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THE DECAY OF HEAVY QUARKONIA STATES
AND THE TRIPLE GLUON VERTEX

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A B S T R A C T

We present calculations, based on summing leading logarithms, describing the strong decay of heavy resonances ($M \geq M_Y$) into other heavy flavours ($m \geq m_c$), in particular the corresponding multiplicities. The results are derived in (i) QCD, where allowance is made also for fix point coupling, and (ii) in a quark-gluon theory with no gluon self-coupling. It is shown that due to calculational shortcomings inherent in the method when applied to case (ii), one needs at least three heavy resonances $[Y, (tt), ?]$ to decide rigorously between cases (i) and (ii).

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1. - INTRODUCTION

It has been suggested again and again by numerous authors ¹⁾ that heavy quarkonia states formed in e^+e^- annihilation offer excellent opportunities to test ideas which are developed on possible theories of the strong interaction, in particular QCD. In most cases it was suggested to measure global quantities, like total cross-sections or widths, and also more detailed but still inclusive quantities relating to the production of hadronic jets. Yet such jet studies are hampered by the fact that, so far, one cannot reliably calculate how quark and gluon properties which - as it is commonly argued - can be computed in QCD perturbation theory, are transformed into properties which show up in the finally observed hadrons. It can be argued with some force that also this fragmentation process can be calculated up to corrections which are increasingly small at sufficiently high energies ; yet some (admittedly crude) estimates indicate that fairly large energies are required, $E \gtrsim 30$ GeV ; furthermore it is quite desirable to design tests of QCD which do not depend so crucially on the quark fragmentation process.

One idea was put forward in this direction by Fritzsche and Streng ²⁾ : to observe and measure quarkonium decay into heavy quark flavours like $Y \rightarrow c\bar{c} + X$, or $(t\bar{t}) \rightarrow c\bar{c} + X$, $b\bar{b} + X$, if such a toponium resonance is found. The basic point in the argument is to note the following : as soon as a heavy flavour, like charm, is produced with a lot of phase space available to it, like in $Y \rightarrow c\bar{c} + X$, that heavy flavour is very unlikely to "disappear" or to be annihilated although it is produced as a $c\bar{c}$ quark pair : it is neither very probable that the c and $\bar{c}$ quarks receding from each other after their production will reapproach each other again so that they can annihilate nor that the, e.g., rapidity gap between them is filled up completely with $c\bar{c}$ pairs created from the vacuum.

Therefore as soon as charm (or any other heavy flavour) is produced, the event is clearly "flavoured" independent of the details of the quark fragmentation function.

The authors of Ref. 2) found for $Y \rightarrow c\bar{c} + X$ a branching ratio of roughly $\sim 3\%$ in lowest order QCD considering only production of charm close to threshold. It was later pointed out that such a small branching ratio into charm reduces the multiplicity in the decay of a heavy resonance which one expects otherwise since charm decays with large multiplicity ³⁾.

In principle the best way to determine how far one can trust a lowest order calculation consists in computing higher order contributions. This is a rather formidable task which has not been solved in general [see, on the other hand, Ref. 4]. One way out of this dilemma is to determine higher order contributions in an approximate scheme ; such a scheme has been used extensively, namely the leading log approximation.

Furmanski, Petronzio and Pokorski ⁵⁾ have applied this method to a similar problem, namely the charm multiplicity generated in the continuum region of e^+e^- annihilation, where charm is produced via gluon decay into a $c\bar{c}$ pair. They find a rather small effect for energies below ~ 70 GeV, which is not surprising since one looks for an effect of order α_s^2 . Furthermore the effect has to compete with a very large background, namely direct, electromagnetic production of charm by the virtual photon.

The situation is different and (apart from the obvious fact that one has to find heavy resonances) more advantageous for quarkonia decays : in QCD the "direct" strong decays of heavy quarkonia are expected to proceed via gluons (at least two for the para- and at least three for the ortho-states) ; therefore the branching ratio into charm via gluon decay constitutes a larger effect $\sim O(\alpha_s)$ and furthermore in the "direct" decays there is no large background of electromagnetically produced charm.

We shall therefore apply quite analogous leading log methods to quarkonia and we shall not restrict ourselves to the production of heavy flavours close to their threshold where application of perturbation theory might not be a safe procedure.

The paper will be organized as follows : in Section 2 we shall discuss the decay of heavy para-states into charm in lowest order ; in Section 3 we will include all higher orders of QCD in the leading log limit, and in Section 4 we shall apply the same procedure to the decays of heavy ortho-states ; finally in Section 5 we will describe a strategy by which one can try not only to confirm or reject leading log QCD results, but also to distinguish them from predictions derived in other models, in particular such with no triple gluon vertex.

2. - DECAY OF PARA-STATES INTO CHARM IN LOWEST ORDER

Assuming that the quarkonium state is "pure" or "unmixed", meaning that its content of lighter quarks (namely u, d, s and c for the Y) can be neglected, charm can be produced only via gluon fragmentation. Therefore one has to determine the multiplicity of off-shell gluons with a virtual mass $k^2 \geq 4m_c^2$, denoted by $n_g(k^2)$. From that quantity one derives the charm multiplicity in the usual fashion for a resonance of mass M_R :

$$N_c(M_R^2) \approx \frac{1}{2\pi} \frac{2}{3} T_F \int_{4m_c^2}^{M_R^2} \frac{dk^2}{k^2} \alpha_s(k^2) n_g(k^2) \quad (2.1)$$

$$T_F[SU(3)] = \frac{1}{2}$$

where it will be explained below why one uses the "running" coupling. Also $n_g(k^2)$ is obtained in the usual fashion :

$$n_g(k^2) = \frac{1}{\Gamma_R} \int dx_g \frac{d\Gamma(R \rightarrow g(k^2) + X)}{dx_g} \quad (2.2)$$

We shall always make the usual assumption : like in positronium, the bound state character of the heavy resonance can be adequately described by some hadronic wave function which is factored off from the annihilation of the "free" constituents into on- and off-shell gluons [see also Ref. 4]. This wave function thus drops out from $n_g(k^2)$, Eq. (2.2), which can therefore be determined in perturbation theory. First we will treat the decays of para states, which in lowest order leads to a two-gluon final state (Fig. 1).

To simplify matters we will furthermore assume that the heavy quarks Q and $\bar{Q}$ move non-relativistically inside the resonance ; present potential models show that this is a very good approximation already for the Y.

A straightforward calculation yields for off-shell gluons

$$n_g(k^2) = 2 \left(1 - \frac{k^2}{M_R^2} \right) \quad (2.3)$$

where the factor two just reflects the two-gluon final state, i.e., $n_g(k^2 = 0) = 2$.

At first one might wonder why there is no infra-red singularity generated by the heavy quark propagator. Kinematics, of course, do not allow such singularities for the decay into two massless on-shell gluons ; yet they do not rule out infra-red singularities for off-shell gluons, when one gluon carries a virtual mass $\sim M_R^2$ and the other gluon is soft. Performing the calculation one finds that the dynamics do not allow such a singularity to occur. That this has to be the case can be seen from a general argument developed for the decays of ortho- and para-positronium where it rests on the notion of charge conjugation ⁶⁾. It is true that gluons carry colour charge and are therefore not eigenstates of C conjugation, unlike photons ; but as long as in a given order no triple (or quartic) gluon vertex appears, as in our lowest order diagram, the topology and symmetry properties of such a QCD diagram are the same as for a QED diagram. Therefore up to some trivial group factors the expressions have to be the same also.

Infra-red singularities are usually cancelled between real and virtual radiative corrections. Yet for positronium decays virtual (Fig. 2a) and real (Fig. 2b) radiative corrections differ under C conjugation since the photon is odd under C ; therefore they cannot be used to cancel each other's singularities. So there better be no singularities in the radiative corrections to para- and ortho-positronium decays in the first place - a wish which is fortunately borne out by the actual calculation. The same is of course also true for radiative corrections into off-shell photons. To lowest order one thus finds for the charm multiplicity :

$$\begin{aligned}
 N_c^{(1)}(M_R^2) &= \frac{1}{3\pi} \int_{4m_c^2}^{M_R^2} \frac{dk^2}{k^2} \alpha_s(k^2) \left(1 - \frac{k^2}{M_R^2}\right) \\
 &= \frac{4}{33-2N_f} \left[\log\left(\frac{\log \frac{M_R^2}{\Lambda^2}}{\log \frac{4m_c^2}{\Lambda^2}}\right) - \frac{\Lambda^2}{M_R^2} \left(\log\left(\frac{\log \frac{M_R^2}{\Lambda^2}}{\log \frac{4m_c^2}{\Lambda^2}}\right) + \log \frac{M_R^2}{4m_c^2} + \dots \right. \right. \\
 &\quad \left. \left. \dots + \frac{1}{n \cdot n!} \left(\log^n\left(\frac{M_R^2}{\Lambda^2}\right) - \log^n\left(\frac{4m_c^2}{\Lambda^2}\right) \right) + \dots \right) \right] \quad (2.4)
 \end{aligned}$$

where we have used

$$\alpha_s(k^2) \approx \frac{12\pi}{(33-2N_f) \log \frac{k^2}{\Lambda^2}}$$

$N_f \equiv$ number of active flavours

If one had used a coupling globally fixed at M_R^2 instead, one would have obtained

$$\tilde{N}_c^{(1)}(M_R^2) = \frac{\alpha_s(M_R^2)}{3\pi} \left(\log \frac{M_R^2}{4m_c^2} - 1 + \frac{4m_c^2}{M_R^2} \right) \quad (2.5)$$

Since

$$\alpha_s(k^2) \geq \alpha_s(M_R^2) \quad , \quad |k^2| \leq M_R^2$$

one finds

$$N_c^{(1)}(M_R^2) > \tilde{N}_c^{(1)}(M_R^2)$$

Yet one cannot decide, just based on lowest order considerations, which is the more efficient way to treat the coupling. We will come back to this point in the next section. The occurrence of terms like $\sim \log(4m_c^2/\Lambda^2)$ and $\log(M_R^2/4m_c^2)$ shows that the charm multiplicity N_c is not an infra-red "safe" quantity, i.e., the limit $m_c \rightarrow 0$ cannot be performed. The reason for it can already be seen on general grounds : one knows (from explicit calculation or from applying the Lee-Kinoshita-Nauenberg construction ^{7),8)} that after cutting a diagram like the one in Fig. 3, which describes $R \rightarrow c\bar{c} + X$ or $(\mu^+\mu^-) \rightarrow e^+e^-\gamma$ in lowest order, and adding all three cut contributions, one obtains a finite result. Yet the physical quantity we want to calculate, namely real charm production, is described just by cut (b) in the diagram, which produces an infra-red expression $\sim \log(M_R^2/4m_c^2)$.

The assumption which lies at the basis of our argument is the following : although N_c depends crucially on the infra-red regime $4m_c^2 \ll M_R^2$, it does it in a calculable way since the charm production threshold $4m_c^2$ is much larger than the typical scale of strong interactions, characterized by Λ . The situation one has in mind can therefore be described in the following fashion :

$$\Lambda^2 \ll 4m_c^2 \ll M_R^2 \quad (2.6)$$

So far we have basically described a lowest order calculation. Yet, as said in the Introduction, one wants to make some estimates on higher-order contribution be it only to make sure that they can be safely neglected. We will tackle this problem via the leading log approach analogous to the one described in Ref. 5).

3. - DECAY OF PARA-STATES INTO CHARM IN THE LEADING LOG APPROACH

By leading log approach we mean, like in other applications considered in e^+e^- annihilation ^{5),9)}, that one keeps only those contributions which contain two large logarithms per coupling α_s ; one stems from the poles of the vertex function ¹⁰⁾ in the infra-red region

$$P(q \rightarrow qg) \sim C_F \frac{1+x_q^2}{1-x_q} = C_F \frac{1+(1-z_g)^2}{z_g} \quad (3.1a)$$

$$P(g \rightarrow gg) \sim 2C_g \left(\frac{1-z_g}{z_g} + \frac{z_g}{1-z_g} + z_g(1-z_g) \right) \quad (3.1b)$$

$$P(g \rightarrow q\bar{q}) \sim T_F (x_q^2 + (1-x_q)^2) \quad (3.1c)$$

where x_q $\left[\frac{z_g}{g} \right]$ denote the fraction of momentum carried by the quark or antiquark $\left[\text{gluon} \right]$, and C_F , C_g and T_F are the colour group factors :

$$C_F = \frac{4}{3}, \quad C_g = 3, \quad T_F = \frac{1}{2}$$

for $SU(3)$.

The other large logarithm originates in the integration over small angles $\sim \int dk^2/k^2$.

It has been shown by various authors that in a physical gauge like the axial gauge (where $n_\mu \cdot A_\mu = 0$) only uncrossed ladder diagrams can yield the maximal number of large logarithms from the small angle regime.

It was pointed out in Ref. 5) that this is not true in general for the infra-red regime.

When gluons couple to fermion lines, in particular to the initial fermion lines, then also crossed diagrams can yield leading log contributions in a general axial gauge thus destroying a simple probabilistic interpretation and, to a large degree, also the calculability of such diagrams. If one chooses a special axial gauge, namely a planar gauge, this problem is avoided since then again only uncrossed ladders contribute.

The situation is less complicated in the case under study here : we consider emission of on- and off-shell gluons off the annihilating heavy quark line. There are three possible types of large infra-red logarithms in $n_g(k^2)$: (i) $\log(M_R^2/k^2)$, (ii) $\log(M_R^2/\lambda^2)$ and (iii) $\log(k^2/\lambda^2)$, where λ is a small gluon mass introduced to regularize possible infra-red singularities.

As pointed out above there are no such singularities in $R \rightarrow g(k^2)g$.

The same is true for $R \rightarrow g(k^2)gg$, be R a para- or ortho-state (which is allowed, since gluons are not C eigenstates). The argument goes in two steps as follows.

(a) Be $M_R^2 > k^2 > 0$ and $\lambda^2 \rightarrow 0$. Then at most one of the gluons can become soft, since $k^2 \neq M_R^2$. If there were a corresponding infra-red singularity, then it had, according to the Lee-Kinoshita-Nauenberg (KLN) theorem, to be cancelled by a singularity in a process where one of the gluons is virtual ; yet for photons this process would have opposite sign under C and therefore could not contribute to the real production process if, as it is the case for such resonances, C is well defined. Therefore the original real photon diagram cannot contain such a singularity, and neither does the gluon diagram, which to this order is identical to the photon diagram up to trivial group theoretical factors. Thus there can be no singularity in λ^2 and a fortiori, since $k^2 \neq M_R^2$ neither a term $\sim \log(M_R^2/\lambda^2)$ nor $\sim \log(k^2/\lambda^2)$, apart from the combination $\log(M_R^2/\lambda^2) - \log(k^2/\lambda^2) = \log(M_R^2/k^2)$.

(b) Then letting $k^2 \rightarrow 0$ (and $\lambda^2 = 0$) one sees immediately that no term $\sim \log(M_R^2/k^2)$ can occur, since the matrix element for $R \rightarrow 3g$ (or $R \rightarrow 3\gamma$) is finite ; this can be seen either from explicit calculation ¹¹⁾ or by applying an argument similar to the one given above, based on the KLN construction and C symmetry for the photon diagrams ⁶⁾ ; this is best expressed and used by the cut method suggested by Nakanishi ⁸⁾.

Therefore there are, to lowest order, no infra-red singularities or large logarithms in $R \rightarrow g(k^2)gg$. The analogous argument can also be exploited to demonstrate the same result to hold to lowest order also for $R \rightarrow g(k^2)3g$ or in general $R \rightarrow g(k^2)ng$, $n=1,2,\dots$

Thus a gluon coupling to the initial heavy quarks which annihilate does not produce any large logarithm and can therefore be ignored in the leading log approach.

The only allowance has to be made for the lowest order diagram : $R \rightarrow g(k^2)g$ for a para- and $R \rightarrow g(k^2)gg$ for an ortho-state. Although no large logarithm is generated, no "penalty" in form of α_s^2 or α_s^3 has to be paid either, since one normalizes with the total width which is of the same order α_s^2 or α_s^3 .

Thus the diagrams of Fig. 1 contribute to the decay of a para-state also in the leading log limit, but not those of Fig. 4. Furthermore due to the same reasoning typical QCD interference terms like the ones shown in Fig. 5 can be ignored.

Therefore, for the problem under study here, we do not have to use a planar gauge like the authors in Ref. 5) ; instead we can use any light-like axial gauge :

$$n \cdot A = 0 \quad , \quad n^2 = 0$$

Using the same arguments as applied in Ref. 5) - which we are entitled to since we are also working in a light-like gauge - one finds that only uncrossed gluon ladder diagrams (Fig. 7) contribute, but not diagrams as sketched in Fig. 6.

Since the vertex $g \rightarrow q\bar{q}$ [Eq. (3.1c)] does not contain an infra-red pole, real quark production does not contribute in the leading log limit - apart from the last step when a gluon turns into $c\bar{c}$ pairs, since we study real charm production.

So far we have discussed only bare diagrams ; now we include also loops, which can yield large logarithms from the infra-red and from the ultra-violet regime. As has been described most explicitly in Ref. 4), all insertions into the initial heavy fermion line either cancel due to Ward identities

or can be factored into the hadronic wave function describing the bound state ; this wave function drops out from any ratio or normalized quantity like $n_g(k^2)$ [Eq. (2.2)].

As in the case of leptonproduction ¹²⁾, only self-energy insertions into the gluon lines yield large logarithms (in the axial gauge there are no ghost loops). Gluon loops give double logarithms, yet according to the LKN construction logarithmic contributions from real gluon production (Fig. 8a) cancel those from virtual gluon radiation (Fig. 8b) as far as total transition rates are concerned. But the quantity we want to calculate is not the total transition rate, but the multiplicity of gluons with virtual mass k^2 , which is one for the virtual process and two for the real process. Due to this weighting one includes just the one particle cut (Fig. 9) in the calculation of $n_g(k^2)$.

Finally, there is a subset of virtual contributions which we want to include although, diagram by diagram, they give only single logarithms : they are the virtual gluon vertex corrections, whose effect is usually described in terms of a running coupling $\alpha_s(k^2)$. The main reason for including them is the fact that the renormalization group equation allows us (at least in the deep Euclidean region) to sum all those single log contributions into the well-known expression for $\alpha_s(k^2)$. It was shown in Ref. 5) that it is consistent to include just the subset of single logarithms coming from the vertex renormalization.

Now we are in a position to write down the equation, which has to be satisfied by $n_g(k^2)$ (Fig. 10) :

$$n_g(k^2) \approx 2\left(1 - \frac{k^2}{M_R^2}\right) + \int_{k^2}^{M_R^2} \frac{d\ell^2}{\ell^2} \frac{\alpha_s(\ell^2)}{2\pi} \int_{k^2/\ell^2}^1 dz_g \frac{2c_g}{z_g} n_g(\ell^2) \quad (3.2)$$

The triple gluon vertex function [Eq. (3.1b)] contains two poles, a "soft" one at $z_g \approx 0$ and a "hard" one at $z_g \approx 1$; yet only the soft one has to be considered, the hard one at $z_g \approx 1$ just reflects the infrared singularity coming from the lower on-shell gluon line (like in Fig. 9), which is cancelled by a corresponding virtual correction (like in Fig. 8b) as was pointed out above.

Later we will integrate over k^2 to obtain the charm multiplicity [Eq. (2.1)] ; in a leading log computation we can therefore drop the expression k^2/M_R^2 in the first term on the right-hand side of Eq. (3.2) and thus simplify it further :

$$n_g^{para}(k^2) \approx 2 + \int_{k^2}^{M_R^2} \frac{dl^2}{l^2} \frac{\alpha_s(l^2)}{2\pi} \int_{k^2/l^2}^1 dz_g \frac{2c_g}{z_g} n_g(l^2) \quad (3.3)$$

Equation (3.3) is solved by the same procedure as the corresponding equation in Ref. 5), just the boundary conditions are different. Rewriting Eq. (3.3) in terms of new variables

$$\eta \equiv \log \frac{l^2}{\Lambda^2}, \quad y \equiv \log \frac{k^2}{\Lambda^2}, \quad Y = \log \frac{M_R^2}{\Lambda^2} \quad (3.4)$$

as

$$n_g(y, Y) \approx 2 + \frac{12c_g}{b} \int_y^Y d\eta \frac{\eta - y}{\eta} n_g(\eta, Y) \quad (3.5)$$

with $b \equiv 33 - 2N_f$, N_f = number of active flavours, one obtains after differentiating twice with respect to y :

$$y \frac{d^2}{dy^2} n_g(y, Y) = \frac{12c_g}{b} n_g(y, Y) \quad (3.6)$$

with boundary conditions :

$$n_g(Y, Y) = 2 \quad (3.7a)$$

$$\left. \frac{d}{dy} n_g(y, Y) \right|_{y=Y} = 0 \quad (3.7b)$$

Defining

$$\tau \equiv \frac{48c_g}{b}, \quad z \equiv \sqrt{\tau y}, \quad z \omega(z) \equiv n_g(y, Y) \quad (3.8)$$

we obtain

$$z^2 \omega''(z) + z \omega'(z) - (z^2 + 1) \omega(z) = 0 \quad (3.9)$$

There exist two linear independent solutions, namely the modified Bessel functions $I_1(z)$ and $K_1(z)$. Imposing the boundary conditions (3.7a,b) one finds

$$n_g(y, Y) = \Phi \sqrt{\frac{Y}{Y}} \left[K_0(z_0) I_1(z) + I_0(z_0) K_1(z) \right] \quad (3.10)$$

with

$$z_0 \equiv \sqrt{\tau Y}$$

$$\Phi \equiv \frac{2}{I_1(z_0) K_0(z_0) + K_1(z_0) I_0(z_0)}$$

Using Eq. (2.1) we get for the charmed multiplicity

$$N_c(M_R^2) \approx \frac{T_F}{3\pi} \frac{12\pi}{b} \Phi \int_{\frac{M_c^2}{4m_c^2}}^{\frac{M_R^2}{4m_c^2}} \frac{dk^2}{k^2} \frac{1}{\log \frac{k^2}{\Lambda^2}} \frac{z}{z_0} \left[K_0(z_0) I_1(z) + I_0(z_0) K_1(z) \right]$$

$$= \frac{8 T_F \Phi}{b z_0} \left[I_0(z_0) K_0(\xi) - I_0(\xi) K_0(z_0) \right] \quad (3.11)$$

with

$$z_0 \equiv \sqrt{\frac{48 c_g}{b} \log \frac{M_R^2}{\Lambda^2}}, \quad \xi = \sqrt{\frac{48 c_g}{b} \log \frac{4m_c^2}{\Lambda^2}} \quad (3.12)$$

For $M_R^2 \gg 4m_c^2$ we can drop the second term and set

$$\Phi \approx \frac{1}{K_1(z_0) I_0(z_0)}$$

Then

$$N_c(M_R^2) \approx \frac{8 T_F}{b z_0} \frac{K_0(\xi)}{K_0(z_0)} \quad (3.13)$$

Substituting $SU(3)$ values for the group factors we finally obtain

$$N_c(M_R^2) \approx \frac{1}{316} \frac{1}{\sqrt{\log \frac{M_R^2}{\Lambda^2}}} \frac{K_o(12 \sqrt{(\log \frac{4m_c^2}{\Lambda^2})/16})}{K_o(12 \sqrt{(\log \frac{M_R^2}{\Lambda^2})/16})} \quad (3.14)$$

In Fig. 11 we have plotted $N_c^{para}(M_R^2)$ [Eq. (3.11)], using $b \equiv 33 - 2N_f \simeq 23$ and $4m_c^2 = 9 \times (2\Lambda)^2$ [20 $\times (2\Lambda)^2$] for curve I [II], while the mass M_R ranges between $10 \times (2\Lambda)$ and $100 \times (2\Lambda)$. Λ is the scale to define the "running" coupling $\alpha_s(k^2)$; presently $\Lambda \sim O(500 \text{ MeV})$ seems to be favoured.

Also the lowest order contributions are included (broken line): one sees that higher order contributions are rather sizeable.

One can also ask how the result is affected if one uses a fixed coupling instead of a "running" one, be it $\alpha_s(M_R^2)$ or a fix point $\alpha_s(\omega)$. Solving Eq. (3.3) one finds

$$\tilde{N}_c(M_R^2) \approx \sqrt{\frac{T_F^2 \alpha_s}{9 C_F \pi}} \left[\exp\left(\sqrt{\frac{\alpha_s C_F}{\pi}} \log \frac{M_R^2}{4m_c^2}\right) - \exp\left(-\sqrt{\frac{\alpha_s C_F}{\pi}} \log \frac{M_R^2}{4m_c^2}\right) \right] \quad (3.15)$$

We will further discuss this expression in Section 5.

4. - DECAY OF ORTHO-STATES INTO CHARM IN THE LEADING LOG APPROACH

So far we have mainly discussed the decay of para-states, i.e., of such states that can decay via two gluons in lowest order. Now we will consider the decay of ortho-states, i.e., heavy vector meson resonances like the Y , which decay via three gluons in lowest order QCD. Since they are produced directly in e^+e^- annihilation, they can be studied more easily than the para-states.

The calculation of N_c^{ortho} is almost identical to that of N_c^{para} in the leading log approximation: as discussed in Sections 2 and 3, gluons attached to the initial heavy quark line do not generate large logarithms; their contribution is therefore ignored, with the only exception of the lowest order decay $R \rightarrow g(k^2)gg$, yielding for the off-shell gluon multiplicity $n_g^{(0)}(k^2) \simeq 3$ as the leading log result.

Using the same arguments as in Section 3 [and as in Ref. 5] one can also rule out contributions from crossed diagrams and thus one arrives at an equation (Fig. 12) :

$$n_g^{\text{ortho}}(k^2) \approx 3 + \int_{k^2}^{M_R^2} \frac{d\ell^2}{\ell^2} \frac{\alpha_s(\ell^2)}{2\pi} \int_{k^2/\ell^2}^1 dz_g \frac{2c_g}{z_g} n_g^{\text{ortho}}(\ell^2) \quad (4.1)$$

Comparing Eq. (4.1) and Eq. (3.3) one sees immediately

$$n_g^{\text{ortho}}(k^2) \approx \frac{3}{2} n_g^{\text{para}}(k^2) \quad (4.2a)$$

$$N_c^{\text{ortho}}(M_R^2) \approx \frac{3}{2} N_c^{\text{para}}(M_R^2) \quad (4.2b)$$

Equations (4.2a,b) just reflect the fact that the leading log approach here amounts to assuming an independent fragmentation to three versus two off-shell gluons, irrespective of its details. To test it experimentally would be very interesting indeed ; deviations from Eq.(4.2b) would show that the leading log approach is - at least - not quite applicable in the case under study.

5. - COMPARISON WITH OTHER MODELS

Up to this moment we have stayed within the realms of "canonical" QCD. The difference between Eq. (3.2) and Eq. (3.15) for example is only due to differences in treating the single logarithms which arise in the vertex renormalization. Yet in both cases the charmed multiplicity is calculated taking into account mainly the triple-gluon vertex. It seems natural to wonder therefore how the results are affected if there is no gluon self-coupling. As a model theory one can take an Abelian gauge theory with colour as a global symmetry only, in which case the structural identity with QED diagrams is complete ; only the vertex functions of Eqs. (3.1a) and (3.1c) have to be considered.

First we will try to identify the diagrams which contribute to the leading log result in such a theory.

- (i) As before "gluons" attached to the initial heavy quark line do not produce large logarithms.
- (ii) Again one can use the KLN theorem to demonstrate that as soon as a $c\bar{c}$ (or more generally, any $q\bar{q}$) pair is produced higher radiative corrections to this process do not generate further logarithms, if one sums over all cut diagrams as in Fig. 13a - provided those radiative corrections do not involve the production of further (or any) $c\bar{c}$ pairs as in Fig. 13b. They have to be included since they contribute a larger charm multiplicity than the other corresponding cut diagrams.

In the leading log approach their contribution is of the form $\sim \alpha_s^2 (\log(M_R^2/4m_c^2))^3$; they are ignored in the QCD computation, since corresponding diagrams involving the triple gluon vertex give

$$\sim \alpha_s^2 (\log(M_R^2/4m_c^2))^4.$$

- (iii) Unfortunately disaster strikes at the next step : just using the arguments given before one cannot prove any more that it is still only the ladder diagrams which contribute in the leading log limit. Previously the argument went as follows ⁵⁾ : when leading log means that one obtains two large logarithms per coupling α_s , then a diagram like in Fig. 14a cannot contribute. This is not true a priori for a diagram like in Fig. 14b. Yet choosing the gauge fixing vector n_μ in a judicious fashion, namely $n_\mu = \bar{p}_\mu$, also such a diagram does not contribute in this gauge.

A real problem arises at this point because in the case under study it is not true anymore that one can always obtain two large logarithms per coupling α_s : in a diagram like in Fig. 15a one still gets two large logarithms at any quark bremsstrahlung vertex [Eq. (3.1a)], but only one at a gluon decay vertex [Eq. (3.1c)].

Therefore interference terms do contribute, at least in principle : although one can again suppress contributions from diagrams like in Fig. 15b by choosing $n_\mu = \bar{p}_\mu$, one can still not ignore other interference diagrams (for example Figs. 15c, d).

It is beyond the scope of this paper to calculate such contributions. Yet their structure as far as large logarithms are concerned should be analogous to the corresponding ladder diagrams which we can calculate. The procedure which we adopt is therefore the following : in order to get a comparison with QCD predictions we calculate $N_c(M_R^2)$ just using ordinary ladders ; we then argue that inclusion of possible contributions from interference terms can well change the coefficients, which we therefore regard as free parameters, but not the structure of the $\log(M_R^2/4m_c^2)$ terms. It is clearly not quite trivial to assume that this possible change in the coefficients of the logarithmic terms proceeds in a uniform fashion at any order in α_s , independent of M_R .

Even so - as follows from point (ii), listed above - the calculation of the charm multiplicity consists of two steps.

- a) One considers diagrams which do not contain charmed quarks, i.e., only the other $N_F - 1$ flavours contribute (Fig. 16a). One finds :

$$n_g^{para}(k^2) \approx 2 + \frac{C_F a}{2\pi} \int_{k^2}^{M_R^2} \frac{d\ell^2}{\ell^2} \alpha_s(\ell^2) \int_{k^2/\ell^2}^1 \frac{dz}{z} n_q^{para}(\ell^2) \quad (5.1a)$$

$$n_q^{para}(k^2) \approx \frac{T_F d(N_F - 1)}{3\pi} \int_{k^2}^{M_R^2} \frac{d\ell^2}{\ell^2} \alpha_s(\ell^2) n_g^{para}(\ell^2) \quad (5.1b)$$

where $n_g(k^2)$ [$n_q(k^2)$] gives the multiplicity of off-shell gluons [quarks] ; "a" and "d" are free, i.e., so far uncalculable parameters : $a = d = 1$ for pure ladder diagrams.

The situation is similar to the one in the case of the singlet structure function in deep inelastic scattering, which depends both on the quark and on the gluon distributions.

An Abelian gauge theory, which we are using here, does not exhibit asymptotic freedom ; the best one can hope for is the existence of a fix point α_s at a small value. This will be assumed to be the case.

Solving (5.1a,b) by the same procedure as the one employed in Section 3, we find :

$$n_g^{para}(k^2) \simeq \frac{2}{3} \left[e^{-Ky} + e^{\frac{1}{2}K(1-i\sqrt{3})y} + e^{\frac{1}{2}K(1+i\sqrt{3})y} \right] \quad (5.2)$$

with $K = \sqrt[3]{ad(N_f-1) \alpha_s^2 / 9\pi^2}$ and a, d assumed to be unknown, yet fixed numbers.

b) Finally one has to compute the conversion $g(k^2) \rightarrow c\bar{c} + X$ (Fig. 16b) :

$$N_c(M_R^2) \simeq \frac{\alpha_s}{6\pi} \int_{4m_c^2}^{M_R^2} \frac{dk^2}{k^2} n_g^{para}(k^2) + \frac{2\alpha_s^2 ad}{9\pi^2} \int_{4m_c^2}^{M_R^2} \frac{dk^2}{k^2} \int_{k^2}^{M_R^2} \frac{d\ell^2}{\ell^2} \int_{k^2/\ell^2}^1 \frac{dz}{z} N_c(\ell^2) \quad (5.3)$$

Solving (5.3), one obtains :

$$N_c^{para}(M_R^2) \simeq \frac{\alpha_s}{9\pi} \left\{ \frac{1}{K(1-\frac{1}{N_f-1})} \left[e^{KY} - \frac{1}{2}(1+i\sqrt{3})e^{-\frac{1}{2}K(1-i\sqrt{3})Y} - \frac{1}{2}(1-i\sqrt{3})e^{-\frac{1}{2}K(1+i\sqrt{3})Y} \right] + \frac{1}{\tilde{K}} \left(1 - \frac{1}{1-\frac{1}{N_f-1}} \right) \left[e^{\tilde{K}Y} - \frac{1}{2}(1+i\sqrt{3})e^{-\frac{1}{2}\tilde{K}(1-i\sqrt{3})Y} - \frac{1}{2}(1-i\sqrt{3})e^{-\frac{1}{2}\tilde{K}(1+i\sqrt{3})Y} \right] \right\}, \quad (5.4a)$$

with

$$Y \equiv \log \frac{M_R^2}{4m_c^2}, \quad \tilde{K} \equiv \sqrt[3]{\frac{2\alpha_s^2}{9\pi^2} ad}, \quad K \equiv \tilde{K} \sqrt[3]{N_f-1}, \quad N_f \geq 3$$

Also, like in Section 4 :

$$N_c^{ortho}(M_R^2) \simeq \frac{3}{2} N_c^{para}(M_R^2) \quad (5.4b)$$

We have therefore three different expressions for N_c , which we list for $\Lambda^2 \ll 4m_c^2 \ll M_R^2$:

(i) QCD with a "running" coupling

$$N_{c,ortho}^I \approx \frac{1}{2\sqrt{b}} \frac{1}{\sqrt{\log \frac{M_R^2}{\Lambda^2}}} \frac{K_o(12\sqrt{(\log \frac{4m_c^2}{\Lambda^2})/b})}{K_o(12\sqrt{(\log \frac{M_R^2}{\Lambda^2})/b})}$$

$$\approx \sqrt{\frac{6}{\pi}} \frac{1}{b} \sqrt{\frac{b}{\log \frac{M_R^2}{\Lambda^2}}} \exp(12\sqrt{\frac{\log \frac{M_R^2}{\Lambda^2}}{b}}) K_o(12\sqrt{\frac{\log \frac{4m_c^2}{\Lambda^2}}{b}}) \quad (5.5)$$

with $b = 33 - 2N_f$;

(ii) QCD with a coupling fixed for each resonance, be it $\alpha_s(M_R^2)$ or a fixed point $\alpha_s(\omega)$:

$$N_{c,ortho}^{II} \approx \sqrt{\frac{\alpha_s}{48\pi}} \exp\left(\sqrt{\frac{3\alpha_s}{\pi}} \log \frac{M_R^2}{4m_c^2}\right) \quad (5.6)$$

(iii) An Abelian vector gluon theory with colour as a global symmetry only, Eq. (5.4b), which simplifies for $\log(M_R^2/4m_c^2) \ll 1$:

$$N_{c,ortho}^{III} \approx \frac{\alpha_s}{6\pi} \left[\frac{1}{K(1-\frac{1}{N_f-1})} e^{K \log \frac{M_R^2}{4m_c^2}} + \frac{1}{\tilde{K}} \left(1 - \frac{1}{1-\frac{1}{N_f-1}}\right) e^{\tilde{K} \log \frac{M_R^2}{4m_c^2}} \right] \quad (5.7)$$

with $K = \sqrt[3]{N_f-1} \tilde{K}$ a (so far) uncalculable number.

The first observation to be made is the following : m_c is not necessarily the mass of a charmed quark ; a priori it is just a parameter to set the scale above which one can apply a perturbation theory based on leading logarithms. On the other hand, one can argue, using some duality ideas, that for heavy quarks this parameter should be close to the corresponding quark mass. That is the reason why we let $4m_c^2$ only vary between $9 \times (2\Lambda)^2$ and $20 \times (2\Lambda)^2$ in Fig. 11, causing N_c^I to change by roughly 50%. We do not know of a very convincing argument to impose more stringent limits on $4m_c^2$ which would be quite desirable to distinguish clearly between the expressions in Eqs. (5.5)-(5.7).

One important point to note is that in the QCD result [Eq. (5.5)] the $4m_c^2$ dependence is nicely factored out ; if one normalizes N_c^I with the value found at the Y resonance, $N_c^I(M_Y)$, this dependence is therefore cancelled. That means also that one needs at least one more heavy resonance to differentiate between Eqs. (5.5)-(5.7).

We propose the following procedure.

- a) First one tries to establish whether the total widths of heavy vector mesons like the Y and hopefully a toponium state are correlated to each other as suggested by an asymptotically free theory or by a theory exhibiting an ultra-violet stable fix point.
- b) If the former is the case, one can write $\alpha_s = \alpha_s(M_R)$ in Eqs. (5.6), (5.7) - also K and $\tilde{K}$ depend on it - and finds :

$$N_{c,ortho}^I \approx \frac{1}{\sqrt{4b \log \frac{M_R^2}{\Lambda^2}}} \exp\left(6 \sqrt{\frac{1}{b \log \frac{M_R^2}{\Lambda^2}}} \log \frac{M_R^2}{4m_c^2}\right) \quad (5.8)$$

$$N_{c,ortho}^{III} \approx \frac{1}{\sqrt{4ad b \log \frac{M_R^2}{\Lambda^2}}} \left[\frac{1}{\sqrt[3]{N_f-1 \left(1 - \frac{1}{N_f-1}\right)}} \exp\left(\sqrt[3]{\frac{32ad(N_f-1)}{b^2 \left(\log \frac{M_R^2}{\Lambda^2}\right)^2}} \log \frac{M_R^2}{4m_c^2}\right) \right. \\ \left. + \left(1 - \frac{1}{1 - \frac{1}{N_f-1}}\right) \exp\left(\sqrt[3]{\frac{32ad}{b^2 \left(\log \frac{M_R^2}{\Lambda^2}\right)^2}} \log \frac{M_R^2}{4m_c^2}\right) \right] \quad (5.9)$$

By writing down Eq. (5.9) we have cheated a bit : as mentioned above, an Abelian gauge theory does not exhibit asymptotic freedom, i.e., $\alpha_s(M_R) \sim 1/\log(M_R^2/\Lambda^2)$. Therefore, Eq. (5.9) is logically not quite consistent. We have written it down anyhow just to illustrate the difference relative to expressions (5.5) and (5.8).

With two heavy resonances one can decide between (5.5) and (5.8), m_c being the only free parameter left ; we have plotted, in Fig. 17, the normalized multiplicities $N_c^I(M_R)/N_c^I(M_Y)$, curve I, and $N_c^{II}(M_R)/N_c^{II}(M_Y)$, curve II.

To differentiate also against (5.9), one needs of course a third heavy resonance as long as one cannot calculate a and d .

It is also worth while to note the differences in the asymptotic behaviour, as $M_R \rightarrow \infty$:

$$N_c^I \sim \frac{1}{\sqrt{\log \frac{M_R^2}{\Lambda^2}}} \exp\left(12 \sqrt{\log \frac{M_R^2}{\Lambda^2}}\right) \quad (5.10)$$

$$N_c^{II} \sim \frac{1}{\sqrt{\log \frac{M_R^2}{\Lambda^2}}} \exp\left(6 \sqrt{\log \frac{M_R^2}{\Lambda^2}}\right) \quad (5.11)$$

$$N_c^{III} \sim \frac{1}{\sqrt[3]{\log \frac{M_R^2}{\Lambda^2}}} \exp\left(\sqrt[3]{\frac{16 a d \log \frac{M_R^2}{\Lambda^2}}{g^2}}\right) \quad (5.12)$$

i.e., N_c^I increases much faster than N_c^{II} or N_c^{III} .

c) If, on the other hand, the dependence of the total width of a vector meson resonance on its mass points towards a fix point theory, one is left with the choice between Eqs. (5.6) and (5.7), which show a similar algebraic structure, namely a multiplicity increasing like a power of M_R^2 . One needs two heavy vector mesons to fix m_c and $K - \alpha_s$ is taken over from the total width - and then a third one to decide between Eqs. (5.6) and (5.7).

A final remark : we have described a calculation for charmed multiplicities ; of course, all the results given can immediately be copied to describe strong decays like $(t\bar{t}) \rightarrow b\bar{b} + X$ with the obvious substitution of the parameter m_c by $m_b \sim 0(5 \text{ GeV})$.

6. - SUMMARY AND CONCLUSIONS

We have discussed how QCD allows to calculate, in the leading log approach, the charmed multiplicity produced in the strong decays of a heavy resonance.

In this approximative scheme one obtains identical results for all vector (or ortho-) states of a flavour generation, like Υ , Υ' and Υ'' ; the corresponding quantities for the η_Q (or para-) states differ from these only by a factor $2/3$, reflecting the dominance of two versus three gluon decays. Experimental information on this relationship, although not easy to obtain, would provide a valuable test on the validity of the leading log approach in the case under study.

The results themselves depend on a parameter connected with the charm production threshold, which we denoted by m_c and which cannot be rigorously calculated in a leading log approach. Yet one can make "reasonable" estimates for m_c : we picked $9 \text{ GeV}^2 \leq 4m_c^2 \leq 20 \text{ GeV}^2$. The charm multiplicity on the Υ resonance can then be calculated up to an uncertainty of $\sim 50\%$. And if one can study the decay into charm of an even heavier resonance, like toponium, this uncertainty is completely compensated when one compares the two multiplicities, $N_c(\Upsilon)$ and $N_c((t\bar{t}))$.

The situation is more complicated if one wants not only to check whether data are consistent with QCD, but if one wants also to determine whether they require a gluon self-coupling. Unfortunately due to some computational complexities encountered in this case when there is no gluon self-coupling, one needs a third heavy resonance of yet another flavour generation to perform this test.

Therefore it would be rather desirable to overcome these technical complexities in the computation. It has been justly emphasized by many authors ¹³⁾ that it is a prime objective to establish, or reject, the existence of the gluon self-coupling.

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FIGURE CAPTIONS

- Fig. 1 Decay of a heavy $(Q\bar{Q})$ state into a massless on-shell gluon and an off-shell gluon with virtual mass k^2 .
- Fig. 2a,b Radiative corrections to $(Q\bar{Q})$ decay.
- Fig. 3 Radiative corrections to $(Q\bar{Q})$ decay involving a virtual $c\bar{c}$ pair, cuts (a) and (c), or a real one, cut (b).
- Figs. 4,5,6 Higher radiative corrections which do not contribute in the leading log approach.
- Fig. 7 Ladder diagrams contributing to $(Q\bar{Q})$ decay.
- Fig. 8a,b Real and virtual gluon production.
- Fig. 9 One-particle cut in the gluon propagator.
- Fig. 10 Bethe-Salpeter equation for the gluon multiplicity $n_g(k^2)$ in a para-state.
- Fig. 11 Charm multiplicity N_c as a function of the "rescaled" resonance mass M_R/Λ . The solid curve I [II] gives the leading log result for $4m_c^2 = 9 \times (2\Lambda)^2$ [$20 \times (2\Lambda)^2$]; the broken lines show the corresponding lowest order contributions. The numbers refer to the decays of a para-state.
- Fig. 12 Bethe-Salpeter equation for the gluon multiplicity $n_g(k^2)$ in an ortho-state.
- Fig. 13a,b Radiative corrections to $(Q\bar{Q})$ decay.
- Fig. 14a,b Radiative corrections involving the triple-gluon vertex.
- Fig. 15a-d Radiative corrections without triple gluon vertex.
- Fig. 16a Coupled Bethe-Salpeter equation for the gluon multiplicity $n_g(k^2)$ involving no internal $c\bar{c}$ production.

Fig. 16b Bethe-Salpeter equation for the charm multiplicity.

Fig. 17 Normalized charm multiplicity $R_c \equiv (N_c(M_R))/(N_c(M_Y))$ with
a "running" coupling, curve I, or with a coupling fixed for
each resonance $\alpha_s = \alpha_s(M_R)$, curve II.

FIG. 1

FIG. 2

FIG. 3

FIG. 4

FIG. 5

FIG. 7

FIG. II

FIG. 12

FIG. 13

FIG. 14

FIG. 15

FIG. 16

FIG. 16

FIG.17