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THE SMOOTHNESS OF THE UNIVERSE

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ABSTRACT

We study the evolution of primordial fluctuations in the Big Bang in the context of grand unified theories of elementary particle interactions. We show that dissipation due to grand unified viscosity strongly damps rotational modes of all frequencies, compressional modes of all except very low frequencies, and radiative modes of high frequencies. The damping mainly occurs at temperatures $> 10^{15}$ GeV, and does not significantly increase the entropy. Compressional modes which originate at high temperatures and have very low frequencies persist down to much lower temperatures and could eventually evolve into galaxies.

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Grand unified theories of particle interactions^{1),2)} provide us with a coherent and plausible framework for discussing the very early evolution of the Universe before the start of the hadron era. They provide us with a hard equation of state with properties that are in principle calculable at all temperatures up to 10^{19} GeV. To date, the main cosmological application of grand unified theories has been to discuss the generation of baryon number³⁾ in homogeneous and isotropic cosmologies. At a temperature around $T \sim 10^{15}$ GeV a C and CP violating component in the baryon number violating interactions present in such theories is believed to have generated an excess of quarks over antiquarks, which subsequently evolved into a universe with an essentially uniform baryon to entropy ratio of order $10^{-9 \pm 1}$. Indeed, no experimental observations⁴⁾ suggest there is any significant amount of antimatter in the universe. There are other observables whose origins may also go back to the very early epoch of the universe to which grand unified theories are relevant. These include the net number L (possibly occurring in more than one variety: $L_e, L_\mu, L_\tau, \dots$), residual black body gravitational radiation, and the observed homogeneity and isotropy of the universe. The lepton number of the universe has been discussed in several recent papers⁵⁾. Our purpose here is to give a discussion within the context of grand unified theories²⁾ of the evolution of fluctuations and perturbations from homogeneity and isotropy in the very early universe.

Evidence is that the universe is homogeneous and isotropic now, and was at the time when the black body photon radiation dissociated⁶⁾. Recent discussions³⁾ of the baryon number of the universe have assumed that the universe was homogeneous and isotropic when a quark excess was formed at a temperature $T \sim 10^{15}$ GeV, but leave open⁷⁾ the possibility that it was originally chaotic and that some mechanism smoothed it out before T fell to 10^{15} GeV. In this paper we study the smoothing that could be achieved by grand unified particle interactions. Our starting point is the observation that at very early epochs the mean free time of an elementary particle $\sim 1/T$ should have been large compared to the age of the universe $\sim 1/T^2$, leading to a large "grand unified viscosity"⁸⁾ and thermal conductivity^{*}). These would have led to dissipation which could have damped primordial irregularities, according to the intuition indicated in the figure. Eventually, one would like to analyze the evolution of gross primordial inhomogeneities and anisotropies. Such a programme would be very complicated, and here we mainly limit ourselves to considering the fate of "small" fluctuations and perturbations about a homogeneous and isotropic background.

^{*}) Henceforth, all dimensional quantities will be expressed in Planck units unless the contrary is stated explicitly.

Such fluctuations may be grouped⁹⁾ into three classes of normal modes: radiative, rotational and compressional. Each of these may be characterized by an essentially constant wave number Q such that the frequency of oscillation is $\omega = QT$. We find that grand unified viscosity at T near 1 damps radiative modes with $Q \geq 0(10^{-1})$ and strongly damps rotational modes of all frequencies. At $T < 10^{-4}$, the dominant dissipative effects come from the long mean free path of gravitons⁹⁾, which results in the further suppression of rotational modes and the strong damping of all compressional modes except those with very low frequencies. Practical consequences of these results are a suppression of the high energy tail of the primordial black body gravitational radiation, an absence of rotational irregularities, and the possibility that compressional fluctuations of the type believed to lead eventually to galaxy formation could have grown unchecked throughout the period when $T > 10^{-19}$. How these fluctuations originated (primordially? at one of the gauge theory phase transitions at $T \sim 10^{-4}$ or 10^{-17} or 10^{-19} ?) and the details of their spectrum are of course, open questions. At the end of our paper we make some comments on the effect of grand unified viscosity on gross primordial anisotropy in homogeneous cosmologies. We find that there is no reason to expect that the end product would be isotropic.

In an imperfect fluid such as the very early universe, the dissipative effects come from components with the longest mean free times¹⁰⁾. In grand unified theories²⁾ one may estimate the mean free time of an elementary particle as

$$\tau_p \approx \frac{1}{n\sigma} \quad (1)$$

where n is the number density:

$$n = Ag_T T^3 \quad (2)$$

with $A = 0.12$ for a full thermal equilibrium distribution for bosons, and g_T a helicity counting factor ($g_T = 161$ in the simplest $SU(5)$ grand unified model²⁾ if one includes the factors of $7/8$ appropriate to fermions in an equilibrium distribution), and σ is the cross-section for collisions between an average pair of elementary particles, which one may estimate¹¹⁾ as

$$\sigma = \phi \frac{\alpha_{GU}^2}{T^2} : \alpha_{GU} \approx \frac{1}{42}, \phi = O(2) \quad (3)$$

Inserting (2) and (3) into the formula (1) for the mean free time we find

$$\tau_{GU} \approx \left(\frac{1}{g_r A \phi \alpha_{GU}^2} \right)^{\frac{1}{2}} \approx \frac{46}{T} \quad (4)$$

For comparison, the age of the universe during this epoch is

$$t \approx \left(\frac{3}{32\pi g_r B} \right)^{\frac{1}{2}} \frac{1}{T^2} \approx \frac{1}{44 T^2} \quad (5)$$

where B is defined by the energy density

$$\rho = B g_r T^4 \quad (6)$$

and $B \approx 3A$ for a full equilibrium distribution. It is plausible⁹⁾ to treat dissipative effects to first order in τ when one considers perturbations with a frequency ω :

$$\frac{2\pi}{\omega} \gg \tau \quad (7)$$

In terms of the wave number $Q \equiv \omega/T$, inserting τ_{GU} (4) into Eq. (7) yields

$$Q \ll 0.14 \quad (8)$$

as the condition for a first order treatment to be valid. In the absence of any better approximation, we will also use the first order formalism^{9),10)} even for larger Q than allowed by the condition (8), for which Q our results will be even more qualitative than usual. In a first order formalism with g_i helicity states of particles with mean free time τ , the shear viscosity

$$\eta = \frac{4}{15} B g_i T^4 \tau \quad (9)$$

while the bulk viscosity is zero for ultra-relativistic particles and the thermal conductivity

$$\chi = \frac{4}{3} B g_i T^3 \tau \quad (10)$$

Inserting Eq. (4) for the mean free time τ_{GU} and $g_T = 161$ we get

$$\left. \begin{aligned} \eta_{GU} &\approx \frac{4}{15} \frac{B}{A} \frac{T^3}{\phi \alpha_{GU}^2} \approx 650 T^3 \\ \chi_{GU} &\approx \frac{4}{3} \frac{B}{A} \frac{T^2}{\phi \alpha_{GU}^2} \approx 3200 T^2 \end{aligned} \right\} \quad (11)$$

which are large in the very early universe when T is near to 1. For comparison, it should be noted⁹⁾ that gravitons in a viscous medium have a mean free time

$$\tau_g = \frac{1}{16\pi\eta} \quad (12)$$

If the gravitons were themselves the main source of dissipation, one would insert (12) on the right-hand side of the expressions (9) and (10) for the shear viscosity and thermal conductivity, obtaining

$$\eta_g \approx \frac{\sqrt{\pi}}{30} T^2, \quad \chi_g = \frac{\sqrt{\pi}}{6} T \quad (13)$$

Comparing the expressions (11) and (13) we see that grand unified viscosity is dominant for

$$T > 10^{-4} \quad (14)$$

while graviton viscosity is dominant for lower temperatures. We will now study¹²⁾ the evolution of small perturbations using the dissipative quantities (11) and (13) in the appropriate temperature regimes.

Radiative Modes

According to the standard analysis⁹⁾, the amplitude h of a radiative mode of high frequency is suppressed in the WKB approximation by a factor

$$S_{Rad} = \exp(-8\pi \int \eta dt) \quad (15)$$

Using the expression (11) for η_{GU} between two temperatures T_i and T_f , and using the usual time-temperature relation (5) we get

$$S_{Rad} = \exp - \left[\frac{64\pi}{15} \left(\frac{B}{A} \right) \left(\frac{1}{\phi \alpha_{GU}^2} \right) (T_i - T_f) \sqrt{\frac{3}{32\pi g_T B}} \right] \quad (16)$$

This suppression factor is valid for frequencies and times:

$$Q \gg \frac{16\pi\gamma}{T} \quad (17)$$

which corresponds, using Eq. (11), to

$$Q \gg 3 \cdot 10^4 T^2 \quad (18)$$

Using as T_i in the suppression factor (16) the smaller of 1 or $\sqrt{Q}/170$ (cf. Eq. (18)), and $T_f \sim 10^{-4}$ (14), we find a numerical suppression in amplitude of

$$S_{\text{rad}} = \exp(0.07 - 4\sqrt{Q}) \quad (19)$$

As emphasized earlier (Eq. (8)) the formalism used here is not appropriate for very large Q . However, the results (19) suggest that radiative modes (gravitational waves, gravitons) with $Q > 0(1/10)$ will be damped by a significant factor ($S_{\text{rad}} \leq 1/10$) by grand unified viscosity in the very early universe. This means that in principle, there is a high frequency cut-off on the spectrum of primordial black body gravitational radiation. Unfortunately, it is not clear how this could ever be detected !

Rotational Modes

According to the standard analysis⁹⁾, the transverse part $\underline{U}_T$ of the perturbation in the velocity field in the fluid obeys an equation

$$\left(\frac{\partial}{\partial t} + 16\pi\gamma\right) \left[T^{-5} \left(\frac{4}{3}\rho - \chi \dot{\tau} \right) \underline{U}_T \right] = -\frac{\gamma}{T^3} Q^2 \underline{U}_T \quad (20)$$

We see from Eqs (6), (11) and (20) that the thermal conductivity term on the left-hand side is dominant for $T > 0(10^{-3})$ and that amplitudes of low frequency are suppressed by a factor.

$$S_{\text{rot}} = \exp - \left[\frac{128\pi}{15} \left(\frac{B}{A} \right) \left(\frac{1}{\phi \alpha_{\text{qu}}^2} \right) \sqrt{\frac{3}{32\pi g_{\text{TB}}}} \right] \approx \exp(-1600) \quad (21)$$

For modes with high frequency $Q \geq 0(2000)T^{3/2}$ there is an additional suppression due to the term on the right-hand side of Eq. (20). We conclude that rotational modes of all frequencies are very strongly damped during the very early universe. There may also¹²⁾ be significant damping when gravitation dominates viscous effects.

Compressional Modes

According to the standard analysis⁹⁾, compressional modes with frequencies much larger than the Jeans limit¹³⁾:

$$\omega \gg k_J = (12\pi\rho)^{\frac{1}{2}} \quad (22)$$

are damped by viscous effects. In terms of $Q \equiv \omega/T$, the condition (22) becomes

$$Q \gg \sqrt{12\pi g_{TB}} T \approx 45 T \quad (23)$$

For high modes obeying this condition (23), the amplitude damping is

$$S_{comp} = \exp\left(-Q^2 \int \frac{T^2 \eta}{4\rho} dt\right) \quad (24)$$

Inserting our previous expressions for t (5), ρ (6) and η_{GU} (11) into Eq. (24), we get a damping factor

$$S_{comp} \approx \exp\left[-\frac{2Q^2}{15} \sqrt{\frac{3}{32\pi g_{TB}}} \left(\frac{1}{g_{TA} \phi \alpha_{GU}^2}\right) \left(\frac{1}{T_f} - \frac{1}{T_i}\right)\right] \quad (25)$$

$$\approx \exp\left[-\frac{Q^2}{6} \left(\frac{1}{T_f} - \frac{1}{T_i}\right)\right] \quad (26)$$

Taking $T_i = Q/50$ (cf. the Jeans¹³⁾ limit (23)) and $T_f = 10^{-4}$ (corresponding (14) to the end of the very early epoch where grand unified particles dominate the generation of dissipation), we find

$$S_{comp} \approx \exp\left[-\alpha(10^3)Q^2\right] \quad (27)$$

for $Q \gg 1/200$, and hence significant damping for $Q > 0(0.05)$.

Compressional modes with lower values of Q are not significantly damped during the period when grand unified viscosity is significant. However, many of them are damped at lower temperatures by gravitational viscosity (13). Inserting η_g (13) into the suppression formula (24) we find

$$S_{\text{comp}} \approx \exp \left[- \frac{Q^2 \sqrt{\pi}}{120 g'_T B} \sqrt{\frac{3}{32 \pi g'_T B}} \left(\frac{1}{T_f^2} \right) \right] \quad (28)$$

where we have introduced a reduced number of helicity states g'_T corresponding to the low mass ($m < 10^{-4}$) particles in the grand unified theory ($g'_T = 107$ in the simplest SU(5) model²⁾). If we can apply the Eq. (28) down to $T_f = 10^{-19}$ (i.e., to $T \sim 1$ GeV) then we get significant damping for

$$Q > 4 \cdot 10^{-17} \quad (29)$$

However, the compressional modes which are believed⁹⁾ to lead to galaxy formation have still lower values of Q ($\approx 10^{-24}$ to 10^{-25}). This should not surprise us, since the standard analysis⁹⁾ shows that the observed sizes of galaxies (masses $\approx 10^{11} M_\odot$) are consistent with the Jeans limit at temperatures $T \sim 10^{10} \text{ K} \sim 1 \text{ MeV} \sim 10^{-22}$.

We therefore conclude that the compressional fluctuations which led to galaxies could have originated at any time during the very early stages of the universe, and grown undamped $\propto t \propto T^{-2}$. The time at which they originated is of course, an open question: primordially at $T \sim 1$? At the grand unified phase transition temperature $\sim 10^{-4}$ ¹⁴⁾? At the weak and electromagnetic phase transition temperature $\sim 10^{-17}$? At the strong interaction phase transition temperature $\sim 10^{-19}$? Or at some other temperature? All these possibilities are consistent with our analysis¹²⁾.

At the end of this analysis of small fluctuations in a homogeneous and isotropic background, we would like to make a few remarks about other dissipative phenomena. The first remark is that since entropy generation is proportional to bulk viscosity ζ in the first order formalism, and since $\zeta^{\frac{1}{2}} \propto (1/3 - \partial p / \partial \rho|_n)$ = 0 for ultra-relativistic fluids such as the one we discuss here at $T \gg$ the grand unification mass $\sim 10^{-4}$, there can be no significant entropy generation in the very early universe - at least within the first order formalism we have been using. The second remark concerns the possible isotropization of an initially homogeneous but anisotropic universe of the type discussed by Misner⁸⁾ in the

context of neutrino viscosity. A treatment by Matzner and Misner¹⁵⁾ shows that if one considers a metric of the form

$$ds^2 = dt^2 - e^{2\alpha(t)} [e^{2\beta(t)}]_{ij} dx^i dx^j \quad (30)$$

with β_{ij} a traceless 3×3 matrix, then one may under certain circumstances have isotropization ($\beta \rightarrow 0$) due to dissipative effects. Analysis of their formalism using grand unified particle or graviton effects indicates¹²⁾ that $\beta \rightarrow 0$ asymptotically only for a subset of measure zero of possible initial conditions. Hence grand unified viscous effects are not in general able to isotropize an initially anisotropic cosmology. The observed⁶⁾ isotropy therefore remains as an open problem.

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FIGURE CAPTION

An illustration of the transfer of energy-momentum from one part of the universe to another by particles with a long mean free time, leading to dissipation and eventual smoothing.