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QUARK-GEOMETRODYNAMICS: A NEW APPROACH TO
HADRONS AND THEIR INTERACTIONS

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1. THE EMERGING PICTURE OF STRONG INTERACTIONS

There remains little doubt today that the quark idea must be taken as the starting point of any serious attempt to build a theory of hadrons and their interactions. But which direction one should take appears at the present level of knowledge highly uncertain; even though the successes of the gauge theories of weak interactions have given a strong impulse to the belief that the "best" candidate for a theory of strong interactions is Quantum Chromo-Dynamics (QCD), which has the virtue of extending the gauge principle to the realm of hadrons. Direct *quantitative* tests will decide which of the proposed approaches, if any, should survive; for the time being it is necessary to stress the importance that all possible avenues be tried and confronted with the evergrowing experimental information on the most diverse hadrodynamical aspects.

Be as it may, all attempts to understand hadrons seem to agree that the following are essential notions:

i) Extended dynamical systems

Hadron extension in space-time is a property suggested by basic facts of particle physics, like Regge trajectories, form factors

and the structure of high-energy scattering. All these facts differentiate hadrons from "point-like" objects like leptons in a most far-reaching fashion. On the other hand, it is a feature of known extended systems (like nuclei) that they are arranged in Regge trajectories, have form factors fastly decreasing in the momentum transfer, and "diffractively" scatter at high energy.

ii) Quarks

From building blocks of hadronic symmetries [SU(2), SU(3), now SU(4), and maybe SU(6) tomorrow] quarks have become important tools first to classify hadronic states in the naive quark model¹⁾ and after as the elementary scatterers of deep inelastic phenomena²⁾. Thus the dynamics of the strong interactions from its low-energy aspects (spectrum) to the highly inelastic phenomena (Bjorken scaling) finds a powerful unification through the notion of point-like spin $\frac{1}{2}$ constituents whose internal symmetry properties are the same as the originally proposed quarks.

iii) Colour³⁾

The idea that quarks, besides flavour [SU(3), SU(4) ...], are endowed with colour [SU(3)_{colour}] helps us in correlating at least two hadrodynamical puzzles. The first puzzle is why all known hadronic states are made out of $q\bar{q}$ and qqq configurations and their combinations (triality puzzle), the second is why the wave function of the lowest baryons (N, Δ , ...) is symmetric in quark spin and isospin and not, as suggested by Pauli principle, antisymmetric. The two puzzles can be resolved at once by assuming the existence of an exact SU(3) colour symmetry and that observable hadrons are *singlets* under this symmetry; it then follows that all singlets must have the configurations $q\bar{q}$ and qqq and all their combinations, and that the qqq colour singlet is totally antisymmetric. There are other puzzles like the value of $R = [\sigma(e^+e^- \rightarrow \text{hadrons})] / [\sigma(e^+e^- \rightarrow \mu^+\mu^-)]$ and the $\pi^0 \rightarrow \gamma\gamma$ decay amplitude which could find a solution in colour, but the situation here is much less clear and more subject to criticism.

It is easy to understand why an approach like QCD is more and more looked upon as the only candidate for a theory of strong interactions. It combines the theoretical paradigm of our times, i.e. Quantum Field Theory (QFT), with two of the pillars of hadrodynamics: quarks and colour, and it does this in an extremely elegant and appealing way, which discloses the possibility of a future "grand-unification" of all the basic interactions.

If this is so, what is then the motivation to look for some alternative theory? The reason, as far as I can see it, lies in the fact that the pillar (i), i.e. extension, is not naturally embodied in QCD, and is related to the problem of "quark confinement" which is recognized to be an extremely difficult problem⁴), whose solution is nowhere near in sight. It may well be that nature is like that, and that we have to climb the hard way the high wall of quark confinement. But then, what about the world of hadrons, which seems so beautiful in its simplicity, and where the results of difficult experiments can be predicted by playing with the simple toys of the quark parton model? Is it really true that this wall has to be frontally assaulted and not circumvented by changing our ideas about unobservable (confined) quark fields?

It is the necessity to make theoretical contact with the great experimental achievements of the last years which motivates some of us to pursue approaches alternative to QCD. These "bag" approaches start where QCD has so far badly failed, i.e. from point (i). They are developed by putting "confinement" in from the beginning as a *primitive* notion. It cannot be excluded *a priori* that some of them will in fact be related to physical QCD solutions, even though such a possibility seems at present quite remote.

In the "bag" models one assumes that hadronic matter (i.e. quarks and gluons) can be found only in well-defined finite space-time regions (bags) and that making these regions unbounded (i.e. seeing free quarks and gluons) requires an infinite amount of energy, implying that hadronic constituents are *permanently* confined.

The approaches which have been proposed so far are essentially of two types:

- the MIT-Budapest bag⁵⁾,
- the geometrodynamical theory of quarks and hadrons⁶⁾.

The basic aspects of the MIT-Budapest approach have been presented at this school two years ago by Prof. Weisskopf⁷⁾, while the first attempts and developments of the geometrodynamical ideas have formed the arguments of my Erice lectures in 1975⁸⁾ and 1976⁹⁾. In these lectures I shall report on the further progresses of the geometrodynamical theory and shall endeavour to show that it has reached a rather mature stage in which one can *quantitatively* attack several hadrodynamical problems of profound interest.

As already emphasized the starting point of the two descriptions is the same, but their basic strategy is quite opposite. In the MIT-Budapest bag the geometry of bags originates from a QCD Lagrangian to which vacuum pressure (and surface tension) terms are added; in the geometrodynamical bag the situation is reversed, it is the simple bag geometry which determines the dynamics of the hadrons (see Fig. 1).

Fig. 1 The basic difference between the MIT-Budapest and the geometrodynamical bags

2. THE GEOMETRODYNAMICAL PRINCIPLES

Geometrodynamics is not a new word in physics; it was coined by Wheeler to describe his own extension of Einstein's General Relativity¹⁰⁾. It correctly stresses the prominent role that the geometry of space-time plays in determining the dynamical behaviour of matter, and embodies that momentous train of thought which from Riemann to Hertz¹¹⁾ led to the formulation of the theory of General Relativity. It is the primary role that geometry should play in the quark hadron microcosm (bag) that the word geometrodynamics intends to stress; therefore it should not be confused with some extension of the theory of gravitation to the elementary particle realm.

The basic idea of this approach is that the geometry of hadrons is not the final product of a sophisticated and very involved chain of dynamical interplays, but that it is a fundamental notion out of which dynamics follows in the simplest fashion. Quarks are thus to be viewed as *hadronic coordinates* describing not only the *external* hadron geometry (the bag) but also the *internal* (the symmetries). Besides the three notions of extension, quarks, and colour, this theory rests on two basic principles.

2.1 Simplicity

The *fundamental hadrons* are quantum mechanical systems, whose structure is described by the minimum number of quark coordinates. All hadronic states can be obtained from the fundamental hadrons by the usual Fock-space construction (+ final state interactions).

2.2 Freedom

Inside the hadronic space-time domains (bags) the quark coordinates carry a wave motion which is the closest to a free motion.

According to (a) and the colour singlet postulate the fundamental hadrons are:

- MESONS, i.e. $q\bar{q}$ systems,
- BARYONS, i.e. qqq systems.

The "derived" hadronic states fall into two classes:

- HADRONIC MOLECULES, i.e. nuclei, $N\bar{N}$ - $B\bar{B}$ states, charm-anticharm states,
- MULTIPARTICLE STATES, i.e. Fok-constructed from meson and baryons.

In order to build the theory we must consider three steps:

- i) Characterize the fundamental states from principles.
- ii) Introduce currents and normalize the states.
- iii) Introduce couplings among hadrons.

After this has been accomplished, the description of the dynamics of hadrons becomes a matter of *feasible* calculations.

3. THE MESONIC STATES AND THEIR SPECTRUM

According to our principles, we can picture a general mesonic state as in Fig. 2. It is characterized by a four-momentum p , a bounded space-time region (bag) $R^4(p)$ in the relative quark coordinate $x = x_1 - x_2$, and a set of indices; Dirac (α, β) , colour, and flavour (a, b) . Thus to describe such a state we can introduce a (translation invariant) wave function

$$\psi_{\alpha a}^{\beta b}(p; x_1, x_2) = e^{ip \frac{x_1 + x_2}{2}} \psi_{\alpha a}^{\beta b}(p; x_1 - x_2) \quad (3.1)$$

which, from the geometrodynamical principles just discussed, can be characterized as follows:

Fig. 2 The geometrodynamical picture of a meson

i) Confinement

We require that

$$\psi(p; x) = 0 \quad \forall x \notin R^4(p) \quad (3.2)$$

where $R^4(p)$ is a relativistically invariant four-dimensional region, whose boundary can in general be represented as

$$B^4(p) \equiv \begin{cases} -x^2 + \frac{(px)^2}{p^2} = R_s(p^2)^2 \\ \frac{(px)^2}{p^2} = R_t^2(p^2) \end{cases} \quad (3.3)$$

ii) Continuity

The wave function (3.1) can be generally decomposed into Dirac covariants

$$\psi_{aa}^{\beta b}(p; x) = \sum_r \bar{\mathbb{P}}^{(r)}(p, -i \frac{\partial}{\partial x})_a^\beta \varphi^{(r)}(p; x)_a^b. \quad (3.4)$$

We require that the scalar functions $\phi^{(r)}(p, x)_a^b$ be continuous functions, i.e.

$$\varphi^{(r)}(p; x) = 0 \quad \forall x \in B^4(p). \quad (3.5)$$

iii) Wave motion

The principles of simplicity and freedom lead us to postulate the following wave equation for the wave function:

$$\vec{D}_1 \psi(p; x_1, x_2) \vec{D}_2 = 0 \quad \forall x \in R^4(p), \quad (3.6)$$

where $D_i = [-i\partial_i + m_i]_{\alpha}^{\beta}$ is the Dirac operator acting on the i^{th} quark, and makes it possible to give a precise meaning to the notion of mass of the quarks.

iv) Maximum freedom

The principle of freedom has not been exploited fully in the wave equation (3.6). Denoting by $\psi^{(0)}$ the solution of the free equations

$$\vec{D}_1 \psi^{(0)} = \psi^{(0)} \vec{D}_2 = 0, \quad (3.7)$$

which fulfills the maximum possible number of continuity conditions (3.5), we further require that only those solutions of (3.6) be retained for which the norm^{*})

$$\|\psi - \psi^{(0)}\| \quad (3.8)$$

is minimum.

We now briefly discuss the physical meaning of the four conditions which characterize the discrete "fundamental" meson states.

^{*}) After the "conventional" normalizations $\|\psi\| = \|\psi^{(0)}\| = 1$ has been imposed. The norm can be defined as

$$\|\psi\|^2 = \int_{R^4(p)} dx \text{Tr}(\psi^* \psi)$$

or as

$$\|\psi\|^2 = \text{Max}_{x \in R^4(p)} \text{Tr}(\psi^* \psi).$$

Condition (i) requires that the quark degree of freedom, or better coordinate, be confined in *finite* space-time regions (bags) whose structures can be characterized by the two functions $R_s(p)^2$ and $R_t(p)^2$ appearing in (3.3). Note that no complicated "bag" surface configuration can appear in this formulation, because here, differently from other "bag" approaches, the dynamics of the meson systems is totally described by their quark coordinates. We shall discuss later the important criteria which constrain the p^2 dependence of R_s and R_t .

Condition (ii) allows us to define a *finite* quark (antiquark) momentum operator

$$\frac{p}{2}\mu - i \frac{\partial}{\partial x_\mu} \quad \left(\frac{p}{2}\mu + i \frac{\partial}{\partial x_\mu} \right).$$

In fact were (3.5) not satisfied the momentum operators applied to the $\phi(r)$'s would yield infinity at the boundaries. Note that (3.5) is the strongest continuity condition which can be imposed on the wave function compatible with the wave equations (3.6).

Condition (iii) specifies the differential equations obeyed by the wave function when x is inside the "bag" region $R^4(p)$. The choice (3.6) is suggested by the simplicity postulate and the idea, central to this approach, that geometry [i.e. the structure of $R^4(p)$] plays the fundamental role. Inside $R^4(p)$ the "quark motion" is the simplest one compatible with the boundary conditions (3.5). It is perhaps worth recalling that (3.6) is the form to which a Bethe-Salpeter equation for two spin- $\frac{1}{2}$ objects reduces when the kernel (or potential) vanishes.

As for (iv) it will play a crucial role in determining the structure of the solutions of (3.6); it deserves therefore some more extended comments. Equations (3.7) are not only the simplest set of equations which can be written for a meson wave function, but also embody an aspect of quark dynamics which is of paramount importance, i.e. the free behaviour of quarks inside hadrons. The

puzzling successes of the simple parton models testify to the physical relevance of free quark behaviour. Equations analogous to (3.7) were initially proposed⁶⁾ in the context of scalar and isoscalar quarks; however, it was noticed that the continuity requirement could not be implemented on the time boundary $[(px)^2/p^2 = R_t(p^2)^2]$ owing to the first-order character of the relative time equation. The idea was then to construct equations for the meson wave functions, which generalized (3.7) while allowing compatibility with (3.5). Equation (3.6) is one such generalization, but it has the property of admitting a class of solutions which have no counterpart in the class of "free solutions" $\psi^{(0)}$ and seem physically unacceptable. The condition of approximate freedom thus gives a precise meaning to a solution of the simple equation (3.6) which is continuous [Eq. (3.7)], and resembles as much as possible the free wave function $\psi^{(0)}$.

We shall now proceed to exploit the four previous conditions to derive the meson spectrum and the corresponding wave functions. We begin by analysing the solutions $\psi^{(0)}$ of (3.7). The procedure we adopt is to develop $\psi^{(0)}$ in a Dirac basis:

$$\psi^{(0)\beta}_\alpha = \sum_R \phi_R (\Gamma_R)_\alpha^\beta, \quad (3.9)$$

where Γ_R are the 16 Dirac matrices [$R = S, P, V, A, T$]. By projecting on the various Dirac covariants Γ_R , we obtain 10 equations which the ϕ_R 's have to satisfy. A straightforward analysis gives us two families of solutions:

i) the pseudoscalar family

$$P^{(0)\beta}_\alpha = [(\not{p}_1 + m_1) \gamma_5 (\not{p}_2 + m_2)]_\alpha^\beta \phi_5^{(0)}(p; x) \quad (3.10)$$

where $\not{p}_i = -1/i(\partial/\partial x_\mu) \gamma^\mu$, and $\phi_5^{(0)}$ obeys the two equations

$$\begin{aligned} K_1 \phi_5^{(0)} &= 0 \\ K_2 \phi_5^{(0)} &= 0, \end{aligned} \quad (3.11)$$

K_i being the Klein-Gordon operators $(\square_i + m_i^2)$.

ii) *The vector family*

$$V_\alpha^{(0)\beta} = [(\gamma_1 + m_1) \gamma_\mu (\gamma_2 + m_2)]_\alpha^\beta \phi^\mu(p; x) \quad (3.12)$$

where $\phi_\mu^{(0)}$, in addition to obeying Eqs. (3.11) satisfies the transversality condition:

$$p^\mu \phi_\mu^{(0)} = 0 \quad (3.13)$$

The solutions of the system (3.11) have been discussed at length in Refs. 6, 8, and 9. We only recall that in the rest frame $[p_\mu \equiv (M, \vec{0})]$ a function $\phi^{(0)}(p; x)$ satisfying (3.11) has the following structure (we set for simplicity $m_1 = m_2$):

$$\phi^{(0)}(M, \vec{0}; \vec{x}, t) = \phi_M(\vec{x}) \psi_M(t) \quad (3.14)$$

with

$$\left(\frac{M^2}{4} + \vec{\nabla}^2 - m^2\right) \phi_M(\vec{x}) = 0 \quad (3.15)$$

and

$$\frac{d}{dt} \psi_M(t) = 0. \quad (3.16)$$

The solutions of (3.15) which vanish at the space-boundary $|\vec{x}| = R_s(M)$ are described by two quantum numbers, ℓ angular momentum, n radial quantum number, and are given by

$$\phi_{n\ell}(\vec{x}) = N_{n\ell} j_\ell(k_{n\ell} |\vec{x}|) Y_\ell^m(\Omega_{\vec{x}}) \quad (3.17)$$

where $N_{n\ell}$ is a normalization factor, $j_\ell(x)$ the spherical Bessel function of order ℓ , and

$$k_{n\ell} = \frac{\beta_{n\ell}}{R_s(M^2)}, \quad (3.18)$$

$\beta_{n\ell}$ being the n^{th} positive zero of $j_\ell(x)$. From (3.15) it also follows that

$$\frac{M^2}{4} - m^2 = k_{n\ell}^2, \quad (3.19)$$

which gives us a discrete spectrum in terms of the quantum numbers n and ℓ . As for $\psi_M(t)$ the only non-trivial solution is

$$\psi_M(t) = \text{const},$$

which cannot meet the continuity requirement at the time-boundary $|t| = R_t(M)$.

Summarizing, the solutions of (3.7) can be grouped in two families, the lowest energy members of which are a 0^- and a 1^- state, respectively. Their spectrum is labelled by the two quantum numbers ℓ and n according to (3.19), once $R_s(M)$ has been given the corresponding energy levels are predicted. The physical relevance of these solutions for the observed meson spectrum needs no further comment.

We are now in the position of finding which are the mesonic states. The results just derived for the $\psi^{(0)}$'s lead us immediately to look for those solutions of (3.6) which can be cast in the forms (3.10) and (3.12). Thus we obtain:

i) The pseudoscalar family, which is given by

$$P_\alpha^\beta = [(\gamma_1 + m_1) \gamma_5 (\gamma_2 + m_2)]_\alpha^\beta \phi_5(p; x) \quad (3.20)$$

where ϕ_5 , according to (3.6), obeys the equation

$$\vec{k}_1 \phi_5 \vec{k}_2 = 0. \quad (3.21)$$

ii) The vector family,

$$V_\alpha^\beta = [(\gamma_1 + m_1) \gamma_\mu (\gamma_2 + m_2)]_\alpha^\beta \phi^\mu(p; x) \quad (3.22)$$

where ϕ_μ , in addition to obeying (3.21), satisfies the transversality condition

$$p^\mu \phi_\mu(p; x) = 0 \quad (3.23)$$

We study now the solutions of (3.21) which, in the sense of (iv) above, are as close as possible to the solutions of the system (3.7). Taking

$$k_0 = -\frac{1}{i} \frac{\partial}{\partial t}, \quad \vec{k} = \frac{1}{i} \vec{\nabla}, \quad m^2 = \frac{m_1^2 + m_2^2}{2} \quad \text{and} \quad \delta = m_1^2 - m_2^2,$$

we can write (3.21) in the rest frame

$$\left[\left(\frac{M^2}{4} + k_0^2 - \vec{k}^2 - m^2 \right)^2 - \left(M k_0 - \frac{\delta}{2} \right)^2 \right] \phi_M(\vec{x}, t) = 0 \quad (3.24)$$

Decomposing $\phi(\vec{x}, t) = \phi_M(\vec{x}) \psi_M(t)$, as indicated by (3.14), (3.24) becomes

$$\vec{k}^2 \phi_M(\vec{x}) = \lambda \phi_M(\vec{x}) \quad (3.25)$$

and

$$\left[\left(\frac{M^2}{4} + k_0^2 - \lambda - m^2 \right)^2 - \left(M k_0 - \frac{\delta}{2} \right)^2 \right] \psi_M(t) = 0, \quad (3.26)$$

with the boundary condition [see (3.5)]

$$\begin{aligned} \phi_M(|\vec{x}| = R_s) &= 0 \\ \psi_M(|t| = R_t) &= 0. \end{aligned}$$

The solution of (3.25) is immediate, and coincides with (3.17) if we set

$$\lambda = k_{n\ell}^2 = \frac{\beta_{n\ell}^2}{R_s^2}. \quad (3.27)$$

The general solution of (3.26) is of the form

$$\psi_M(t) = \sum_{i=1}^4 c_i e^{i\omega_i t} \quad (3.28)$$

where ω_i are the four solutions of the quartic equation

$$\left[\left(\frac{M^2}{4} + \omega^2 - \lambda - m^2 \right)^2 - \left(M\omega - \frac{\delta}{2} \right)^2 \right] = 0. \quad (3.29)$$

We get

$$\begin{aligned}\omega_{1,2}^{(+)} &= -\frac{M}{2} \pm E_1 \\ \omega_{1,2}^{(-)} &= \frac{M}{2} \pm E_2 ,\end{aligned}\tag{3.30}$$

where we have introduced the "quark energies" $E_i = \sqrt{k_{n\ell}^2 + m_i^2}$.

Approximate freedom eliminates immediately $\omega_2^{(+)}$ and $\omega_1^{(-)}$, which give rise to rapid oscillations in $\psi_M(t)$. The condition $\psi_M(\pm R_t) = 0$ yields

$$\psi_M(t) = N [e^{i\omega_1 t} + (-1)^{p-1} e^{i\omega_2 t}]\tag{3.31}$$

where

$$\begin{aligned}\omega_1 &= -\frac{M}{2} + E_1 \\ \omega_2 &= \frac{M}{2} - E_2\end{aligned}\tag{3.32}$$

and the eigenvalue condition (p integer positive)

$$\omega_1 - \omega_2 = p \frac{\pi}{R_t(M)} .\tag{3.33}$$

Approximate freedom again comes to our help by excluding all the solutions with $p > 1$. We finally get

$$\psi_M(t) = N (e^{i\omega_1 t} + e^{i\omega_2 t})\tag{3.34}$$

and combining (3.32) and (3.33) we can write the very simple eigenvalue equation for the mass of the state ($n\ell$):

$$M_{n\ell} = \sqrt{k_{n\ell}^2 + m_1^2} + \sqrt{k_{n\ell}^2 + m_2^2} - \omega_{n\ell} ,\tag{3.35}$$

where

$$\omega_{n\ell} = \omega_1 - \omega_2 = \frac{\pi}{R_t(M_{n\ell})} ,\tag{3.36}$$

and $k_{n\ell}$ is given in (3.18).

The physical interpretation of (3.35) is particularly suggestive; it tells us that the mass of the meson state is given by the sum of the quantized energies of its two quark waves minus a contribution, stemming from the finite extension of the time bag, which resembles a binding energy term.

The extension of $\phi_M(\vec{x}, t)$ to moving frames is straightforward and has been discussed in last year's course⁹⁾.

Equations (3.35) and (3.36) just derived predict the spectrum for the two meson families, the pseudoscalar and the vector, once $R_s(M^2)$ and $R_t(M^2)$ are specified. It is clear that through such equations R_s and R_t are functions of the geometrical constants β_{nl} only. The Massive Quark Model criterion of asymptotic universality for the distributions of "quark masses", discussed at length in previous courses^{8,9)} gives the asymptotic behaviour

$$R_s, R_t \xrightarrow{n \text{ large}} R^2 M. \quad (3.37)$$

The simplest parametrizations of $R_{s,t}(\beta_{nl})$ which, via (2.27) and (2.28), are compatible with (3.1) are

$$R_s^2 = R_0^2 \beta_{nl} \quad (3.38)$$

$$R_t^2 = R_0^2 \beta_{nl} + C \quad (3.39)$$

with $R_0^2 = 2R^2$, and C constant.

It should be noted that inserting (3.37) and (3.38) in the spectrum equations (3.35) and (3.36) yields the unperturbed energy levels. The unitarity corrections coming from self-energy diagrams such as

which must be considered once we introduce the three-bag coupling (see later), will be responsible for mass shifts as well as for finite widths of the meson states. Thus a precise determination of the parameters entering in (3.38) and (3.39) must await the completion of the fairly involved program of computing such effects. The previous results should be looked at having this always in mind.

3.1 The Pseudoscalar Nonet

This very important subfamily deserves special discussion. So far our outlook has been typical of a quark model approach, in which the mass differences between the $|\vec{S}| = 0$ and the $|\vec{S}| = 1$ states should be attributed to fine structure effects such as the just mentioned unitarity corrections.

However, there is another aspect of pseudoscalar dynamics which must be taken into account, i.e. the chiral structure of the hadronic world. We shall check later that when $m_q \rightarrow 0$ the axial currents become conserved [with the possible exception of the ninth^{*)}] and in order to avoid parity doubling the pseudoscalar octet must choose the Nambu-Goldstone mode. Thus according to (3.35) in the limit $m_q \rightarrow 0$, $\omega \rightarrow 2|\vec{k}|$ for the eight pseudoscalar mesons (π, K, η) while all other states are expected to be relatively unaffected by small variations in the quark masses.

In order to implement $\omega \rightarrow 2|\vec{k}|$ in our approach we have three choices: to change R_s in (3.38), to modify R_t in (3.39), or to change them both from the value they assume in the vector meson subfamily. It seems natural, however, to maintain the quark model outlook (π - ρ degeneracy) in the space dynamics and to modify time dynamics, i.e. to let R_t vary. For it is the time coordinate that, being conjugate to masses, responds to the symmetry-breaking effects. Needless to say, understanding the mechanism of chiral as well as

*) In order to overcome the famous U(1) problem one must assume that there is some additional piece to the ninth axial current¹²⁾.

SU(3) breaking in the context presented here is of the utmost importance, but so far such an insight is unfortunately lacking.

Accordingly, Eqs. (3.35) and (3.36) assume the role of determining R_t of the wave functions of the pseudoscalar octet, once R_s has been fixed at the same value as the vector nonet. In the case of the η - X_0 system, SU(3) breaking requires the introduction of two ω 's according to the equations

$$M_\eta = 2E_p - \omega_\eta^{(p)} = 2E_\lambda - \omega_\eta^{(\lambda)} \quad (3.40)$$

$$M_{X_0} = 2E_p - \omega_{X_0}^{(p)} = 2E_\lambda - \omega_{X_0}^{(\lambda)}, \quad (3.41)$$

where, as usual, $E_{p,\lambda} = \sqrt{k^2 + m_{p,\lambda}^2}$.

The requirement that the masses of the pseudoscalar octet depend *linearly* on the chiral symmetry-breaking parameters (m_p, m_λ), as we shall see later on, through the spectrum equations, determines the low-mass behaviour of $\omega(M)$ as

$$\omega(M) = \omega_0 - M + \beta M^2 + O(M^3) \quad (3.42)$$

where $\omega_0 = 2|\vec{k}_{10}|$. This has the consequence that the pseudoscalar octet obeys a quadratic Gell-Mann-Okubo relation; in fact to first order in m_p^2 and m_λ^2 :

$$\beta M_\pi^2 = \frac{m_p^2}{|k_{10}|} \quad (3.43)$$

$$\beta M_K^2 = \frac{m_p^2 + m_\lambda^2}{2|k_{10}|} \quad (3.44)$$

$$\beta M_\eta^2 = \frac{m_p^2 + 2m_\lambda^2}{6|k_{10}|} \quad (3.45)$$

where the last equation follows from (3.42) by setting, as required by the η -wave function,

$$\omega_\eta = \frac{1}{3} \omega_\eta^{(p)} + \frac{2}{3} \omega_\eta^{(\lambda)},$$

where $\omega_\eta^{(p)}$ and $\omega_\eta^{(\lambda)}$ are defined in (3.40).

3.2 The Other Meson Nonets, before Fine Structure Effects

All other meson states have no special role to play with regard to the patterns of hadronic symmetries and their spectrum must therefore follow the simple structures of Eqs. (3.35) and (3.36).

We predict that all meson families are, before unitarity corrections, in the ideal mixing configurations with the $I = 0$ non-strange meson degenerate with the $I = 1$ member of the octet. In Table 1 the calculated meson masses are reported with the following parametrizations in Eqs. (3.38) and (3.39):

$$R_0^2 = 4 \text{ GeV}^2 \quad (3.46)$$

$$C = -3 \text{ GeV}^2 \quad (3.47)$$

$$m_\phi^2 = 0.01 \text{ GeV}^2 \quad (3.48)$$

$$m_\lambda^2 = 0.22 \text{ GeV}^2 \quad (3.49)$$

Table 1

The calculated spectrum for SU(3) states

$n \backslash \ell$	0	1	2	3	4	5	6	7	Particle type
1	0.76 0.88 1.00	1.32 1.41 1.51	1.71 1.79 1.88	2.02 2.10 2.18	2.29 2.37 2.44	2.53 2.60 2.67	2.75 2.81 2.88	2.94 3.01 3.07	ρ - ω K^* ϕ
2	1.85 1.93 2.02	2.19 2.27 2.34	2.48 2.55 2.62	2.73 2.79 2.86	2.95 3.01 3.08	3.16 3.22 3.28	3.35 3.40 3.46	3.52 3.58 3.63	ρ - ω K^* ϕ
3	2.54 2.61 2.68	2.81 2.88 2.94	3.05 3.12 3.18	3.27 3.33 3.39	3.47 3.52 3.58	3.65 3.70 3.76	3.82 3.87 3.93	3.99 4.04 4.09	ρ - ω K^* ϕ
4	3.09 3.15 3.21	3.32 3.38 3.44	3.53 3.59 3.64	3.73 3.78 3.83	3.91 3.96 4.01	4.08 4.13 4.17	4.24 4.28 4.33	4.39 4.43 4.48	ρ - ω K^* ϕ
5	3.56 3.62 3.67	3.77 3.82 3.87	3.96 4.01 4.06	4.13 4.18 4.23	4.30 4.35 4.39	4.46 4.50 4.55	4.61 4.65 4.89		ρ - ω K^* ϕ
6	3.98 4.03 4.08	4.16 4.21 4.26	4.34 4.38 4.43	4.50 4.55 4.59	4.56 4.70 4.74				ρ - ω K^* ϕ
7	4.35 4.40 4.45	4.52 4.57 4.61							ρ - ω K^* ϕ

One obtains a set of approximately linear and parallel Regge trajectories without odd daughters. The agreement on the highest J members of the ℓ, n families is remarkable. The $SU(3)$ symmetry pattern comes out in very good agreement with experimental information. In Fig. 3 we give the energy levels in a spectroscopic notation; one should note the mass bands making up distinct fire-sausage structures as we shall discuss later. No attempt has

Fig. 3 The spectroscopic scheme for the meson states of the ρ - ω type

been made to optimize the values of the four parameters (3.46) to (3.49), because such an operation would assume a precise physical meaning only after the first-order unitarity corrections have been properly calculated. In view of this, the lack of $\vec{L} \cdot \vec{S}$ splitting exhibited by the calculated spectrum cannot be taken at this stage as a legitimate cause for worry.

3.3 Hidden and Apparent-Charmed States

In order to calculate the spectrum for both the charmed states and the members of the J/ψ family we need only to introduce a value of the charmed quark mass m_c . The pseudoscalar mesons are expected to show some trace of an otherwise badly broken chiral symmetry and our inability to grasp this problem prevents us from making definite predictions. We expect, however, the chiral symmetry-breaking effects to be much less severe on this state and that the vector-pseudoscalar splitting can be correctly given by the unitarity effects.

The calculated spectrum is presented in Table 2 for the various (ℓ, n) by taking

$$m_c = 1.86 \text{ GeV} . \quad (3.50)$$

It should be noted that the mass of the D^* is 1.93 GeV against the experimental value ≈ 2.0 GeV; but again shifts of this order of magnitude are no cause for worry. Of particular interest is the closeness of the vector states belonging to the subfamilies $[(\ell, n)]$ $(0, n)$ and $(2, n-1)$. Such states as an effect of unitarity will mix quite thoroughly, thus giving rise to mixtures of S- and D-waves.

Thus we predict the $\psi'(3.7)$ and the $\psi''(4.02)$ to be almost perfect mixtures of the $(S, 2)$ and $(D, 1)$ states and both have a substantial coupling (see Section 7) to the photon, in spite of the fact that the D-wave state coupling to the photon is quite suppressed.

Table 2

The calculated spectrum for charmed states

ℓ n	0	1	2	3	4	5	Particle type
1	1.93	2.39	2.71	2.97	3.20	3.40	D
	2.03	2.48	2.79	3.04	3.27	3.46	F
	3.10	3.47	3.72	3.93	4.11	4.28	ψ
2	2.83	3.12	3.36	3.57	3.76	3.94	D
	2.90	3.18	3.42	3.63	3.82	3.99	F
	3.82	4.04	4.24	4.41	4.57	4.72	ψ
3	3.41	3.64	3.84	4.04	4.21	4.37	D
	3.47	3.70	3.90	4.09	4.26	4.42	F
	4.28	4.48	4.65	4.81	4.95	5.09	ψ
4	3.88	4.08	4.27	4.44	4.60	4.75	D
	3.94	4.14	4.32	4.48	4.64	4.79	F
	4.67	4.85	5.00	5.15	5.29	5.42	ψ
5	4.29	4.47	4.64	4.80	4.85	5.09	D
	4.34	4.52	4.68	4.84	4.99	5.13	F
	5.02	5.18	5.33	5.47	5.60	5.72	ψ
6	4.66	4.83	4.98	5.13	5.27		D
	4.70	4.87	5.02	5.17	5.31		F
	5.34	5.49	5.63	5.76	5.89		ψ

A similar situation obtains for the states (S,3) and (D,2), which we predict at 4.28 and 4.24 GeV, respectively. They can account for the bumps at 4.10 and 4.4 observed at SPEAR. Finally the mass differences between D and F states come out around 100 MeV.

What happens if there exists another heavy quark with mass

$$m_{\mu} = 5.2 \text{ GeV ?} \quad (3.51)$$

In Table 3 we can read predictions which may be relevant for the spectroscopy of the T's.

Table 3

The calculated spectrum of the T family ($m_H = 5.2$ GeV)

$\begin{matrix} \ell \\ n \end{matrix}$	0	1	2	3	4	Particle type
1	9.53 5.15 5.26	9.80 5.56 5.65	9.97 5.84 5.92	10.10 6.06 6.13	10.21 6.25 6.32	T (H + $\bar{H}$) $\mathcal{D}$ (H + $\bar{p}$) $\mathcal{F}$ (H + λ)
2	10.03 5.94 6.02	10.17 6.18 6.25	10.28 6.38 6.45	10.38 6.56 6.62	10.47 6.71 6.77	T $\mathcal{D}$ $\mathcal{F}$
3	10.31 6.43 6.49	10.42 6.62 6.68	10.51 6.78 6.84	10.60 6.94 6.99	10.68 7.07 7.13	T $\mathcal{D}$ $\mathcal{F}$
4	10.53 6.81 6.87	10.62 6.97 7.03	10.71 7.12 7.17	10.79 7.26 7.31	10.87 7.39 7.44	T $\mathcal{D}$ $\mathcal{F}$
5	10.72 7.14 7.19	10.81 7.29 7.34	10.89 7.42 7.47	10.97 7.55 7.60	11.04 7.67 7.71	T $\mathcal{D}$ $\mathcal{F}$
6	10.90 7.44 7.48	10.98 7.57 7.62	11.06 7.70 7.74	11.13 7.82 7.86	11.21 7.93 7.97	T $\mathcal{D}$ $\mathcal{F}$

4. BARYONS AND THEIR SPECTRUM

The application of the geometrodynamical ideas to the qqq system has been worked out in collaboration with Szegö¹³). The baryon wave function shall be written as

$$\psi_{\alpha a \beta b \gamma c}(p; x_1 x_2 x_3) = e^{ipX} \psi_{\alpha a \beta b \gamma c}(p; x, y), \quad (4.1)$$

where α, β, γ are Dirac indices, a, b, c carry flavour and colour, and

$$\begin{aligned} X &= \frac{1}{3} (x_1 + x_2 + x_3) \\ x &= \frac{1}{\sqrt{2}} (x_2 - x_3) \\ y &= \sqrt{\frac{2}{3}} \left(x_1 - \frac{x_2 + x_3}{2} \right). \end{aligned} \quad (4.2)$$

The characterization of (4.1) proceeds exactly as in the meson case:

i) *Confinement*

$$\psi(p; x, y) = 0, \quad \forall x, y \notin R^8(p; x, y) \quad (4.3)$$

where $R^8(p; x, y)$ is a compact eight-dimensional space-time region with boundary $B^8(p; x, y)$.

ii) *Continuity*

Decomposing into Lorentz covariants (4.1), i.e.

$$\psi_{\alpha\alpha\beta\beta\gamma\gamma}(p; x, y) = \sum_s P_{\alpha\beta\gamma}^{(s)}(p, -i\partial_x, -i\partial_y) \phi_{abc}^{(s)}(p; x, y) \quad (4.4)$$

we require that the scalar functions $\phi_{abc}^{(r)}(p; x, y)$ be continuous functions, i.e.

$$\phi_{abc}^{(s)}(p; x, y) = 0 \quad \forall x, y \in B^8(p; x, y). \quad (4.5)$$

iii) *Wave motion*

For $x, y \in R^8(p, x, y)$ ψ obeys the simple differential equation

$$D_1 D_2 D_3 \psi = 0 \quad (4.6)$$

where D_i is again the Dirac operator.

iv) *Maximum freedom*

The "distance" of ψ from the "free solution" $\psi_{\alpha\alpha\beta\beta\gamma\gamma}^{(0)}(p; x, y)$ determined by the free equations

$$D_i \psi^{(0)} = 0 \quad i=1,2,3 \quad (4.7)$$

is minimum, in the sense of (3.8).

In solving the problem for $\psi^{(0)}$ we get a big surprise. In fact we can find no solution of (4.7) in a bounded space region, unless the space coordinates (in the rest frame) are constrained by the relations:

$$\begin{aligned} \vec{x} &= \vec{R} \cos \frac{\phi_0}{2} \\ \vec{y} &= \vec{R} \sin \frac{\phi_0}{2} . \end{aligned} \quad (4.8)$$

Such a constraint corresponds to a spatial configuration where the three quarks are aligned along the direction of the vector $\vec{R}$. This constitutes an important difference between the geometrical and the field theoretical approaches to the quark degree of freedom, it being very hard in the latter to understand how such a *constrained* motion could arise. Choosing $\phi_0 = 0$ we obtain for the spatial structure of the baryon a quark-diquark configuration which seems to be favoured by present experimental information.

Proceeding now in a way completely analogous to the meson case we obtain the following results.

- a) For a completely symmetric baryonic wave function the physical states can be organized in $[SU(6), L^P]$ multiplets following the pattern of Fig. 4. Note that the representations $(20, L^P)$ and $(70, 0^+)$ are absent, and that the parity of the states is always $(-1)^L$. These are all consequences of the suppression of three degrees of freedom dictated by our approach, and seem to have experimental support.
- b) The mass of a physical baryon state is given by the equation

$$M = \sum_{\substack{abc \\ ijk}} |\psi_{aibjck}^{(SU_8)}|^2 (E_{1a} + E_{2b} + E_{3c}) - \omega \quad (4.9)$$

Fig. 4 $[SU(6), L^P]$ pattern of physical baryon states

where $\psi^{[SU(8)]}_{(abc = 1, \dots, 4; i, j, k = 1, 2)}$ are the normalized SU(8) wave functions (m_i are the quark masses);

$$E_{1a} = \sqrt{\vec{Q}^2 + m_a^2} ; E_{2b} = E_{3b} = \sqrt{\frac{\vec{Q}^2}{4} + m_b^2} \quad (4.10)$$

$$|\vec{Q}| = \frac{\beta n L}{R_{s, nL}} \quad (4.11)$$

$$\omega = \frac{\pi}{R_{t, nL}} , \quad (4.12)$$

where R_s and R_t have the same meaning as in the preceding paragraph, and can be parametrized similarly to (3.38) and (3.39). See Table 4.

Table 4
The mean masses of the $[SU(6), L^P]$ multiplets

$[SU(6), L^P] \backslash n$	1	2	3	4	5
(56, 0^+)	1.14	2.07	2.71	3.24	3.69
(70, 1^-)	1.60	2.39	2.97	3.46	3.89
(56, 2^+) (70, 2^+)	1.94	2.65	3.20	3.66	4.07
(70, 3^-)	2.23	2.89	3.41	3.85	4.24
(56, 4^+) (70, 4^+)	2.48	3.10	3.60	4.03	4.41
(70, 5^-)	2.70	3.30	3.78	4.19	4.56
(56, 6^+) (70, 6^+)	2.91	3.48	3.94	4.34	4.71
(70, 7^-)	3.10	3.65	4.10	4.49	
(56, 8^+) (70, 8^+)	3.27	3.81	4.25	4.64	

5. THE INTERACTIONS OF MESONS

In this section we shall construct the interactions among mesons starting from their geometry. The general strategy of how to compute hadrodynamics starting with the fundamental hadron interactions has been described at length in the Erice courses of the last two years and need not be repeated here.

We shall begin by exhibiting explicitly the structure of the meson wave functions, which shall enter in an essential way in the construction of meson interactions.

5.1 Meson Wave Functions

In Section 3 we have seen that as a result of the general characterization of meson states their wave functions are given by

$$P = (\not{p}_1 + m_1) \gamma_5 (\not{p}_2 + m_2) \phi_S(p; x) \quad (5.1)$$

$$V = (\not{p}_1 + m_1) \gamma_\mu (\not{p}_2 + m_2) \phi^\mu(p; x) \quad (5.2)$$

for the pseudoscalar and the vector families, respectively. Both $\phi_S(p; x)$ and $\phi^\mu(p; x)$ obey the equation

$$K_1 K_2 \phi(p; x) = 0 \quad (5.3)$$

and we can write

$$\phi_S(p; x) = \phi(p; x) \quad (5.4)$$

$$\phi_\mu(p; x) = \epsilon_\mu(p) \phi(p; x), \quad (5.5)$$

where $\phi(p; x)$ is the solution of (4.3) which is "closest" to the free solution (see Section 2), and $\epsilon_\mu(p)$ is a polarization vector obeying $p^\mu \epsilon_\mu(p) = 0$.

According to (3.14) in the rest frame $p \equiv (M, \vec{0})$, we have

$$\phi(M, \vec{0}; t, \vec{x}) = \phi_M(\vec{x}) \psi_M(t) \quad (5.6)$$

where

$$\phi_{M, ne}(\vec{x}) = \left(\frac{4\beta_{ne}}{\pi R_{s, ne}^3} \right)^{1/2} \frac{1}{J'_{\frac{1}{2}+1}(\beta_{ne})} j_{\frac{1}{2}} \left(\beta_{ne} \frac{|\vec{x}|}{R_{s, ne}} \right) Y_{\frac{1}{2}}^m(\Omega_{\vec{x}}),$$

which has been *conventionally* normalized in such a way that

$$\int d^3x |\phi_{M, ne}(\vec{x})|^2 = 1$$

and

$$\psi_{M,ne}(t) = \frac{1}{2R_{t,ne}k_z} (e^{i\omega_1 t} + e^{i\omega_2 t}) \quad (5.7)$$

with

$$\begin{aligned} \omega_1 &= -\frac{M}{2} + \sqrt{k_{ne}^2 + m_1^2} \\ \omega_2 &= \frac{M}{2} - \sqrt{k_{ne}^2 + m_2^2} \end{aligned} \quad (5.8)$$

which has also been *conventionally* normalized such that

$$\int_{-R_t}^{R_t} dt |\psi_{M,ne}(t)|^2 = 1.$$

The quantity ω_{nl} defined as

$$\omega_{ne} = \omega_1 - \omega_2 = -M + \sqrt{k_{ne}^2 + m_1^2} + \sqrt{k_{ne}^2 + m_2^2} \quad (5.9)$$

is related to $R_{t,ne}$ by the equation

$$\omega_{ne} = \frac{\pi}{R_{t,ne}}. \quad (5.10)$$

The wave function $\phi(p;x)$, in a general Lorentz frame in which the meson is moving, can be obtained by (5.6), (5.7), and (5.8) by the following identifications:

$$|\vec{x}| = \sqrt{-x^2 + \frac{(px)^2}{M^2}} \quad (5.11)$$

$$t = \frac{(px)}{M} \quad (5.12)$$

and $\Omega_{\vec{x}}$ is given by the direction of the spatial part of the four-vector obtained from x by applying the Lorentz transformation $\Lambda^{-1}(\vec{p})$ which brings the meson to its rest frame; i.e.

$$\Omega_{\vec{x}} = \text{direction } [\Lambda^{-1}(\vec{p})x]_i. \quad (5.13)$$

We shall now calculate the Fourier transform $\phi(p;k)$ of $\phi(p;x)$:

$$\phi(p;k) = \int d^4x e^{ikx} \phi(p;x) \quad (5.14)$$

As a result of a simple computation, we have in the rest frame:

$$\begin{aligned} \phi_{n\ell}(k_0, \vec{k}) &= \left(\frac{4\pi}{|\vec{k}|}\right)^{1/2} R_s 2\pi \beta_{n\ell} \frac{J_{\ell+1/2}(R_s |\vec{k}|)}{(R_s |\vec{k}|)^2 - \beta_{n\ell}^2} \cdot \\ &\cdot Y_{\ell}^m(\Omega_{\vec{k}}) \frac{\pi}{R_{t,n\ell}^{3/2}} \frac{\cos\left[R_{t,n\ell}\left(k_0 + \frac{\omega_{n\ell} z}{2}\right)\right]}{\frac{\omega^2}{4} - \left(k_0 + \frac{\omega_{n\ell} z}{2}\right)^2}, \end{aligned} \quad (5.15)$$

where all quantities have been previously defined.

The extension to a general Lorentz frame is again immediate, and consists in identifying in (5.15)

$$|\vec{k}|^2 = -k^2 + \frac{(pk)^2}{M^2} \quad (5.16)$$

$$k_0 = \frac{(pk)}{M} \quad (5.17)$$

and

$$\Omega_{\vec{k}} = \text{direction} [\Lambda^{-1}(p)k]_i. \quad (5.18)$$

A convenient approximation of $\phi_{n\ell}(p;k)$ is

$$\begin{aligned} \phi_{n\ell}(p;k) &\simeq \left(\frac{2}{\pi R_{s,n\ell}}\right)^{1/2} (2\pi)^2 \delta_{R^2}\left(\vec{k}^2 - \frac{\beta_{n\ell}^2}{R_{s,n\ell}^2}\right) \cdot \\ &Y_{\ell}^m(\Omega_{\vec{k}}) \omega_{n\ell}^{3/2} \frac{\cos\left[\frac{\pi}{\omega_{n\ell}}\left(k_0 + \frac{\omega_{n\ell} z}{2}\right)\right]}{\frac{\omega^2}{4} - \left(k_0 + \frac{\omega_{n\ell} z}{2}\right)^2} \end{aligned} \quad (5.19)$$

where $\delta_{R^2}(z) = (R^2/\pi)(\sin z/z)$ is the "fat δ -function" introduced in Ref. 2.

5.2 The Three-Meson Coupling

The general structure of meson couplings has been discussed in last year's lectures⁹). There it was shown that, in order to accommodate all the salient dynamical properties of mesons, only the three- and the four-meson couplings are needed. Here we shall give a general description of the three-meson coupling only; four-meson couplings have not been worked out in detail yet.

The diagram describing the three-meson coupling is depicted in Fig. 5, where the dots represent a two-body "quark operator" denoting the probability amplitude for a quark of one meson to tunnel into another meson in the space-time region where the two mesons overlap. The most general form for such an operator is

$$(\text{---}\bullet\text{---})_{\alpha}^{\beta} = A(p^2) (\not{p} - m_a)_{\alpha}^{\beta} + B(p^2) m_a \delta_{\alpha}^{\beta} \quad (5.20)$$

where $p_{\mu} = (1/2i)(\overleftrightarrow{\partial}/\partial x_{\mu})$. The actual form of the functions A and B is important only for the low-lying mesons; for high masses in fact the operator p^2 has the universal eigenvalue $M\omega = \pi/R^2$.

Several choices of $A(p^2)$ and $B(p^2)$ are, however, possible and in the calculations to follow the particular choice was made that

$$A(p^2) = \frac{\mu^2}{p^2 - m^2}, \quad B(p^2) = 0, \quad (5.21)$$

Fig. 5 The three-meson coupling

which corresponds to an inverse propagator of a quark which cannot propagate through the vacuum, and μ^2 is a constant which has the dimensions $[\text{mass}]^2$ and plays the role of a coupling constant. Its value can be determined by fixing one coupling constant, say $g_{\rho\pi\pi}$, at its experimental value.

The general form of a three-meson vertex after substituting (5.21) is very complicated and need not be given here; we should, however, keep in mind that once the wave functions' normalizations are determined (see later) and a simple parametrization of (5.21) has been supplied, we are in a position to compute all three-meson coupling strengths.

The extension of the meson couplings to the unphysical region has already been discussed at this school⁹), the procedure is tedious, but otherwise well defined and presents no particular difficulty in its implementation.

6. CURRENTS AND WAVE FUNCTION NORMALIZATIONS

In order to determine the physical normalization of the wave functions we must first define the action of the current operators in the space of the meson wave functions and then require that the charge of hadron states be given correctly. In Ref. 9 we have analysed the problem in the context of scalar quark coordinates, and the conclusion was reached that in order to have a consistent description of high-energy quark-quark scattering it is necessary that the physical current operators be made up of two distinct pieces: the vector-meson dominated and the direct one. The different roles of these two contributions, and their important physical consequences have been adequately stressed⁹); in this section we shall apply these ideas to the meson case, work on baryons is still in progress.

6.1 The Vector Dominated Piece

The vector-meson dominated piece of the vector currents is defined through the sum of the diagrams in Fig. 6. The vector dominated form factor is then given by the expression

$$\langle A | J_\mu^i(0) | B \rangle = \sum_n \langle 0 | J_\mu^i(0) | V_n \rangle \frac{1}{m_n^2 - q^2} V(V_n \rightarrow A\bar{B}), \quad (6.1)$$

where $V(V_n \rightarrow A\bar{B})$ is the three-meson coupling discussed in the preceding section, and

$$\begin{aligned} \langle 0 | J_\mu^i(0) | V_n \rangle = & Z_1 N_{V_n} \int \frac{d^4 k}{(2\pi)^4} \text{Tr} \left(\frac{\lambda_i}{2} V_n \right) \Phi_{V_n}(q; k) \\ & \cdot \text{Tr} \left[\gamma_\mu \left(\frac{\not{q} + \not{k} + m_1}{2} \right) \not{q} \left(-\frac{\not{q} + \not{k} + m_2}{2} \right) \right] \end{aligned} \quad (6.2)$$

In (6.2) λ_i are the Gell-Mann matrices, Z_1 is the current normalization factor (in a theory with colour quarks $Z_1 = \sqrt{3}$) and N_{V_n} is the normalization factor of the vector meson V_n . By taking $A = B$ in (6.1) we obtain the vector-meson dominated form factor of the given meson A. Putting $\lambda_i/2 = Q$, selecting the charge form factor, and setting $q^2 = 0$, we get the vector meson contribution to the charge Q_A , to be called $Q_A Z_A^V$, in the form

$$Q_A Z_A^V = \sum_n \langle 0 | J_\mu^Q(0) | V_n \rangle \frac{1}{m_n^2} V(V_n \rightarrow A\bar{A}) \Big|_{q^2=0} \quad (6.3)$$

charge FF

Fig. 6 The vector-current piece dominated by vector-meson intermediate states

Obviously Z_A^V is proportional to N_A^2 ; thus we can write in general

$$Z_A^V = N_A^2 W_A, \quad (6.4)$$

where W_A can be explicitly computed once the normalization problem has been solved for the vector mesons V_n . The calculation of W_A is, in general, a quite difficult non-linear problem; a drastic simplification, however, occurs when we realize that, owing to the structure of the overlap integral $I(V_n \rightarrow A\bar{A})$, the contribution which is definitely dominant in (6.3) is the one corresponding to the vector mesons of lowest mass ρ , ω , ϕ , and J/ψ . This observation enables us to set up an iterative calculational scheme for W_A .

In the following we shall only keep the lowest vector mesons (V) and write approximately

$$W_A \simeq \sum_V \frac{g_{\gamma V}}{m_V^2} \frac{1}{N_A^2} V(V \rightarrow A\bar{A}) \quad (6.5)$$

where

$$\begin{aligned} \langle 0 | J_\mu^Q(0) | V \rangle &= \epsilon_\mu(q) g_{\gamma V} \\ g_{\gamma V} &= \frac{m^2}{2\gamma_V}. \end{aligned} \quad (6.6)$$

6.2 The Direct Piece of the Current Operator

We must now construct a vector current operator which couples directly to the quark coordinates and meets the requirements

- i) that it is conserved [in the SU(3) limit];
- ii) that it is additive in the quark charges.

In Fig. 7 the diagram corresponding to a vector current is reported where the operational definition of the symbols requires a more detailed discussion which has been carried out in Ref. 14. In particular, by taking two equal particles, one obtains at zero momentum transfer:

Fig. 7 The action of the direct piece of the current operator $J_\mu^i(0)$ on meson states

$$\langle A | J_\mu^Q(0) | A \rangle \Big|_{\text{direct}} = Q_A Z_A^D (2P_A)_\mu . \quad (6.7)$$

Writing, as suggested by the structure of the direct current,

$$Z_A^D = N_A^2 X_A \quad (6.8)$$

the normalization factor N_A is then determined by the equation

$$N_A^2 = \frac{1}{W_A + X_A} . \quad (6.9)$$

In this way we have achieved a well-defined procedure to normalize the meson wave functions. After this is accomplished every hadronic property can be computed starting from the basic parameters of our theory $[R_0^2, c; m_p, m_\lambda, m_c, \dots; \mu^2]$. In the next section the first preliminary results on meson properties will be reported.

7. THE CURRENT-PARTICLE MATRIX ELEMENTS

In this section we shall compute matrix elements of a current between the vacuum and the one-meson states, and compare the results with experiments.

7.1 The Coupling between a Vector Current and a Vector State

We consider the matrix element

$$\langle 0 | J_\mu^i(0) | \psi^j \rangle = \text{diagram} \quad \psi_j(p)$$

From (6.2) we readily compute for the $\ell = 0$ states

$$\epsilon_\mu(p) \frac{M_V^2}{2\gamma_V} \Big|_{\ell=0} = Z_1 \text{Tr} \left(\frac{\lambda^i}{2} \psi^j \right) N_{V_j} 2\epsilon_\mu(p) \phi_{n0}(0) \quad (7.1)$$

$$\left[M_V(M_V + \omega_V) - \frac{4}{3} |k_{n0}|^2 - (m_a - m_b)^2 \right],$$

where from (5.7) we can write

$$\phi_{n0}(0) = \frac{n}{\sqrt{2}} \omega_V^{1/2} R_{S,n0}^{-3/2}. \quad (7.2)$$

Asymptotically $[M_V \rightarrow \infty]$ we have

$$\frac{M_V^2}{2\gamma_V} \Big|_{\ell=0} \rightarrow Z_1 N_{V_j} \text{Tr} \left(\frac{\lambda^i}{2} \psi^j \right) \frac{4}{3} M_V^2 \frac{1}{R_0^2} \frac{1}{(2\pi)^{1/2}}. \quad (7.3)$$

For the $\ell = 2$ states we calculate a much smaller coupling, given by

$$\frac{M_V^2}{2\gamma_V} \Big|_{\ell=2} = \left(\frac{8}{9\pi} \right)^{1/2} Z_1 N_{V_j} \frac{2\vec{k}_{n2}^2}{15} \left(\frac{2\vec{p}_{n2}}{R_{3n2}^3 R_{t,n2}} \right)^{1/2} \text{Tr} \left(\frac{\lambda^i}{2} \psi^j \right), \quad (7.4)$$

which becomes asymptotically

$$\frac{M_V^2}{2\gamma_V} \Big|_{\ell=2} \rightarrow Z_1 N_{V_j} \text{Tr} \left(\frac{\lambda^i}{2} \psi^j \right) \frac{2M_V^2}{45} \frac{1}{R_0^2} \frac{1}{\sqrt{\pi}}. \quad (7.5)$$

which is a factor ~ 20 down from (7.3).

The coupling of $\ell = 2$ states to the photon does not vanish, as in the non-relativistic limit, but it is nevertheless very suppressed with respect to the $\ell = 0$ states.

Choosing the value $Z_1 = \sqrt{3}$, as suggested from a colour theory of quarks (note, however, that in this approach Z_1 is essentially

arbitrary), we can determine N_ρ by requiring that γ_ρ be given by its experimental value*)

$$\frac{\gamma_\rho^2}{4\pi} \Big|_{\text{exp}} = 0.55 ;$$

and we obtain

$$N_\rho^2 = 1.6 \text{ GeV}^{-4} . \quad (7.6)$$

From (7.1) we can also compute γ_ω and γ_ϕ and obtain

$$\gamma_\rho : \gamma_\omega : \gamma_\phi \simeq 1 : 3 : \frac{3}{\sqrt{2}} . \quad (7.7)$$

This agrees with experiments; and it is quite remarkable that the γ 's follow a SU(3) pattern, and not m^2/γ or any other combination of m^2 and γ . Incidentally m^2/γ deviates from the SU(3) pattern by some 70%.

7.2 The Ratio $R = \sigma(e^+e^- \rightarrow \text{hadrons})/\sigma(e^+e^- \rightarrow \mu^+\mu^-)$

We compute $\sigma(e^+e^- \rightarrow \text{hadrons})$ by considering the sum of the vector meson contributions of arbitrarily large masses. (See Fig. 8.) In Ref. 9 this problem was studied in the case of scalar quarks; here we shall proceed in a completely analogous way.

We define the vacuum polarization tensor

$$\begin{aligned} \Pi_{\mu\nu}(Q) &= (Q_\mu Q_\nu - g_{\mu\nu} Q^2) \Pi(Q^2) = \\ &= \sum_n (\pi)^\mu S^\nu(Q - P_n) \langle 0 | J_\mu^Q(0) | n \rangle \langle n | J_\nu^Q(0) | 0 \rangle , \end{aligned} \quad (7.8)$$

$$\sum_n \left| \text{---} \bigcirc \text{---} \right|_n^2$$

Fig. 8 The diagrams contributing to $\pi(Q^2)$ [Eq. (7.9)]

*) This requirement is not independent of the value of the coupling $g_{\rho^0 \pi^+ \pi^-}$ as in standard vector dominance.

where

$$\pi(Q^2) = \frac{1}{Q^2} 2 \text{Im} \left\{ \sum_n \sum_i \left(\frac{M^2}{2\gamma} \right)_{n,i}^2 \frac{1}{s - M_n^2 + i M_n \Gamma_n} \right\} \quad (7.9)$$

and $(M\Gamma)_n \rightarrow \text{constant}^{15})$ at high energy. In the high mass region we have (for $\ell = 0$)

$$M_n^2 \xrightarrow{n \rightarrow \infty} \frac{4\pi}{R_0^2} n, \quad (7.10)$$

and we can easily evaluate the sum (7.9), which gives

$$\pi(Q^2) \xrightarrow{Q^2 \rightarrow \infty} Z_1^2 \frac{4}{9\pi} \frac{Q^2}{R_0^2} N_v^2 \sum_i Q_i^2. \quad (7.11)$$

From the normalization condition neglecting, as it is legitimate being of order $1/M$, the vector dominated piece, we obtain

$$N_v^2 \rightarrow \frac{3}{2\pi} \frac{R_0^2}{Q^2} \quad (7.12)$$

Putting all factors together, we finally obtain

$$\pi(Q^2) \xrightarrow{Q^2 \rightarrow \infty} Z_1^2 \frac{2}{3\pi^2} \sum_i Q_i^2, \quad (7.13)$$

and recalling that for $e^+e^- \rightarrow \mu^+\mu^-$

$$\pi_{\mu^+\mu^-}(Q^2) \xrightarrow{Q^2 \rightarrow \infty} \frac{1}{6\pi}, \quad (7.14)$$

we obtain the remarkable result

$$R \rightarrow \frac{4}{\pi} Z_1^2 \sum_i Q_i^2 \quad (7.15)$$

Setting $Z_1^2 = 3$ the constant in front of $\sum_i Q_i^2$ has a value 3.82; thus we calculate

$$\begin{aligned} R &\rightarrow 2.55 && 3 \text{ quarks } (p, n, \lambda) \\ &\rightarrow 4.25 && 4 \text{ quarks } (p, n, \lambda, c). \end{aligned}$$

This also looks quite good; however, one remarks that the effect of the direct γ -coupling as well as the finite mass corrections have been left out. All these problems will be analysed in due course.

7.3 The e.m. Widths of the J/ψ Family

Given the γ_V 's whose expressions are exhibited in (7.2) for $\ell = 0$ states and in (7.3) for $\ell = 2$ states, the vector meson leptonic widths are computed through the well-known formula

$$\Gamma(V \rightarrow \ell^+ \ell^-) = \frac{\pi \alpha^2}{3 \gamma_V^2} M_V. \quad (7.16)$$

Accordingly, we can write for the ratios of e.m. widths

$$\frac{\Gamma(V \rightarrow \ell^+ \ell^-)}{\Gamma(V' \rightarrow \ell^+ \ell^-)} = \frac{M_V}{M_{V'}} \frac{\gamma_{V'}^2}{\gamma_V^2}. \quad (7.17)$$

By use of (7.2) we have for any radial recurrence of the J/ψ particle, $\psi^{(n)}$:

$$\frac{\gamma_p^2}{\gamma_{\psi^{(n)}}^2} = \frac{8}{9} \frac{N_{\psi^{(n)}}^2}{N_p^2} \frac{\omega_n}{\omega_p} \left(\frac{R_{s,10}}{R_{s,n0}} \right)^3 \left[\frac{1 + \frac{\omega_n}{M_n} - \frac{4}{3} \frac{k_{n0}^2}{M_n^2}}{1 + \frac{\omega_p}{M_p} - \frac{4}{3} \frac{k_{10}^2}{M_p^2}} \right]^2. \quad (7.18)$$

For the J/ψ we then compute $[N_{\psi} = 0.23]$:

$$\frac{\Gamma(J/\psi \rightarrow \ell^+ \ell^-)}{\Gamma(\rho \rightarrow \ell^+ \ell^-)} = 0.6, \quad \text{Exp} \approx 0.6$$

To compute the higher widths we need only to compute N_{ψ_n} , which gets determined by the direct coupling only (the J/ψ coupling is substantially suppressed by an unfavourable overlap). We get

$$\frac{\Gamma(\psi^{(n)} \rightarrow \ell^+ \ell^-)}{\Gamma(J/\psi \rightarrow \ell^+ \ell^-)} \approx \sqrt{n} \frac{M_{J/\psi}^2}{M_n^2} \frac{1 + \frac{\omega_n}{M_n} - \frac{4}{3} \frac{k_{n0}^2}{M_n^2}}{1 + \frac{\omega}{M_{J/\psi}} - \frac{4}{3} \frac{k_{10}^2}{M_{J/\psi}^2}}, \quad (7.19)$$

which gives for $n = 2$:

$$\frac{\Gamma(\psi \rightarrow \ell^+ \ell^-)}{\Gamma(\psi' \rightarrow \ell^+ \ell^-)} \simeq \frac{1}{.79}$$

i.e. $\Gamma(\psi^{(1)} \rightarrow \ell^+ \ell^-) \simeq 3.3 \text{ keV}$. This is not in disagreement with the reported $\sim 1.8 \text{ keV}$ for the ψ' , for we have argued in Section 3 that substantial mixing should occur between the $\ell = 0$ and the $\ell = 2$ vector state. In the maximal mixing configuration the figure 3.3 keV should be halved and attributed to each of the (3.69) and (4.02) states. This agrees with observation.

7.4 The Coupling between an Axial Current and a Pseudoscalar State

The matrix element to be computed is

$$\langle 0 | A_\mu^i(0) | P^j \rangle = Z_1 \gamma_\mu \gamma_5 \lambda_{1/2}^i \text{ (diagram) } P^j(p)$$

By using our wave function we compute

$$\langle 0 | A_\mu^i(0) | P^j \rangle = f_{PS} \text{Tr} \left(\frac{\lambda_i}{2} P_j \right) P_\mu \quad (7.20)$$

with

$$f_{PS} = Z_1 2 \left[(m_a + m_b) - (m_a - m_b) \frac{E_a - E_b}{M_{PS}} \right] N_{PS} \left(\frac{\omega_{PS}}{2R_{S, \pi 0}} \right)^{1/2} \quad (7.21)$$

For the π^+ we get:

$$f_{\pi^+} = Z_1 N_\pi 2m_p \frac{(2\omega_\pi)^{1/2}}{R_{S, \pi 0}^{3/2}} \quad (7.22)$$

and for K^+

$$f_{K^+} = Z_1 (m_p + m_\lambda - (m_p - m_\lambda) \frac{E_p - E_\lambda}{M_K}) N_K \frac{(2\omega_K)^{1/2}}{R_{S, K 0}^{3/2}} \quad (7.23)$$

Evaluating the various normalization factors following the procedure outlined in the preceding section, and recalling that $m_\lambda^2 = 0.22 \text{ GeV}^2$ and $m_p^2 = 0.01 \text{ GeV}^2$, we obtain

$$\frac{f_{K^+}}{f_{\pi^+}} = 1.18 \quad \text{Exp: } \frac{f_{K^+}}{f_{\pi^+} f_{\pi^0}} = 1.24.$$

We can furthermore compute the absolute value of f_{π^+} . The result is

$$f_{\pi^+} = 0.13 \text{ GeV}.$$

Experimentally, we have

$$f_{\pi^+} = 0.96 m_\pi$$

The agreement is very good.

We end this section by stressing that several very demanding tests have been passed by the approach presented here with great ease. However, whether or not we possess an accurate theoretical tool to deal with hadrodynamics will be answered only by a much wider and more refined analysis.

8. GEOMETRODYNAMICS AT HIGH ENERGY¹⁶⁾

In this section I will review the results so far obtained in the description of high-energy hadronic scattering processes. Some of the important points of the high-energy behaviour in the geometrodynamical theory have been touched upon in my previous Erice courses; in this section I would like to emphasize the coherence and the interdependence of the many different aspects of high-energy scattering which emerge in this approach.

8.1 The Fire-sausage: The Prototypical High-Energy Hadronic State

We have seen that for "low-mass quarks" (p, n, λ) the spectrum of fundamental hadrons [mesons ($q\bar{q}$) and baryons (qqq)] turns out as

comprising an infinite number of almost linear and parallel Regge trajectories without "odd daughters".

In this treatment, which is just a first approximation, I shall consider only the meson states; the baryon states in fact turn out to be less coupled at high energy¹⁷⁾.

The spectrum of such states is depicted in Fig. 9; the line $[R_T(M) \sim \log M]$

$$l_{\max} = R_1(M) \frac{M}{2} \quad (8.1)$$

defines the boundary between the meson states which have a wave function of order 1 ($l \leq l_{\max}$) and those which have an exponentially

Fig. 9 The meson Regge trajectories in the (orbital-angular momentum)- M^2 plane

small wave function ($\ell > \ell_{\max}$)¹⁸⁾. Thus at a given high mass M we have a set of $\sim \ell_{\max}/2$ states approximately degenerate in mass which couple strongly to their quark-antiquark constituents (large wave functions). By means of these states we can construct a set of "coherent states" by taking the following superposition:

$$|\Omega_0, M\rangle = \frac{1}{\ell_{\max}^2} \sum_{\ell=0}^{\ell_{\max}} \sum_{m=-\ell}^{\ell} Y_{\ell}^m(\Omega_0) \psi_{n\ell m}(p; k), \quad (8.2)$$

where $\psi_{n\ell m}(p, k)$ is the wave function describing the meson state (n, ℓ) whose mass is given asymptotically by

$$M^2 \approx \frac{\pi}{R^2} (2n + \ell). \quad (8.3)$$

These states have a very simple structure in configuration space. It is, in fact, very easy to show that the states (8.2) correspond to a cylindrical space domain (see Fig. 10), which we shall call a fire-sausage (FS)¹⁵⁾ of height $R_{\ell} = 2R^2 M$ and width $R_T(M)$, containing stationary "quark waves" of momentum $|k| \approx M/2$ moving with small divergences $[\delta \approx R_T/R^2 M]$ along the direction Ω_0 .

Why do we choose these seemingly arbitrary superpositions of states? The reason is just because the particular structure of the FS is preserved in the process by which stable hadrons are produced in the observed final states¹⁵⁾.

Let us in fact suppose that we produce in some way (which does not interest us here) one of the high-mass meson states. Such a state

Fig. 10 The space structure of a fire-sausage of mass M

is not stable, but it has a finite probability amplitude for decaying into two other such states of lower mass (see Section 5). Calculation¹⁵⁾ shows that the configuration of the final state which is by far the most probable, is the one in which one bag has a very low mass (μ) and the other a mass (M) which is a large fraction of the original one. Thus the most probable (canonical) decay configuration of the initial mesonic state (1) is given by the cascade chain depicted in Fig. 11.

To find out the structure of the final state we must square the amplitude of Fig. 11 and sum over all possible states $M_1, M_2, \dots$ and integrate over the full phase space. In carrying out this calculation we make two important observations:

- a) in the square of the probability amplitude interferences for $M_1 \neq M'_1, M_2 \neq M'_2, \dots$, turn out to be negligible, so we can visualize the cascade process as a classical one;
- b) fixing the mass of M_1 the phase of the transition amplitude for the various (quasi-) degenerate states of different angular momenta are fixed in such a way that the state M_1 can be considered a FS as in (8.2) along the direction of the momentum of μ_1 .

From these observations it follows that also $M_2, M_3, \dots$ are fire-sausages along the directions of $\mu_2, \mu_3, \dots$. From the structure of the FS and the fact that their interactions are simply given

Fig. 11 The "canonical" decay pattern of a high mass meson (1)

by space-time overlaps it follows very easily that the low mass hadrons $\mu_1, \mu_2, \dots$ are produced with very low momentum transverse to the direction of the initial FS.

By use of the observations (a) and (b) we can dramatically simplify the treatment of the cascade decay of the FS and, as has been shown in Ref. 15, it all boils down to solving an integral equation of a very simple type¹⁹⁾, the solution of which shows the following interesting characteristics.

i) Scaling

The invariant one-particle inclusive distribution

$$2E \frac{d^3N}{d^3p}$$

shows strong interaction scaling, i.e.

$$2E \frac{d^3N}{d^3p} = f(x, p_\perp) \quad , \quad x = \frac{2p_z}{M} \quad , \quad (8.4)$$

and the function $f(x, p_\perp)$ is peaked for small values of $p_\perp$, the momentum component transverse to the FS direction.

ii) Logarithmic multiplicities

As a consequence of (8.4) the particle multiplicities increase logarithmically with the FS mass, i.e.

$$\langle n_i \rangle = c_i \log M_i^2 + d_i \quad . \quad (8.5)$$

iii) Short-range correlations

Owing to the peculiar character of the cascade, particle correlations can only arise when the final stable hadrons emerge from low mass resonances. This bars the possibility of having long-range correlations from *single FS decay*. In the ideal case where $\mu_1, \mu_2, \dots$ are all π 's, the distribution would be of a Poisson type.

iv) Clustering

By inserting the meson wave functions, which are known explicitly, we can compute the ratios of multiplicities for "direct" production of low-lying meson states. Calculation gives

$$\langle n_{\pi} \rangle : \langle n_{\kappa} \rangle : \langle n_{\rho} \rangle : \langle n_{f_0} \rangle \approx 1 : 1 : 5 : \frac{1}{2} . \quad (8.6)$$

These surprising results are the consequence of a fairly tricky interplay of quark spins and meson masses, and show that even though direct production of π 's is strongly favoured by the multiperipheral character of the dynamics of FS decay, quark spins and wave-function factors end up "favouring" direct production of vector mesons and higher resonances. This finding seems to be confirmed by very recent observations and constitutes a *derivation* of the cluster model, which seems to be very adequate in describing the short-range part of particle correlations at high energy²⁰).

From the previous properties it is clear that FS decay is a strong candidate to account for the most important aspects of hadronic final states. Thus we can set up the correspondence:

$$\text{FIRE-SAUSAGES} \rightleftharpoons \text{FINAL STATES} .$$

In this way the extraordinary complication of high-energy inelastic processes reduces to the problem of finding the mechanisms by which, in the different reactions, one produces a small²¹) number of FS's which then decay most of the time according to the canonical pattern just described.

8.2 High-Energy Scattering and the "Pomeron"

After having described how the FS concept emerges from the spectrum and the space-time structure of high-mass meson states, we must ask how FS's get produced in high-energy collisions. In fact, we can conceive of inelastic high-energy scattering as a two-step process:

$$\text{initial state} \rightarrow \text{FS's} \rightarrow \text{final state (FS's decay)} .$$

Fig. 12 The single FS production in e^+e^- annihilation

In our approach the natural place to look for the production of a single FS is e^+e^- annihilation at high energy. We can visualize this process as in Fig. 12; from the annihilation of the lepton, through the intermediate heavy photon, a FS is created at an angle θ with probability $P(\theta) \propto (1 + \cos^2 \theta)$ due to the quark spin $\frac{1}{2}$ nature. The FS then decays according to the canonical pattern described in the previous section, thus generating a "jet" along the direction θ . According to recent results from SPEAR and DESY²²⁾, we can conclude that this description is strongly supported by the experimental observations.

Thus to produce the many particle states which are so familiar in all high-energy collisions we need not produce a large number of FS's. In fact, as already remarked, at the very root of the geometrodynamical approach there lies the idea that once we have taken into account the rich structure of hadrons inherent in their extended nature, the residual hadron-hadron interactions are quite weak and can be treated perturbatively. Thus, in general, even at the very high energies available at present machines we can visualize the collisions as giving rise to a small number of FS's which then decay into the final detected hadrons. For definiteness, let us now consider $\pi\pi$ scattering at high energy. According to our

Fig. 13 (1) One- and (2) two-FS contributions to the $\pi\pi$ cross-section

perturbative approach we consider to successive orders the contributions to high-energy inelastic scattering as reported in Fig. 13, where the "black boxes" have their explicit representation in terms of quark lines^{8,9}).

At high energy, the process (1) can be shown to die off like $1/s^2$, owing to form-factor damping⁹) and can be neglected. Process (2), however, can be shown⁹) to yield a cross-section which increases like R_T^2 . One can, in fact, compute

$$\sigma_2(s) \propto \pi R_1^2(s). \quad (8.7)$$

The production of 4, 6, ... FS's gives logarithmic corrections to (8.7) and unless we go to extremely high energies they can be treated as perturbations. It thus follows that the main mechanism for producing highly inelastic states at high energy involves the production and subsequent decay of *two* FS's. This means that asymptotically the following relation holds between the multiplicity in hadron-hadron scattering and e^+e^- annihilation¹⁵)

$$\langle n \rangle_{HH} \rightarrow 2 \langle n_{e^+e^-} \rangle. \quad (8.8)$$

This relation has been checked by Ferbel and Stix²³) and found consistent with data; its correctness constitutes a good support for the idea that in the geometrodynamical framework "strong interactions" can be treated as perturbations.

Fig. 14 The production of two FS's in pp collisions

For proton-proton scattering the situation is a bit more complicated owing to the fact, mentioned above, that baryonic FS's can be neglected at high energies. Diagram (2) of Fig. 12 must now be replaced by the one in Fig. 14, which shows that the dominant final states in baryon-baryon scattering comprise two leading clusters and two FS's. It is very interesting to note that it is precisely the non-existence of baryonic FS's that forces the baryon number to emerge in the final states always in the form of a low-mass cluster which, for kinematical reasons, takes a fair fraction ($\sim \frac{1}{2}$) of the incident baryon momentum. On the other hand, inelastic production originates from the scattering of the virtual mesons left behind by the leading clusters.

In this fashion a baryon interacting at high energy must necessarily give off about half of its momentum to the leading cluster. This structure leaves the interaction region simply as a spectator of the collision, which involves thoroughly only the virtual meson carrying the remaining fraction of the initial baryon momentum. All this strongly suggests that the "missing momentum" of deep inelastic scattering²⁴⁾ rather than going into gluons goes into something much less elusive, i.e. the leading cluster.

8.3 Particle Production at Large p_T

After having discussed the leading mechanism of high-energy particle production, we can now investigate the production of particles with large p_T .

We have seen that the dominant mechanism for particle production at high energy consists in the production of two FS's and in their subsequent "canonical" decay into many particles. Among the features of the "canonical" decay we have quite a strong cut-off in transverse momentum, which renders this decay configuration extremely inefficient (much more than experimentally observed) to produce particles at large p_T . We must then ask whether there are non-canonical decay configurations of the FS which yield more abundant production of particles carrying a large p_T .

The answer, as was shown in previous work, is indeed positive. The dominant mechanism is depicted in Fig. 15, which yields particles at large p_T through the "tilting" of one of the FS's of the decay chain at some finite angle θ .

The process can be described as follows: one of the two FS's produced at high energy starts decaying canonically until at one link of the chain one FS changes the direction of its axis by a *finite* angle θ and initiates another canonical decay process along

Fig. 15 The "tilting" of a FS along the decay chain

Fig. 16 The diagram determining the "tilting" probability (8.9)

the new direction, thus yielding particles at large p_T . The tilting probability for a FS of mass M has been computed for scalar quarks (see Ref. 9) from the diagram in Fig. 16, a similar calculation can be carried out for spinning quarks with the result

$$P(M, \theta) \rightarrow \lambda_1 \frac{f(\theta)}{M^4} \quad (8.9)$$

where

$$f(\theta) = \frac{1 + \frac{1}{4} (1 + \cos \theta)^2}{(1 - \cos \theta)^3}$$

and λ_1 is a "coupling constant" whose value can be, with some further labour, calculated. It is interesting to note that (8.9) is similar, but not identical, to the Møller scattering cross-section for two quarks which has been invoked by people working with QCD and the parton model²⁵). It has the same mass dependence (M^{-4}), but a slightly different angular dependence, thus leading to results which are similar to the parton model calculations. If we want to compute the one-particle inclusive cross-section we must juxtapose three separate pieces, as indicated in Fig. 17.

The tilting probability and the one-particle inclusive decay of a FS has just been discussed, so in order to complete the picture we must find out what is the probability of producing a high-mass $q\bar{q}$ pair (FS) in the hadronic high-energy collision. According to the previous discussion, apart from over-all normalization the first diagram in Fig. 16 can be decomposed according to Fig. 18.

Fig. 17 The elements needed to calculate the one-particle large p_T cross-section

Fig. 18 The inclusive production of a $q\bar{q}$ pair

In deep inelastic scattering we encounter a similar diagram (see Fig. 19), so that it is not difficult to see that the "quark distribution functions" measured in deep inelastic scattering naturally appear also in the description of the diagram in Fig. 17, in agreement with the postulates of the parton model.

Fig. 19 The diagram giving rise to the structure functions in deep inelastic scattering

An important question which has recently been drawing a great deal of attention is related to the quark transverse momenta. In the parton model it has always been assumed that partons carry only a small transverse momentum, of the same order as the one observed in hadronic collisions, i.e. $\langle p_T \rangle \approx 300$ MeV. According to recent analyses, it appears that such a value should be more than doubled, thus creating some difficulty for the parton model. How do we fare in this respect? It should be noticed that the quark degrees of freedom in this approach do carry even on the low-lying hadronic states a momentum of the order of 1 GeV²⁶⁾ and that on average $2/3$ of it is transverse. How about the small p_T observed in single-particle distributions at high energy? If we recall the result of Section 2 that the great majority of π 's are generated by resonance decay, that also can be naturally understood. Thus in our approach we have $\langle p_T \rangle_{\text{quark}} \approx 700$ MeV.

Putting everything together we arrive (after some simple calculation) at the following expression:

$$\frac{2E}{\sigma} \frac{d\sigma}{d^3p} \approx \frac{\lambda_1}{p_{\perp}^4} \sum_{q\bar{q}} \int dx_1 dx_2 [f_{H \rightarrow q}(x_1) f_{H \rightarrow \bar{q}}(x_2) + f_{H \rightarrow \bar{q}}(x_2) f_{H \rightarrow q}(x_1)] g(\omega\theta^*) f(z) = \frac{1}{p_{\perp}^4} F\left(\frac{2p_{\perp}}{\sqrt{s}}, \theta\right) \quad (8.10)$$

where

$$f_{H \rightarrow q, \bar{q}}(x)$$

are the $q\bar{q}$ distribution functions for the hadron H:

$$g(\omega\theta) = \frac{(1+\cos\theta)^2}{1-\cos\theta} \left[1 + \frac{1}{4} (1+\cos\theta)^2 \right] \quad (8.11)$$

and θ^* is the angle of the particle momentum in the $q\bar{q}$ centre-of-mass; and finally $f(z)$ is the one-particle distribution function in the FS decay which, according to Section 2, coincides with the one measured at SPEAR and DORIS.

The most remarkable aspect of (8.10) is the famous p_T^{-4} "scaling behaviour", which was suggested a long time ago²⁷⁾, and confirmed by several calculations. Although it is not supported by the experimental data, it seems hardly avoidable from the theoretical standpoint, and failure to observe evidence of it at higher energies and transverse momenta would constitute a serious difficulty for the approach advocated here. Thus there should be another mechanism which, even though asymptotically irrelevant, can in fact in some intermediate p_T range ($p_T \lesssim 6$ GeV) dominate over the "tilting" process. Indeed, the existence of such a mechanism was ascertained in previous work⁹⁾, where it was shown that the direct decay of a FS into a low-mass cluster and another smaller FS at large angle (see Fig. 20) lead in fact to a faster drop for the inclusive cross-section.

Going through the same steps as before, we can write:

$$\begin{aligned}
 2E \frac{d\sigma}{d^3p} \Big|_{\text{non leading}} &= \frac{\lambda_2}{p_T^8} \sum_{q\bar{q}} \int dx_1 dx_2 [f_{H_1 \rightarrow q}(x_1) f_{H_2 \rightarrow \bar{q}}(x_2) + \\
 &+ f_{H_1 \rightarrow \bar{q}}(x_1) f_{H_2 \rightarrow q}(x_2)] g'(\omega s \theta^*) f'(z) = \\
 &= \frac{1}{p_T^8} G\left(\frac{2p_T}{\sqrt{s}}, \theta\right).
 \end{aligned} \tag{8.12}$$

Fig. 20 The next leading mechanism for high p_T particle production

where

and $f'(z)$ is a fairly complicated function of the particle fractional momentum in the $q\bar{q}$ centre of mass²⁸⁾.

Thus, depending on the value of λ_2 (which can in principle be computed), we may have (8.12) dominate over (8.10) in some limited p_T range, and give the observed p_T^{-8} behaviour for the one-particle inclusive cross-section.

To conclude, our theory of large p_T processes predicts that asymptotically the one-particle cross-section is described by the formula (8.10), and that in an intermediate p_T and energy range the next leading contribution (8.12) may dominate the cross-section behaviour. The cross-section dependence on different beams and targets is practically the same as in the parton fusion model of Landshoff and Polkinghorne²⁹⁾. All this is nicely compatible with present experimental knowledge.

8.4 Structure of Final States

In this last section I shall briefly report on an analysis of the structure of final states arising in large p_T collisions done in collaboration with Rossi³⁰⁾. From the physical picture assumed in this report, it follows that the high p_T events originate in the two-step process

$$\begin{aligned} H_1 + H_2 &\rightarrow L_1 + L_2 + FS_1 + FS_2 \\ &\quad \searrow \\ &\quad R(p_T) + FS_3(-p_T) \end{aligned} \quad (8.13)$$

where $L_{1,2}$ are the leading clusters and $R(p_T)$ denotes a low mass (~ 1 GeV) resonance which gets emitted at large p_T through one of the two mechanisms analysed in the previous section. It is quite obvious that an analytical description of such complicated configurations is out of the question; thus we decided to follow a strategy based on generating high p_T events on the large CERN

computer using Monte Carlo methods. A detailed account of our method is reported in Ref. 30, where we show that several of the features observed in the CERN ISR experiments³¹⁾ are naturally reproduced.

Let me conclude this section by stressing that this methodology is likely to be the most efficient one to analyse and to compare with theory the amazing kinematical complexity of the multiparticle states produced in high-energy collisions.

9. CONCLUSIONS

In these lectures I have tried to give a presentation, as extensive as possible, of the new approach to hadrons and their interaction which has been called Quark-Geometrodynamics. I hope I have convinced you that this theory is capable of answering a great number of questions related to hadronic structure and high-energy behaviour. The principles are clearly stated, but the theoretical framework still suffers from some residual flexibility which could be removed by means of "theoretical experimentation". By this I mean that there are some aspects like three-bag overlaps and high-energy couplings whose precise quantitative evaluation needs further analysis in order to fix some of their structural elements. Only after this is done will one be able to make definite predictions on the enormously complex field of particle production and decays.

Work is in progress and I hope to be able to report on it at some future Erice School.

REFERENCES AND FOOTNOTES

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- 4) See the discussion of K.G. Wilson, Proc. 1975 International School of Subnuclear Physics, Erice, Italy, *in* New phenomena in subnuclear physics, part A (ed. A. Zichichi) (Plenum Press, New York, 1977), p. 127.
- 5) The original paper on the MIT-Bag is by A. Chodos, R.L. Jaffe, K. Johnson, C.B. Thom and V.F. Weisskopf, *Phys. Rev. D* 9, 3471 (1974); its extension is due to J. Kuti and collaborators; see, for example, J. Kuti, Proc. of the European Conference on Particle Physics, Budapest, 1977.
- 6) This approach has been proposed by G. Preparata and N. Craigie, *Nuclear Phys. B* 102, 478 (1976). Among its precursors the works of Yukawa and his group in the early fifties are the most noteworthy.
- 7) V.F. Weisskopf, Proc. 1975 International School of Subnuclear Physics, Erice, Italy, *in* New phenomena in subnuclear physics, part A (ed. A. Zichichi) (Plenum Press, New York, 1977), p. 241.
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- 9) G. Preparata, Proc. 1976 International School of Subnuclear Physics, Erice, Italy, *in* Understanding the fundamental constituents of matter (ed. A. Zichichi).
- 10) J.A. Wheeler, *Geometrodynamics* (Academic Press, New York, 1962).
- 11) I have in mind the remarkable posthumous book by H. Hertz, *The principles of mechanics presented in a new form* (Dover, New York, 1956).

- 12) For a discussion of the $U(1)$ -problem, see S. Coleman's lectures at this School.
- 13) G. Preparata and K. Szegö, Phys. Letters 68B, 239 (1977).
- 14) F. Csikor and G. Preparata, Geometrodynamics for quarks and hadrons; the definition of currents, CERN TH 2396 (1977).
- 15) N. Craigie and G. Preparata, Nuclear Phys. B102, 497 (1976).
- 16) I am following closely the presentation in G. Preparata, Proc. 8th Internat. Symposium on Multiparticle Dynamics, Kayzersberg, France, 1977 (C.R.N., Strasbourg, 1977), p. B-65.
- 17) This follows from the particular structure of baryonic wave functions; see Section 4.
- 18) This is a consequence of our normalization procedure; see Ref. 9.
- 19) This equation has been introduced first in A. Krzywicki and B. Petersson, Phys. Rev. D 6, 2606 (1972).
- 20) For a recent review of the situation, see A. Wroblewski, Proc. 8th Internat. Symposium on Multiparticle Dynamics, Kayzersberg, France, 1977 (C.R.N., Strasbourg, 1977), p. A-1.
- 21) The smallness of the number of produced FS's is a consequence of the perturbative nature of hadronic interactions (see Section 5).
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- 29) See the review of J. Polkinghorne, Proc. of the European Conference on Particle Physics, Budapest, 1977.
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