitschrift für Physik).
- 12) We have taken these data from H. Anderson's talk at the Highly Specialized Seminar on "Probing Hadrons with Leptons", held in Erice, March 1979.
- 13) ABCLOS Collaboration, Oxford University Preprint, Ref. 16/78.

FIGURES CAPTIONS

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- Figure 2 : The MQM diagram for the DIS cross-section.
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- Figure 13 : CDHS-data for $F_2^{\nu N}$, for different x-bins and Q^2 -values.
- Figure 14 : CDHS-data for $\times F_3^{\nu N}$, for different x-bins and Q^2 -values.

Figure 15 : The log-log plot of different moments as observed by the BEBC-collaboration [Ref. 13)] compare with our predictions.

Figure 16 : Comparison of our predictions for $M_N(Q^2)$ with the FNAL data on $\mu - p$ scattering [Ref. 12)].

FIG.1

$$q^2 = -Q^2$$

$$p \cdot q = \nu$$

$$W^2 = (p+q)^2 = M^2 + 2\nu - Q^2$$

FIG.2

FIG.3

FIG.4

FIG.5

FIG. 6

FIG. 7

FIG. 9

FIG.10

FIG. II

FIG. 12

FIG. 13

Q^2 (GeV^2)

FIG. 14

FIG. 15

FIG. 16