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GAUGE INVARIANCE FOR SPIN $\frac{1}{2}$

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A B S T R A C T

We construct a Lagrangian for an antisymmetric tensor spinor field $\chi_{\mu\nu}$, invariant under the gauge transformation $\delta\chi_{\mu\nu} = (\partial_\mu\gamma_\nu - \partial_\nu\gamma_\mu)\epsilon$, and show that it is equivalent to the massless Dirac Lagrangian. This is analogous to the description of spin 0 by an antisymmetric tensor field.

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Gauge invariant field theories provide a natural description of massless particles since the addition of a mass term destroys the gauge invariance. This applies not only to spins ≥ 1 but also to spin zero particles if they are described by means of antisymmetric tensors rather than scalars¹⁾. Consider the Lagrangian

$$\mathcal{L} = B_{\mu\nu} \epsilon^{\mu\nu\rho\sigma} \partial_\rho A_\sigma - \frac{1}{2} A_\mu A^\mu \quad (1)$$

Eliminating A_μ produces the standard free antisymmetric tensor theory invariant under $\delta B_{\mu\nu} = \partial_\mu \xi_\nu - \partial_\nu \xi_\mu$, whereas eliminating $B_{\mu\nu}$ implies that $A_\mu = \partial_\mu \phi$, with ϕ an arbitrary scalar and $\mathcal{L} = -\frac{1}{2}(\partial_\mu \phi)^2$. Thus the first order Lagrangian (1) shows nicely the equivalence between the antisymmetric tensor theory and the scalar theory. This model can be generalized to include interactions and the resulting theory is equivalent to a non-linear σ model²⁾.

The question addressed in this paper is whether these ideas can be applied also to spin $\frac{1}{2}$. That is, can we describe a massless spin $\frac{1}{2}$ particle by a gauge invariant Lagrangian of an antisymmetric tensor-spinor field $\chi_{\mu\nu}$? The answer to this question is yes, but the construction is not completely obvious. Consider the following direct analogy to (1)³⁾

$$\mathcal{L} = \bar{\chi}_{\mu\nu} (\gamma^\mu \phi^\nu - \gamma^\nu \phi^\mu + \gamma_5 \gamma_\rho \phi_\sigma \epsilon^{\mu\nu\rho\sigma}) - \frac{1}{2} \bar{\chi}_\mu \gamma_5 \gamma_\nu \partial_\rho \phi_\sigma \epsilon^{\mu\nu\rho\sigma} \quad (2)$$

First of all, if we add a "mass term" $\bar{\chi}_{\mu\nu} \chi_{\mu\nu}$ to (2) we can eliminate $\chi_{\mu\nu}$ to get a massive spin $\frac{3}{2}$ Lagrangian for ϕ_μ . Similarly if $B_{\mu\nu}^2$ is added to (1) we can eliminate $B_{\mu\nu}$ to get a massive spin 1 Lagrangian for A_μ . However, with the mass term absent, as in (2), $\chi_{\mu\nu}$ is a Lagrange multiplier and eliminating it imposes a constraint on ϕ_μ whose solution is $\phi_\mu = \gamma_\mu \chi$, with χ an arbitrary spinor field. The Lagrangian (2) then becomes a massless Dirac Lagrangian for χ . Up to this point (1) and (2) are very analogous but the difference is that we cannot use the ϕ_μ field equation to solve for ϕ_μ algebraically as a function of $\chi_{\mu\nu}$. So we cannot obtain a Lagrangian for $\chi_{\mu\nu}$ alone. The origin of the problem is that there are two spin $\frac{1}{2}$ projections of ϕ_μ , $\phi_\mu = \gamma_\mu \epsilon$ and $\phi_\mu = \partial_\mu \epsilon$. If we use the former then the term quadratic in ϕ_μ must contain a derivative, as in (2), whereas if we use the latter then the term quadratic in ϕ_μ will produce

a final Lagrangian with two derivatives instead of one. To get around this we must use both spin $\frac{1}{2}$ projection, and the way in which this can be done is suggested by some of the formulae of conformal supergravity ⁴⁾. The Lagrangian is

$$\mathcal{L} = \frac{1}{2} \bar{\chi}_{\rho\sigma} \gamma_5 \epsilon^{\rho\nu\sigma} (\partial_\mu \psi_\nu - \partial_\nu \psi_\mu + \gamma_\mu \phi_\nu - \gamma_\nu \phi_\mu) + \bar{\psi}_\mu \sigma^{\mu\nu} \phi_\nu \quad (3)$$

The constraint imposed by $\chi_{\mu\nu}$ has the solution

$$\psi_\mu = \partial_\mu \alpha + \frac{1}{\sqrt{3}} \gamma_\mu \chi \quad , \quad \phi_\mu = \frac{1}{\sqrt{3}} \partial_\mu \chi \quad (4)$$

with α and χ arbitrary spinors. Substituting (4) into (3) gives

$$\mathcal{L} = -\frac{1}{2} \bar{\chi} \not{\partial} \chi + \text{total derivative} \quad (5)$$

which shows that the model describes a single spin $\frac{1}{2}$ particle. Notice that the term quadratic in derivatives in (5) is fortunately a total derivative and may be dropped.

Now we notice that ϕ_μ occurs only linearly in (3). It can therefore be eliminated to give a constraint on ψ_μ ,

$$\psi_\mu = 2\gamma_5 \left(\gamma_\lambda \tilde{\chi}_{\lambda\mu} + \frac{2}{3} \gamma_\mu \sigma_{\alpha\beta} \tilde{\chi}_{\alpha\beta} \right) \quad ; \quad \tilde{\chi}_{\alpha\beta} = \epsilon^{\alpha\beta\mu\nu} \chi_{\mu\nu} \quad (6)$$

Substitution for ψ_μ gives the desired Lagrangian

$$\mathcal{L} = -\partial_\mu \tilde{\chi}^{\mu\nu} \left(\gamma_\lambda \tilde{\chi}_{\lambda\nu} + \frac{2}{3} \gamma_\nu \sigma_{\alpha\beta} \tilde{\chi}_{\alpha\beta} \right) \quad (7)$$

which is invariant under the gauge transformation

$$\delta \chi_{\mu\nu} = (\partial_\mu \gamma_\nu - \partial_\nu \gamma_\mu) \epsilon \quad (8)$$

which corresponds to the original invariance of (3) under the transformation $\delta\chi_{\mu\nu} = (\partial_\mu\gamma_\nu - \partial_\nu\gamma_\mu)\epsilon$, $\delta\psi_\mu = 8\partial_\mu\epsilon$. To show that (7) describes a spin $\frac{1}{2}$ particle directly is not easy, but by reversing the steps that led to its construction one can deduce its equivalence to the Dirac Lagrangian.

We have succeeded in constructing a field theory for a massless spin $\frac{1}{2}$ particle in which the masslessness is required by a gauge invariance. It is satisfying to note that massless particles of any spin can be described by a gauge theory. We have not yet been able to find a consistent interacting version of (7), but by analogy with the antisymmetric tensor σ model equivalence one can conjecture that if it exists it will be equivalent to the non-linear supersymmetric model of Volkov and Akulov⁵⁾.

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