ON THE SEMILEPTONIC DECAY DISTRIBUTIONS
FROM THE NEXT HEAVY QUARK⁺

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ABSTRACT

Motivated by the discovery of $T(9.4)$, we study the semileptonic decay correlations of the next heavy quark, and their consequences for the e^+e^- colliding beam and neutrino experiments. A detailed comparison is made with the corresponding correlations from the production and decay of charm and heavy leptons, $\tau^\pm$. Some tests of the $B^0 - \bar{B}^0$ mixing ($B = bq$ with b a $Q = -1/3$ heavy quark) are suggested for e^+e^- colliding beam experiments.

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1. INTRODUCTION

The discovery of T(9.4) by Herb et al.¹⁾ is being generally interpreted as a bound state of a quark-antiquark system consisting of bottom ($Q = -1/3$) and/or top ($Q = +2/3$) quarks. The present data is inadequate for a clear-cut assignment though (model dependent) theoretical attempts²⁾ tend to favour the bottom-onium interpretation. The mass of T(9.4) suggests that there is a rich spectroscopy of new hadrons in the mass range 5-7 GeV, and they will be produced copiously in the next generation of e^+e^- colliding beam experiments at PETRA, PEP and CESR. The lowest lying members of this group of particles will decay only through weak interactions shedding light on the nature of the quark (t versus b) as well as on the new weak interaction. Some of the anticipated properties of the weak interactions associated with the b (or t) decays have been studied by Ellis et al.³⁾ in the context of the six quark model of Kobayashi and Maskawa⁴⁾ (KM). It is not our aim to advocate any particular model at this stage, but we believe that the observations made in Reference 3) in the context of the KM model may have a general validity. It is shown in Reference 3) that the dominant part of the KM current is of the form

$$j_{\lambda}^{\Delta B = \pm 1} = \bar{b} \gamma_{\lambda} (1 - \gamma_5) c \quad \text{for } m_t > m_b \quad (1)$$

$$j_{\lambda}^{\Delta T = \pm 1} = \bar{t} \gamma_{\lambda} (1 - \gamma_5) s \quad \text{for } m_b > m_t \quad (2)$$

Above the threshold of both the b and t quarks, of course, there will be transitions $t \rightarrow b$ or $b \rightarrow t$ depending on the masses of the b and t quarks.

The structures (1) {(2)} of the bottom changing {top changing} charged currents are shared by a large variety of other models, so whereas the KM model itself may not survive in its present form, the structure of the $\Delta B = \pm 1$ (or $\Delta T = \pm 1$) weak currents might. The signatures of (1) are obvious and spectacular³⁾, however (2) needs a detailed analysis to work out the effects of the enormous energy released as compared to the GLM⁵⁾ current which it otherwise imitates. The only other possible assignments for the $\Delta B = \pm 1$, $\Delta T = \pm 1$ weak currents are (apart from chiral structure):

$$j_{\lambda}^{\Delta B = \pm 1} = \bar{b} \gamma_{\lambda} (1 - \gamma_5) u \quad \text{for } m_t > m_b \quad (3)$$

$$j_{\lambda}^{\Delta T = \pm 1} = \bar{t} \gamma_{\lambda} (1 - \gamma_5) d \quad \text{for } m_b > m_t \quad (4)$$

Again, the energy release is enormous like (2), but both (3) and (4) are $\Delta S = 0$ transitions and would be readily distinguished from the assignments (1) and (2) by looking at the final states in the decay process. In this paper we work out the signatures of these assignments quantitatively, study the effects of the energy release through (1) - (4) on the various distributions of experimental interest and compare them with the ones induced by charm and heavy leptons, $\tau^\pm$. Motivated by the observation in Ref. 3) that the lack of enhancement of the non-leptonic part of the GIM weak Hamiltonian for charm decays^{6),7)} may be a feature common to the decays of heavy quarks in general and even more pronounced for the bottom and top decays, we concentrate only on the semileptonic decays.

First, we recall the signatures³⁾ of the $\Delta B = -\Delta C = -\Delta Q$ current (1)⁸⁾. The current (1) has the unique feature (as compared to the rest) of producing weak cascade decays through the subsequent $\Delta C = \Delta S$ decay of the charm quark as shown in Fig. 1(b). The decay

$$\begin{array}{l} b \rightarrow c \ell^- \bar{\nu}_\ell \\ \quad \quad \quad \searrow \\ \quad \quad \quad s \ell^+ \nu_\ell \end{array} \quad (5)$$

where

$$\begin{array}{l} \ell^- = e^-, \mu^-, \tau^- \\ \ell^+ = e^+, \mu^+ \end{array}$$

will result in trilepton and tetralepton events in e^+e^- and ν experiments. One will have, for example, reactions of the type

$$e^+e^- \rightarrow b\bar{b} \rightarrow (\ell^\pm \ell^\mp, \ell^\pm \ell^\pm, \ell^\pm \ell^\mp \ell^\pm, \ell^\pm \ell^\mp \ell^\pm \ell^\mp) + \text{anything}$$

($\ell = e, \mu$). Further, they will be produced approximately in the ratio (a) $1:\frac{1}{4}:\frac{1}{20}:\frac{1}{400}$ assuming a flat 10% semileptonic branching ratio⁷⁾ for charm and bottom decays, for electrons and muons separately (b). Note that the presence of $\ell^\pm \ell^\pm$ events by itself does not constitute evidence of $B^0 - \bar{B}^0$ mixing. We present later (see Section III) a criterion, based on lepton energy distributions, for distinguishing the like sign leptons (c), so that such events may indeed signal $B^0 - \bar{B}^0$ mixing⁹⁾.

Now we discuss briefly the multikaon production in the decays of bottom and top through (1) - (4). First note that although (3) and (4) are $\Delta S = 0$ transitions, they will still induce multikaon events (though at a small rate) through the decays $b \rightarrow u \bar{c} s$ and $t \rightarrow d c \bar{s}$. In order to calculate the multikaon rate, we

estimate:

$$\begin{aligned} \frac{\Gamma(b \rightarrow u + (\bar{c}s))}{\Gamma(b \rightarrow u + \text{anything})} &= \frac{\Gamma(t \rightarrow d + (c\bar{s}))}{\Gamma(t \rightarrow d + \text{anything})} \\ &\simeq \frac{\Gamma(t \rightarrow s + (c\bar{s}))}{\Gamma(t \rightarrow s + \text{anything})} \simeq 1/6 \\ \frac{\Gamma(b \rightarrow c + (\bar{c}s))}{\Gamma(b \rightarrow c + \text{anything})} &\simeq 1/10 \end{aligned} \quad (6)$$

The fraction of multikaon events can then be calculated by estimating the F, F^* content in the $(\bar{c}s)$ pair ^(d). A method of calculating the single particle to inclusive ratio is given in Appendix A. The assignment (1) has the common feature with (2) of producing at least one kaon per decay. In addition, they will induce processes of the type

$$e^+e^- \rightarrow b\bar{b}(t\bar{t}) \rightarrow (K\bar{K}, 2(K\bar{K}), 3(K\bar{K})) + \text{anything} \quad (7)$$

where $K = K^+$ or K^0

A rough estimate for the production ratio is $\sigma(K\bar{K}) : \sigma 2(K\bar{K}) : \sigma 3(K\bar{K}) \simeq 1 : \frac{1}{5} : \frac{1}{100}$ for assignment (1) and $1 : \frac{1}{3} : \frac{1}{36}$ for assignment (2), respectively ^(e). So, if the assignment (1) or {(2)} is correct, as is the case for the KM model, the fraction of events containing multikaons (≥ 3) in e^+e^- experiments will not be insignificant. In addition, if there is $B^0 - \bar{B}^0$ mixing ($T^0 - \bar{T}^0$) mixing, then one will have processes {based on assignments (1) and (2)} of the type

$$e^+e^- \rightarrow b\bar{b}(t\bar{t}) \rightarrow (2K, 2\bar{K}, (2K, 2\bar{K}) + K\bar{K}, (4\bar{K} + 2K), (4K + 2\bar{K})) + \text{anything} \quad (8)$$

where $K = K^+$ or K^0

The relative rate of the various processes in (8) depends on the details of weak mass mixings. Theoretical estimates ³⁾ give rather large mixings.

In Section II, we present our main dynamical assumptions for studying the production and semileptonic decays of the bottom (top) quarks. This in a way is recycling the technology ¹⁰⁾ harnessed in the study of charm decays, albeit in a different kinematic region. For decays, we consider the free quark decay model

with $(V \pm A)$ interaction, and also the analogues of $D \rightarrow K \ell \nu_\ell$ and $D \rightarrow K^* \ell \nu_\ell$. All these models predict very similar distributions for the $b(t)$ decays, and we concentrate only on the free quark decay mechanism. For production, we believe the heavy quark inertial fragmentation picture advocated by Suzuki¹¹⁾ and Bjorken¹²⁾ is appropriate for heavier than charm quarks. In order to take into account the subasymptotic energies, we assume a fragmentation function $D(Z)$ ($Z = \frac{|p_H|}{|p_Q|}$, $H = Q\bar{q}$) of the form

$$D(z) = z^n \quad n=0, 1 \quad (9)$$

In Section III we derive analytic expressions for the lepton and hadron energy distributions valid for any arbitrary fragmentation-decay chain. Some of this already exists in the literature¹⁰⁾. We supplement this by studying the cascade decays that should be increasingly encountered in the future. For this section we assume a mass range $5.0 \text{ GeV} \leq m_B(m_T) \leq 5.5 \text{ GeV}$ for the $B(T)$ meson and study the effect of (1) varying the mass parameters of the final state hadrons, (2) the chirality of the current, and (3) the effect of different fragmentation functions. In selecting the range of these parameters, we have sought guidance from the experimental data on charm production⁷⁾. Also studied in this section are the lepton momentum distributions perpendicular to a plane defined, for example, by the incident ν and the outgoing lepton. Insofar as the $(p_\perp)_\ell$ distribution is universal, we expect the results to be useful in hadron production of the top (bottom) as well.

In Section IV, we discuss the collinearity angle distributions for leptons from $\tau^+\tau^-$, $c\bar{c}$, $b\bar{b}$ and $t\bar{t}$ production for the energy region relevant for the next generation of e^+e^- colliding beam experiments. The collinearity angle distribution for the leptons from the production and decay of charm and $\tau^+\tau^-$ have the well-known feature of showing a forward peak as the beam energy increases, arising from the Lorentz boost. However, for leptons which are the decay products of the bottom (or top) quarks, the Lorentz boost effect will be much smaller near the $b\bar{b}$ ($t\bar{t}$) threshold. The collinearity angle distribution is thus a useful criterion to signal the onset of a new quark threshold through the flattening of the collinearity angle distribution. Much above the threshold, the primary leptons produced through $b\bar{b}$ as shown in Fig. 2(a) ($t\bar{t}$ production is described by a similar diagram) will show characteristics similar to that of $\tau^+\tau^-$ and charm induced leptons. However, the cascade leptons produced from the same b (or $\bar{b}$) through the reaction (5) will show a markedly distinct pattern in populating the

entire $\cos \theta_{\text{coll.}}$ region, with the details depending on the masses, energy and fragmentation function. The quantity $\langle \cos \theta_{\text{coll.}} \rangle$ exemplifies this difference in a distinct way.

We also calculate the invariant mass squared distribution for the di-leptons produced by the cascade decay of the b quark, with both the leptons coming from the same b quark. Section IV contains a summary of the various correlations. Appendix A describes a method of calculating the single particle to the inclusive final state ratio in the free quark decay model. The technical details of the lepton momentum distributions are relegated to Appendix B.

2. PRODUCTION AND DECAY MECHANISM

We here recycle the dynamical models used to study the semileptonic correlations for the decay of charm. Consider the semileptonic decay generically denoted as

$$X \longrightarrow Y \ell \nu_\ell$$

There are three different mechanisms that have been employed in the literature¹⁰⁾.

Model I

x and y both spin $(\frac{1}{2})$ quarks with x decaying like a heavy muon. The model advocated for the charm decay by Gaillard, Lee and Rosner¹⁴⁾ is borne out by the inclusive lepton data in e^+e^- and ν experiments⁷⁾. This model, which could be used as a prototype to estimate heavy quark decays, has to be used in an isobaric sense with m_y a phenomenological parameter, determined in an average way by the masses of the most important physical hadronic states, which in turn depend on the nature of the weak current. For the GIM current⁵⁾, m_y is restricted by the data to lie in the range $m_K \leq m_y \leq m_{K^*}$ ⁷⁾. This is a reflection of the isosinglet nature of the $\Delta C = \Delta S$ part of the GIM current, since low energy pion emission is then suppressed¹⁴⁾. The relevant mass states for the $\Delta B = -\Delta C$ current, (1), should lie in the range $m_D \leq m_y \leq m_D + n\pi$, as far as semileptonic decays are concerned. Note that the current (1) is also isosinglet. We anticipate, in the spirit of the semileptonic decays of charm, that multipion emission will also be suppressed in the semileptonic decays of bottom. Consequently, we do not expect n to be large.

Model II

Here both x and y are treated as the lowest lying pseudoscalar mesons which are compatible with the structure of the weak current. This model, which is the analogue of $D \rightarrow K \ell \nu_\ell$, represents an extreme possibility since there are suggestions

from the non-relativistic quark model that the $0^- \rightarrow 0^-$ semileptonic decays may become increasingly unimportant, as the mass of the heavy quark increases (see below). Nonetheless, the model can be used in an isobaric sense much the same way as the free quark decay model, to study the effect of a different Lorentz structure on the shape of the correlations.

Model III

Alternatively, one may assume that x is a pseudoscalar meson and y a vector one, the analogue of $D \rightarrow K^* \ell \nu_\ell$. The model has some phenomenological support from the e^+e^- experiments which suggests that the charm quark tends to couple more to vector states $\{\sigma(DD^*), \sigma(D^*D^*) \gg \sigma(DD), (D \rightarrow K^* \ell \nu_\ell)/(D \rightarrow K \ell \nu_\ell) \geq 1, \text{ etc.}\}$. The non-relativistic quark model also suggests the increasing importance of this mode. To see this more quantitatively define $(q = p_x - p_y)$

$$\begin{aligned} \langle Y | J_\lambda | X \rangle &= f_+(q^2) (p_x + p_y)_\lambda \quad \text{for } 0^- \rightarrow 0^- \\ \langle Y^* | J_\lambda | X \rangle &= F_A(q^2) \epsilon_\lambda(Y^*) \quad \text{for } 0^- \rightarrow 1^- \end{aligned}$$

then one has,

$$\frac{\Gamma(X \rightarrow Y \ell \nu_\ell)}{\Gamma(X \rightarrow Y^* \ell \nu_\ell)} = \frac{4 f_+^2 m_{Y^*}^2 I_Y}{F_A^2 I_{Y^*}} \quad (10)$$

where

$$I_Y = 1 - 8m^2 + 8m^6 - m^8 - 12m^4 \ln(m^2)$$

$$I_{Y^*} = 1 + 72m^{*4} - 64m^{*6} - 9m^{*8} + (36m^{*4} + 48m^{*6}) \times \ln(m^{*2})$$

$$m = m_Y/m_X, \quad m^* = m_{Y^*}/m_X$$

Assuming the non-relativistic quark model result

$$f_+(0) = 1$$

$$F_A(0) = m_X + m_{Y^*}$$

(11)

the results corresponding to the various $B(T)$ decay modes are listed in Table 1. These values, which suggest a predominance of the $0^- \rightarrow 1^-$ transitions, should be taken with caution, since there are models (e.g. current algebra) which do not reproduce these results [see Ali and Yang in Ref. 10)].

However, we will see that, as the quark mass increases, the distributions due to the models I ... III tend to merge, with only $(m_y - m_x)$ becoming the single most important parameter. This is encouraging since it means that the correlations are characteristic of the weak current itself rather than of a particular decay mechanism. We show this in Fig. 3 for lepton energy distributions for the B decay via current (1) for $m_y = m_D$ and $m_y = m_D^*$. The curve for $m_y = m_D$ also applies to model I(V - A) case. We note that the distributions are rather similar. The same is true of energy distribution of the hadron y and is not shown. We believe that any of the decay modes I ... III is an equally reliable way to study the heavy quark correlations. For its aesthetic appeal we shall adopt the free quark decay model and vary the parameter m_y as we calculate the correlations from the currents (1) ... (4), as well as from $c\bar{c}$ and $\tau^+\tau^-$ production.

Having a decay model at hand, we turn to the production of heavy quarks in e^+e^- and ν experiments. Near the threshold, which we believe for the $b(t)$ quarks to lie in the region $10 \text{ GeV} \leq \sqrt{s} \leq 12 \text{ GeV}$ for the e^+e^- experiments, two-body and quasi two-body processes dominate with the production, for example:

$$e^+e^- \rightarrow H \bar{H}, H \bar{H}^*, H^* \bar{H}^*, \dots$$

$$\quad \quad \quad \downarrow$$

$$\quad \quad \quad H\pi, H\gamma, \dots$$

where H denotes a heavy meson. However, because of the large mass of the heavy quark, the stable hadron ($H = Q\bar{q}$) retains the bulk of the momentum. It is, then, a good first approximation to forget about the smearing of the heavy hadron momentum due to the soft Brehmsstrahlung of pions and photons. Much above the threshold, the inclusive production of the heavy hadrons is presumably the correct framework for studying any detailed effect. If one assumes the inertial picture of the final state evolution advocated by Suzuki¹¹⁾ and Bjorken¹²⁾, the fragmentation function of the heavy quark Q into a heavy hadron H peaks more and more towards $Z (=p_H/p_Q) \simeq 1$, as the mass of the quark and the energy increase. A theoretical analysis of the CDHS data¹⁵⁾ suggests a fragmentation function which is non-negligible near $Z \simeq 1$. The high energy data in e^+e^- annihilation¹⁶⁾ suggest a large high momentum D^{*s} yield giving some credence to this picture. We believe this peaking of $D(Z)$ near $Z = 1$ will be even more pronounced, as compared to charm,

for the case of $b(t)$ quarks with $m_b(m_t) \approx 5 \text{ GeV}$ and $\sqrt{s} \geq 20 \text{ GeV}$.

We assume

$$D(z) = z^n \quad n = 0, 1$$

as a reasonable intermediate fragmentation picture interpolating between the u , d quarks, where the data suggests a $(1 - Z)^2$ behaviour and the regime of giant quarks with $m_Q \geq 100 \text{ GeV}$, $\sqrt{s} > 100 \text{ GeV}$ where the quarks become lepton-like with $D(Z) = \delta(1 - Z)$, in the spirit of the Suzuki-Bjorken fragmentation picture.

3. LEPTON ENERGY DISTRIBUTIONS

We shall consider the following processes

$$e^+ e^- \rightarrow \tau^+ \tau^- \begin{cases} \rightarrow l^- \bar{\nu}_l \nu_\tau \\ \rightarrow l^+ \nu_l \bar{\nu}_\tau \end{cases} \quad (12)$$

$$e^+ e^- \rightarrow c \bar{c} \begin{cases} \rightarrow \bar{D}^+ \dots \rightarrow \times l^- \bar{\nu}_l \\ \rightarrow D^+ \dots \rightarrow \times l^+ \nu_l \end{cases} \quad (13)$$

$$e^+ e^- \rightarrow b \bar{b} \begin{cases} \rightarrow \bar{c} l^+ \nu_l \\ \rightarrow c l^- \bar{\nu}_l \end{cases} \quad (14)$$

$$e^+ e^- \rightarrow b \bar{b} \begin{cases} \rightarrow c l^- \bar{\nu}_l \\ \rightarrow D^+ \dots \rightarrow \times l^+ \nu_l \end{cases} \quad (15)$$

$$e^+ e^- \rightarrow b \bar{b} \begin{cases} \rightarrow \bar{u} l^+ \nu_l \\ \rightarrow u l^- \bar{\nu}_l \end{cases} \quad (16)$$

$$e^+ e^- \rightarrow t \bar{t} \begin{cases} \rightarrow (\bar{d}, \bar{s}) l^- \bar{\nu}_l \\ \rightarrow (d, s) l^+ \nu_l \end{cases} \quad (17)$$

$l = e, \mu$

and calculate the lepton spectra both near the threshold of $b(t)$ quarks and at $\sqrt{s} = 20.0$ GeV so as to illustrate the energy dependence of the various spectra. The lepton spectrum from the heavy leptons $\tau^+\tau^-$ can be calculated from Eqs. (B2) and (B9) for the case of $(V - A)$ interaction with $m_x = m_\tau$, $m_y = 0$. Near the $b\bar{b}(t\bar{t})$ threshold, the lepton energy spectra from the processes (14), (16) and (17) can be calculated from Eq. (B9) of the Appendix B. The function F appearing in (B9) is taken from Eq. (B2) for models I and II and from Eq. (B4) for model III. The $(V + A)$ case is obtained by inserting (B3) in (B9). Much above the threshold, say $\sqrt{s} \geq 20$ GeV, one has to worry about the fragmentation of the $b(t)$ quarks into $B(T)$ mesons. The lepton spectrum is now obtained from Eq. (B11) by inserting (B9) and choosing a fragmentation function. Eq. (B11) also gives the lepton spectrum from the production of a $c\bar{c}$ pair¹⁷⁾, i.e. reaction (13). The lepton energy spectrum for the secondary lepton from the cascade decay of the b quark, i.e. reaction (15), requires somewhat more care. Near the threshold of the $b\bar{b}$ pair, say $\sqrt{s} \approx 12$ GeV, with $m_b \approx 5$ GeV, $m_c \approx 2.0$ GeV, adopting a fragmentation picture for the quark to hadron transition $C \rightarrow D$ may be inappropriate kinematically. However, not knowing of any better way of incorporating the inelasticity in the production and decay we shall continue using the fragmentation picture, assuming that $D(Z)$ takes into account such effects as, for example,

$$B^* \rightarrow B\gamma, B\pi, \dots$$

The choice $D(Z_c) = Z_c^n \{Z_c = (|p_D|/|p_c|)\}$ with $n = 0, 1$ will only overestimate the hardness of the secondary lepton energy spectrum. The lepton energy spectrum is now obtained by folding (B10) with (B11). The resulting expression is:

$$\frac{1}{\Gamma} \frac{d\Gamma(E_b, E_\ell)}{dE_\ell} = \int_{E_c^{\min.}}^{E_c^{\max.}} dE_c J(E_b, E_c) \int_{Z_c^{\min.}}^1 dZ_c H(E_c, E_\ell) D(Z_c) \quad (18)$$

where

$$E_c^{\min.} = m_c \quad \text{for } E_\ell \leq \frac{m_D^2 - m_S^2}{2m_D}$$

$$= \left[\frac{(4m_D^2 E_\ell^2 - (m_D^2 - m_S^2)^2)^2}{4(m_D^2 - m_S^2)^2 m_D^2} + m_c^2 \right]^{1/2} \quad \text{for } E_\ell > \frac{m_D^2 - m_S^2}{2m_D} \quad (19)$$

The functions $H(E_c, E_\ell)$ and $J(E_b, E_c)$, defined through (B9) and (B10), depend on $F(p_x \cdot p_\ell/m_x)$ and $G(p_x \cdot p_\ell/m_x)$ respectively, and are listed in Appendix B for the various decay models under consideration.

Finally, the secondary lepton energy spectrum from the cascade decay process (14) for a beam energy much higher than the $b\bar{b}$ threshold will require two fragmentation processes according to the chain [see Fig. 2(b)]:

$$\begin{aligned}
 e^+e^- &\rightarrow b\bar{b} \\
 &\quad \downarrow \\
 &\quad B+----- \\
 &\quad \quad \downarrow \\
 &\quad \quad c\ell\nu_\ell, c q\bar{q} \\
 &\quad \quad \quad \downarrow \\
 &\quad \quad \quad D+----- \\
 &\quad \quad \quad \quad \downarrow \\
 &\quad \quad \quad \quad s\ell\nu_\ell
 \end{aligned} \tag{20}$$

The spectrum is now obtained by folding Eq. (18) with the fragmentation function $D(Z_b)$, where $Z_b = |p_B|/\sqrt{s/4 - m_B^2}$ and integrating over Z_b . This is left as an amusing exercise for the reader!

Having set up the machinery to calculate the lepton energy spectra from the processes (12) through (17), we now discuss the range of the various phenomenological parameters. For the heavy lepton ($\tau^+\tau^-$) induced spectra we have assumed $m_\tau = 1.87$ GeV, $m_{\nu_\tau} = 0$ and a V - A interaction¹⁸⁾. We remark that none of the distributions associated with heavy leptons depends sensitively on m_{ν_τ} within ± 100 MeV. For the spectra from direct $c\bar{c}$ pair production we have assumed the GIM current⁵⁾ with $m_s = (m_K + m_{K^*})/2$ and

$$D(Z_c) = \left(\frac{|p_D|}{(s/4 - m_D^2)^{1/2}} \right)^n = Z_c^n, \quad \text{with } n = 0, 1.$$

These choices are motivated by the data on the inclusive lepton production in e^+e^- experiments near the charm threshold and the CDHS ν data⁷⁾. We also discuss the effects of taking $m_s = m_{K^*}$ and $D(Z_c) = (1 - Z_c)^{17)$. For the lepton spectra from $b\bar{b}$ production, reaction (14), we assume $m_b = 5.0$ GeV and let B have a mass in the range $5.1 \text{ GeV} \leq m_B \leq 5.5 \text{ GeV}$. Similarly, we vary the final hadron mass m_c for the decay $b \rightarrow c \ell \nu_\ell$ in the range $m_D \leq m_c \leq 2.5 \text{ GeV}$.

We also compute the effect of the chirality of the weak current $\{b_\gamma (1 - \gamma_5)\bar{c}$ versus $b_\gamma (1 + \gamma_5)\bar{c}\}$ on the shape of the lepton energy spectrum using the quark decay model. For the processes (16) and (17) we have assumed $m_u = m_d = 0.5 \text{ GeV}$ and $m_s = 1.0 \text{ GeV}$, as representative of the final state hadron masses in these

processes. For the cascade decay (15) we have combined the parameters stated for (13) and (14).

We now discuss the results. Fig. 4 shows the lepton spectra from the decay of B in its rest frame according to (14) and the effect of varying all the phenomenological parameters $\{m_b, m_c, (V - A) \text{ vs. } (V + A)\}^{(f)}$. The $(V - A)$ spectrum for the primary leptons from bottom decays tends to be harder than the $(V + A)$, much like the μ^- decay⁸⁾. As m_c increases, the spectrum becomes softer. However, for $m_c \leq 2.4$ GeV it still peaks above 1 GeV. In Fig. 5 we compare the lepton spectra from processes (12) - (17) for $\sqrt{s} = 12.0$ GeV and $m_b(m_t) = 5.1$ GeV. Unless otherwise stated the charm induced distributions here and elsewhere are calculated assuming $m_s = (m_K + m_{K^*})/2$, $D(Z_c) = \text{constant}$ and all interactions $(V - A)$. It is obvious from Fig. 5 that reactions (12) - (17) yield characteristic spectra, though (16) and (17) are practically indistinguishable, reflecting that both $(m_u, m_d)/m_b$ and m_s/m_b are very small numbers. The softest lepton energy spectrum emerges for the secondary leptons from the cascade decay of the bottom quark (15).

In Table II through IV, we list the average lepton energy in the rest frame of $B(T)$, for the primary leptons from $b(t)$ decays, the secondary leptons from bottom cascades and the leptons from direct $c\bar{c}$ production for $\sqrt{s} = 11.0$ GeV. It is not difficult to see that there is a hierarchy of average lepton momenta with each decay mode {except (16) and (17)} clearly distinct. The table also shows that for reaction (14) $(V - A)$ has larger average energy than $(V + A)$ ^(g). Table V contains the fraction of events surviving a lepton energy cut of 1 GeV (1.5 GeV) at $\sqrt{s} = 12.0$ GeV, for heavy leptons, charm, bottom primary and bottom secondary leptons.

Of course, we do not anticipate that this hierarchy of lepton energy spectra, namely among the leptons from charm decays and the ones from bottom or top decays, will persist for secondary lepton energy distributions in ν experiments or in very high energy e^+e^- experiments. The Lorentz boost effects then totally overtake, and quark mass differences, which make the lepton spectra qualitatively different near the threshold, become rather insignificant. Also, it is premature to predict that even near the new quark thresholds, the lepton momentum measurements could always be used as a pioneering device to probe the onset of a new quark. However, if the pattern of the observed quark mass differences, namely that m_b (or m_t) - $m_c \gg m_c - m_s$, is maintained, then the inclusive lepton spectrum studies would become a very strong tool in e^+e^- colliding beam experiments.

In Fig. 6 we compare the lepton energy spectra from heavy leptons, charm, bottom primary and bottom cascade decays for $\sqrt{s} = 20.0$ GeV assuming $D(Z) = \text{constant}$ for the first three modes. Again all the spectra can be distinguished, with the marked difference between the primary and secondary leptons persisting in much the same way as for $\sqrt{s} = 12.0$ GeV. The effect of the fragmentation function on the shape of the spectra from the bottom primary and secondary leptons is shown in Fig. 7, which shows that under such extreme variations as from $D(Z) = (1 - Z)$ to $D(Z) = Z$, the two spectra are still distinctly different.

This distinction between the primary and secondary lepton momentum distribution, from the cascade decay of bottom, will play a very important role. This information would be particularly needed, if the assignment (1) turns out to be the choice of nature for the $\Delta B = \pm 1$ weak current, in deciding whether the events of the type

$$e^+e^- \rightarrow l^\pm l^\pm + \text{anything} \quad (21)$$

($l = e, \mu$)

are produced at the primary vertex {Fig. 1(a)}, in which case they signal weak mixing or whether one of them is produced through the subsequent decay of the charm quark {Fig. 1(b)} in which case no $B^0 - \bar{B}^0$ mixing is needed. Note that due to the very soft secondary lepton energy spectrum, the fraction of secondary lepton events surviving a large lepton energy cut, E_c , is very small. For instance, we find, under rather conservative assumptions, that the fraction of secondary leptons with $E_c \geq 1.5$ GeV at $\sqrt{s} = 12.0$ GeV is less than 2%. On the other hand more than 50% of the bottom primary leptons survive this cut. Defining

$$N^{\pm\pm} \equiv \sigma(e^+e^- \rightarrow b\bar{b} \rightarrow l^\pm l^\pm + \text{anything})$$

(and similarly N^{-+}) a non-negligible value for the quantity

$$\gamma_1 = \frac{N^{++} + N^{--}}{N^{++} + N^{--} + N^{-+}} \quad \Bigg| \quad E_{l_1}, E_{l_2} > E_c$$

will be a definite evidence for $B^0 - \bar{B}^0$ mixing.

The contribution of charm to N^{-+} can be eliminated by imposing a more severe cut on the lepton energy or else calculated from low energy e^+e^- data or

ν dimuon data $\{\tau^+ \tau^-\}$ contribution to any process of the type (21) is trivially removed by demanding that "anything" is anything but neutrinos, in which case r_1 measures the $B^0 - \bar{B}^0$ mixing angle directly.

A closely related question is that of CP-violation in the decays of heavy mesons, having a b or t quark, a direct measure of which is the charge asymmetry in e^+e^- experiments defined as

$$\gamma_2 \equiv \frac{N^{++} - N^{--}}{N^{++} + N^{--}}$$

Moreover, except for the cascade decays $b \rightarrow c \rightarrow s$, the denominator in the ratio r_2 arises only from weak mass mixing and the following relation holds:

$$\gamma_2 = 4 \operatorname{Re} \epsilon + O(\epsilon^2)$$

where ϵ is the CP-violating parameter for the top or bottom mesons defined in an analogous way to the $K^0 - \bar{K}^0$ system. For the cascade decay $b \rightarrow c \rightarrow s$, the denominator receives contribution from weak mass mixing as well as from the unmixed semileptonic decay modes. Again, choosing a large E_c judiciously one would remove the secondary leptons. The ratio r_2 then also measures $|\epsilon|$ for the production of a $b\bar{b}$ pair and subsequent cascade decays $b \rightarrow c \rightarrow s$.

The CP-violation in the charm decays which could also contribute to r_2 is very small for the GIM current¹⁴⁾ ($\epsilon_D \sim 10^{-4}$). However, if one assumes that the mass mixings mediated by the $W^\pm$ bosons is the only CP-violating effect, as for example in the KM model with $\delta \neq 0$ ³⁾, or else if it arises only from the exchange of charged Higgs boson, then one anticipates a much larger value for the corresponding quantities $|\epsilon_B|$ and $|\epsilon_T|$ ($|\epsilon_B|, |\epsilon_T| \geq 10^{-3}$ and may even turn out to be $\sim 10^{-2}$!). The semileptonic decays of the heavy mesons, top and bottom, then provide a check on the weak mass mixing and CP-violation through the measurement of the ratios r_1 and r_2 .

Now we discuss the momentum distribution transverse to a plane (defined by the incident neutrino and the outgoing lepton in the ν experiments) of the leptons produced through the production and decay of a heavy hadron (charm, bottom, top, etc.). The transverse momentum distribution, being invariant under a Lorentz boost in the longitudinal direction, can be compared to an experimental situation directly. Taking into account the smearing due to the transverse momentum distribution of the parent hadron, described by the function $f(p_{\perp y})$, we have

$$\frac{dN}{d(p_{\perp})_{\ell}} = \frac{2C_1}{\pi} \times \int_{p_{\perp Y}^{\min.}}^{p_{\perp Y}^{\max.}} dp_{\perp Y} \frac{f(p_{\perp Y})}{E_{\perp Y}} \int_0^{\Lambda} d(p_Y \cdot p_{\ell}) \times \frac{(p_Y \cdot p_{\ell})}{m_Y} F\left(\frac{p_Y \cdot p_{\ell}}{m_Y}\right)$$

$$p_{\perp Y}^{\min.} = \max(0, (p_{\perp \ell}^2 - \Delta^2 m_Y^2)/2\Delta p_{\perp \ell})$$

$$p_{\perp Y}^{\max.} = \infty$$

$$\Lambda = p_{\perp Y} (E_{\perp Y} - p_{\perp Y}) \quad , \quad \Delta = \frac{1}{2m_Y^2} (m_Y^2 - m_X^2)$$

$$E_{\perp Y} = (m_Y^2 + p_{\perp Y}^2)^{1/2}$$

we assume $f(p_{\perp Y}) = e^{-\alpha p_{\perp Y}}$. C_1 and $F(p_Y \cdot p_{\ell}/m_Y)$ are the same expressions as the ones for the lepton momentum distributions appearing in (B2) - (B4). In Fig. 8 we have plotted the typical transverse momentum distribution expected from currents (1) through (4). The distribution corresponding to $B \rightarrow \rho \ell \nu_{\ell}$ describes equally well the distribution for $T \rightarrow \rho \ell \nu_{\ell}$, and is typical of the currents (3) and (4). Since $(m_K^* - m_{\rho})/m_T$ is very small, we expect a similar transverse momentum distribution for the current (2). The curve for $b \rightarrow c \ell \nu_{\ell}$ is representative of the weak current (1). These spectra are compared with the ones from the semileptonic modes of charm decay, namely $D \rightarrow K \ell \nu_{\ell}$, $D \rightarrow K^* \ell \nu_{\ell}$ and $D \rightarrow \pi \ell \nu_{\ell}$. The decay $D \rightarrow \pi \ell \nu_{\ell}$ may be the most serious large $(p_{\perp})_{\ell}$ background to the top (bottom) induced lepton events. The secondary leptons from the cascade decay of the bottom have a transverse momentum distribution very similar to that of the leptons from the $\Delta C = \Delta S$ part of the GIM current, and this distribution is not shown separately.

We find that the $(p_{\perp})_{\ell}$ distribution from the current (1) is distinguishable from the rest of the assignments (2) to (4) as well as from the $(p_{\perp})_{\ell}$ distributions from charm (for both $\cos \theta_c$ and $\sin \theta_c$ transitions). The curves correspond to the choice $\alpha = 2 \text{ GeV}^{-2}$. We have calculated the effect of varying α in the region $2 \leq \alpha \leq 6$, the resulting spectra are very similar and not shown.

Figures 9 and 10 pertain to the current (1) and show the effect of the chirality of the weak current ($V - A$ vs. $V + A$) and the effect of varying the mass parameter, m_c , respectively. In Table VI we list the fraction of lepton events produced through the decay of charm, top and bottom mesons, surviving a certain $(p_{\perp})_{\ell}$ cut, for the various choices of the slope. The entries can be compared to dimuon data with the cut applied to the secondary lepton transverse momentum. We caution that the fraction of events from charm with $(p_{\perp})_{\ell} \geq 1$ GeV can be several percent due to the smearing of the charm hadron transverse momentum. The detection of top and bottom quarks through $(p_{\perp})_{\ell}$ studies will not be easy if the couplings between the (t, b) doublet and the known quarks (u, d) , (c, s) turn out to be small, as is the case in most $(V - A)$ six quark models.

4. COLLINEARITY ANGLE DISTRIBUTIONS

In this section we present the collinearity angle distributions for the leptons produced in the e^+e^- colliding beam experiments through reactions (12)-(17). The kinematic situations for the production of a $b\bar{b}$ pair and decay through (14) and (15) are shown in Figs. 2(a) and 2(b), respectively. The kinematics for the processes (13), (14) and (17) are similar to the one depicted in Fig. 2(a), whereas for the $\tau^+\tau^-$ case the same figure is applicable, but the intermediate fragmentation step is not present. The collinearity angle distributions for the process $e^+e^- \rightarrow \ell^+\ell^- + \text{anything}$ has been discussed extensively for the $\tau^+\tau^-$ decays¹³⁾. In particular, an analytic expression for the collinearity angle distribution is derived by Fujikawa and Kawamoto¹³⁾ without including any experimental cuts. The analogous expressions for the processes (13), (14), (16) and (17) can be obtained likewise adopting any of the decay models I ... III discussed in Section II and folding the resulting expressions with the appropriate fragmentation function. However, because of the limited applicability of such analytic expressions to experimental situations and their unavoidably clumsy forms, we shall refrain from doing so. The expressions derived in Ref. 13) may be used for exploratory purposes for reactions (12) - (17), except (15), assuming a free quark decay model, and with the choice $D(Z) = \delta(1 - Z)$ for the fragmentation function.

There is an essential difference, however, between the cascade decay leptons from bottom, i.e. reaction (15), and the rest in that for cascade decays both the leptons come from the same parent quark (hadron), whereas for the others they originate from different quarks (or $\tau^+\tau^-$), produced back to back in e^+e^- experiments. As the beam energy increases, the leptons in the latter case tend to go more and more forward with respect to the parent quarks (or τ^+), in accordance

with the Lorentz boost. Defining the collinearity angle as $\cos \theta_{\text{coll.}} = -(\mathbf{p}_{\ell_1} \cdot \mathbf{p}_{\ell_2}) / (|\mathbf{p}_{\ell_1}| |\mathbf{p}_{\ell_2}|)$ for the decay processes (12) - (14) and (16) - (17), the $\cos \theta_{\text{coll.}}$ distribution peaks in the small θ region, with the slope determined by the masses, beam energy and fragmentation function. A priori, no such peaking is expected for the leptons from the bottom cascade decays¹⁹⁾. All this is shown quantitatively through Figs. (11) - (15). In Fig. (11) we show the collinearity angle distributions for the leptons from $\tau^+ \tau^-$ {reaction (12)}, $c\bar{c}$ {reaction (13)}, $b\bar{b}$ {reaction (14)} and the cascade decay of bottom (h) {reaction (15)} with both the leptons cascading from the same bottom quark, for $\sqrt{s} = 12.0$ GeV, $m_B = 5.1$ GeV and $m_c = 2.0$ GeV. The collinearity angle distributions for reaction (16) and (17) are very similar to that of (14) for the mass range $0.5 \leq m_y \leq 1.0$ GeV and are not shown separately. There are several comments in order:

i) The collinearity angle distribution for leptons from $b\bar{b}(t\bar{t})$ decay is not sensitive to the precise value of $m_B(m_T)$ in the range $5.0 \text{ GeV} \leq m_B, m_T \leq 5.5 \text{ GeV}$. The same is true of the variation in the parameter m_y in the range $0.5 \text{ GeV} \leq m_y \leq 1.0 \text{ GeV}$. In fact, the effect of going from $m_y = 0.5 \text{ GeV}$ to $m_y = 2.0 \text{ GeV}$ on the distribution is only marginal.

ii) The collinearity angle distributions depend only slightly on the underlying chiral structure of the weak current, a statement also true for leptons from the heavy lepton and charm decays.

iii) The opening of a new quark threshold will be signalled by a flattening of the collinearity angle distribution in e^+e^- experiments, as one crosses the $b\bar{b}(t\bar{t})$ threshold. In principle, it is possible to distinguish, on the basis of integrated collinearity angle distributions, among the decays (14) or (16) and (17), if one remembers in addition that there will be a factor 4 difference between $b\bar{b}$ and $t\bar{t}$ production rates. Consequently the flattening of the distribution will be more pronounced for $t\bar{t}$ production as compared to the $b\bar{b}$ production.

Next, we address ourselves to the experimental cuts. First, we note that there is no coplanarity angle cut needed, as was the case for the heavy leptons ($\tau^\pm$) to subtract the Bhabha events, since almost all the di-lepton (as well as multi-lepton) events from quark decays will be of the form $e^+e^- \rightarrow \ell^\pm \ell^\mp + \text{hadrons}$. This, as noted earlier, also removes the $\tau^+ \tau^-$ induced events. The most important of the experimental cuts is the energy cut on the leptons, which for all practical purposes is non-negligible. As is obvious from the lepton spectra calculated in

the previous section, imposing a cut on the lepton energy in excess of 1 GeV will eliminate most of the secondary leptons from bottom decays for $\sqrt{s} = 10-20$ GeV. The effect of imposing a lepton energy cut of 0.5 GeV for $\sqrt{s} = 12.0$ GeV, on the shape of the collinearity angle distribution is shown in Fig. 12. The effect of the beam energy on the collinearity angle distribution is shown in Fig. 13 for the two representative values $\sqrt{s} = 12$ GeV and $\sqrt{s} = 20$ GeV. The distribution starts peaking in the forward direction for all the processes (12) to (17) except (15).

The collinearity angle distribution is rather sensitive to the fragmentation function employed, which is easy to understand, since these functions act as weightage factors of Lorentz boosts and enter twice, once for each lepton, in the expression for the distribution. This is shown in Fig. 14 for $\sqrt{s} = 20$ GeV for the two cases $D(Z) = Z^n$ ($n = 0, 1$). Finally, we remark that the mean value $\langle \cos \theta_{\text{coll.}} \rangle$ is sensitive to the underlying decay mechanism. In Table VII we have presented $\langle \cos \theta_{\text{coll.}} \rangle$ and $\langle \cos^2 \theta_{\text{coll.}} \rangle$ for $\sqrt{s} = 12.0$ GeV, assuming different lepton energy cuts. The same quantity is tabulated for $\sqrt{s} = 20$ GeV in Table VIII. Figures 15 and 16 show, respectively, the energy dependence of $\langle \cos \theta_{\text{coll.}} \rangle$ and $\langle \cos^2 \theta_{\text{coll.}} \rangle$ in the range $12.0 \text{ GeV} \leq \sqrt{s} \leq 20.0 \text{ GeV}$.

5. INVARIANT MASS DISTRIBUTION OF CASCADE LEPTONS FROM BOTTOM

In Fig. 17 we plot the invariant mass squared distribution for the leptons produced in the cascade decay of the bottom quark through the chain

$$\begin{array}{c} b \rightarrow c e^- \bar{\nu}_e \\ \quad \quad \quad \downarrow \\ \quad \quad \quad s e^+ \nu_e \end{array}$$

The figure corresponds to the choice $m_b = 5.1 \text{ GeV}$, $m_c = 2.0 \text{ GeV}$, $m_s = \frac{m_K + m_K^*}{2}$ and $D(Z_b) = D(Z_c) = \text{constant}$. The distribution may be compared, for example, to the invariant mass squared distribution of the two slow leptons for the trilepton events in neutrino experiments, as a check of the hypothesis of bottom production (i).

DISCUSSION AND CONCLUSIONS

Motivated by the discovery of $T(9.4)$, $T'(10.0)$, ..., we have tried to study the various distributions from the production and the subsequent semileptonic decays of the next heavy quarks and compared them to the corresponding distributions

from the production of charm and heavy leptons $\tau^\pm$. The exercise was undertaken in preparation for the next generation of colliding beam experiments at PETRA, PEP and CERN, though some of the correlations may be directly compared to the on-going ν experiments. We have systematically studied the differences and parallels of the various decay schemes of the $b(t)$ quarks. Our conclusions are summarized below.

1. The lepton energy distributions from the $b(t)$ quarks are very different from the lepton spectra due to $\tau^+\tau^-$ and charm production. This difference is particularly marked in the vicinity of the $b\bar{b}(t\bar{t})$ threshold in e^+e^- experiments. In addition, it is possible to distinguish among the decay modes $b \rightarrow c\ell\nu_\ell$ and the others ($b \rightarrow u\ell\nu_\ell$, $t \rightarrow s\ell\nu_\ell$, $d\ell\nu_\ell$) merely by the shape of the lepton spectra. The lepton spectrum from cascade decay $b \rightarrow c \dots \rightarrow s\ell\nu_\ell$ is calculated and shown to be remarkably soft for the secondary leptons, thus providing an additional handle on the $B^0 - \bar{B}^0$ mixing by distinguishing the primary and secondary leptons. The possibility of measuring CP-violation in e^+e^- experiments, due to the production and mixing of the neutral top and bottom mesons, is discussed. This hierarchy of the lepton energy spectra is also present in the transverse lepton momentum distributions. In principle, the signature of $b(t)$ quarks in the ν experiments is then the observation of dimuon events with the slow muon having a transverse momentum distribution much wider than for the dimuon events expected from charm. This is true even if one takes into account the Cabibbo suppressed $D \rightarrow \pi\ell\nu_\ell$ decay mode for the charm. In practice, the effect may be minuscule due to very small production of top and bottom quarks in ν experiments.

2. The collinearity angle distribution studies for the leptons in the process

$$e^+e^- \longrightarrow \ell^\pm \ell^\mp + \text{anything}$$

can signal the presence of new heavy quarks. We show that the collinearity angle distributions from the $b(t)$ quarks are also different from the ones due to $\tau^+\tau^-$ and $c\bar{c}$. The effect is kinematic and reflects the difference in masses and Lorentz boosts. Particularly interesting is the collinearity angle distribution for the cascade leptons from $b \rightarrow c\ell\nu_\ell \rightarrow s\ell\nu_\ell$, which does not show the peaking under a Lorentz boost for the energies considered $12 \text{ GeV} \leq \sqrt{s} \leq 20 \text{ GeV}$. The effects of energy cuts, energy dependence and different fragmentation functions do not blur the systematic differences between the old and the new quarks induced collinearity angle distributions. We believe that $\langle \cos \theta_{\text{coll}} \rangle$ provides another useful criterion to distinguish the basic weak decay mechanism.

3. The invariant mass distribution of the cascade leptons is calculated so as to provide a check on the hypothesis of bottom production and the subsequent decay $b \rightarrow c \ell \nu_\ell$.

We believe that the systematic differences among the correlations discussed in this paper from the production and decay of the old and new quarks, as well as the heavy leptons, are not excessively demanding on the resolution of the various detectors present, and being built for the PETRA, PEP and CESR energies. However, we emphasize the importance of low lepton energy detection efficiency and planning the experiments not very far from the expected $b\bar{b}(t\bar{t})$ thresholds.

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FOOTNOTES

- (a) The ratio holds for each lepton pair separately, e.g., e^+e^- , $e^\pm\mu^\mp$, $\mu^\pm\mu^\mp$ and not for the sum of all the possible charge assignments.
- (b) The 10% semileptonic branching ratio for charm decays, (for electron and muon separately) is an average of the various e^+e^- and ν experiments⁷⁾. We believe the semileptonic branching ratios for bottom (for electrons and muons) will also be close to 10%. The short distance enhancement effects for the semileptonic decays of bottom over charm³⁾ are compensated by the more numerous decay modes available to bottom as compared to charm.
- (c) We are making here a distinction between the leptons produced at the primary vertex Fig. 1(a) as compared to the leptons coming from the subsequent decay of a charm quark, Fig. 1(b) .
- (d) The non-leptonic decays of charm mesons are very model dependent. However, it does not seem to be a bad approximation to assume that $F^\pm$ mesons have an η or η' in half of the decays, thus not producing any kaons. The inclusive $\bar{c}s$ state will normally evolve into DK , D^*K^* , etc., by picking up a $q\bar{q}$ pair, thus resulting in two kaons per decay. The rearrangement of the $(\bar{c}s)(q\bar{q})$ pairs, produced in the non-leptonic decays, will also yield on average two kaons. One, therefore, need only calculate the ratio of events in which an $(F, F^*, \dots)$ is produced to the total decay rate.
- (e) The estimates quoted above are based on the neglect of single particle states, i.e. $F, F^*, \dots$. The corresponding numbers for the other extreme based on F, F^* dominance, neglecting any DK component in the final state, are: 1 : 1/10 : 1/400 for the assignment (1) and 1 : 1/6 : 1/144 for the assignment (2). The additional factor of 2 for the $2(K\bar{K})$ case is due to the fact that both $b(\bar{t})$ and $\bar{b}(t)$ can produce a $K\bar{K}$ pair through the $\bar{c}s$ and $c\bar{s}$ states, respectively.
- (f) The lepton energy spectrum for the secondary lepton from the cascade decay of the b quark is the same for both the $(V \pm A)$ interactions for the $\Delta B = \pm 1$ current. This is a consequence of the fact that the hadron energy distribution for the process $b \rightarrow c \ell \nu_\ell$ is identical for both the $(V \pm A)$ interactions, and we are assuming the GIM current for the subsequent decay of the charm quark.
- (g) This is to be contrasted with the lepton energy spectra from the semileptonic decay of the charm quark $c \rightarrow s \ell \nu_\ell$ where $(V - A)$ interaction gives softer spectrum as compared to $(V + A)$. One can formulate a general rule based on

the analogy with the decays of a lepton. Defining

$$\Theta \equiv Q \times \lambda$$

where Q is the electric charge of the fermion (quark or lepton) and λ the chirality of the current ($\lambda = \pm 1$ for $V \pm A$), then all the interactions having $\Theta = -1$ generate softer lepton energy spectra for $(V - A)$ while those with $\Theta = +1$ for $(V + A)$.

- (h) The collinearity angle for the leptons from the cascade decay of the bottom quark is defined without the conventional extra minus sign used for the leptons from $\tau^+\tau^-$, $c\bar{c}$, etc. This is to take into account the fact that $p(\tau^+) = -p(\tau^-)$.
- (i) There is a subtle point about the calculation for the invariant mass distributions from the quark production and decays. The procedure of introducing a $Q \rightarrow H$ (or $H \rightarrow Q$) fragmentation in the variable $Z = |p_H|/|p_Q|$ is not Lorentz invariant. The invariant mass distributions so calculated are frame dependent. The situation is remedied by a Lorentz invariant formulation of the fragmentation procedure. R. Barbieri et al.²⁰⁾ have shown explicitly how this is done. The variable $\bar{\xi}$ introduced in Ref. 20) reduces to Z^{-1} in the infinite momentum frame of the quark. The invariant mass distribution shown is done with the variable $\bar{\xi}$ instead of Z . I am grateful to J. Ellis and M. K. Gaillard for emphasizing this point, and to T. Walsh for pointing out an error.

Table I: The ratio $\frac{(x \rightarrow y^* \ell \nu_\ell)}{(x \rightarrow y \ell \nu_\ell)}$ for the semileptonic decays of the B(T) mesons, from non-relativistic quark model. Here y is an 0^- , y^* a 1^- meson, and x either a B or T. $m_B(m_T) = 5.1$ GeV.

Decay modes	Ratio
$\frac{B \rightarrow D^{*} \ell \nu_\ell}{B \rightarrow D \ell \nu_\ell}$	4.5
$\frac{B \rightarrow \rho \ell \nu_\ell}{B \rightarrow \pi \ell \nu_\ell} = \frac{T \rightarrow \rho \ell \nu_\ell}{T \rightarrow \pi \ell \nu_\ell}$	14.5
$\frac{T \rightarrow K^{*} \ell \nu_\ell}{T \rightarrow K \ell \nu_\ell}$	11.3

Table II: Average lepton energy for the primary leptons from the various b(t) semileptonic decay modes, in the rest frame of the B(T) meson. The entries for $b \rightarrow u \ell \nu_\ell$ also apply to the decay mode $t \rightarrow d \ell \nu_\ell$ if one flips the chirality of the current. We use $m_B(m_T) = 5.1$ GeV.

Decay Mode	Units (GeV)					
$b \rightarrow c \ell \nu_\ell$	$m_c = 2.0$ GeV		$m_c = 2.25$ GeV		$m_c = 2.5$ GeV	
	(V - A)	(V + A)	(V - A)	(V + A)	(V - A)	(V + A)
	1.35	1.2	1.25	1.14	1.16	1.1
$b \rightarrow u \ell \nu_\ell$	$m_u = 0.5$ GeV		$m_u = 0.75$ GeV			
	(V - A)	(V + A)	(V - A)	(V + A)		
	1.46	1.37	1.46	1.36		
$t \rightarrow s \ell \nu_\ell$	$m_s = 1.0$ GeV		$m_s = 1.25$ GeV			
	(V - A)	(V + A)	(V - A)	(V + A)		
	1.35	1.45	1.32	1.44		

Table III: Average lepton energy for the secondary leptons from the cascade decay of the bottom quark Fig. 1(b) in the rest frame of the B meson, for the various choice of the fragmentation function $D(Z_c)$, and the parameter m_s . The entries correspond to both the $(V^\pm A)$ decay of the bottom quark assuming the GIM current for the charm decay.

Units (GeV)

Fragmentation function $m_c \downarrow$	$D(Z_c) = \text{constant}$		$D(Z) = Z$	
	$m_s = 0.7$	$m_s = 0.9$	$m_s = 0.7$	$m_s = 0.9$
2.0	0.515	0.45	0.54	0.47
2.25	0.51	0.44	0.53	0.465
2.5	0.5	0.435	0.52	0.46

Table IV: Average lepton energy for the leptons from charm decay for $E_c = 5.5$ GeV, corresponding to the various choices of the fragmentation function $D(Z_c)$ and the mass parameter m_s .

Units (GeV)

Fragmentation function $m_s \downarrow$	$D(Z) = \text{constant}$	$D(Z) = Z$	$D(Z) = (1 - Z)$
0.5	0.84	1.0	0.69
0.7	0.76	0.90	0.62
0.9	0.67	0.79	0.54

Table V: The fraction of leptons surviving a lepton energy cut of 1.0 GeV and 1.5 GeV for $\sqrt{s} = 12.0$ GeV from the heavy lepton, charm and bottom decays. The entries for charm correspond to $D(Z_c) = \text{constant}$ and $m_s = 0.7$ GeV. All interactions are assumed (V - A).

Decay mode \ Fraction of events	$E_\ell \geq E_c = 1.0 \text{ GeV}$		$E_\ell \geq E_c = 1.5 \text{ GeV}$	
	$D(Z) = Z$	$D(Z) = \text{constant}$	$D(Z) = Z$	$D(Z) = \text{constant}$
$\tau^\pm \rightarrow \ell^\pm \nu_\ell \nu_\tau$	0.74		0.60	
$c \rightarrow s \ell \nu_\ell$	0.35		0.18	
$b \rightarrow c \ell \nu_\ell$	0.77	0.75	0.53	0.51
$b \rightarrow c + \dots$ $\quad \downarrow$ $\quad s \ell \nu_\ell$	0.14	0.09	0.03	0.01

Units (%)

Quantity Decay Mode	Fraction of events with $(p_T)_\ell \geq 1.0$ GeV			Fraction of events with $(p_T)_\ell \geq 1.5$ GeV			Fraction of events with $(p_T)_\ell \geq 2.0$ GeV		
	$\alpha=2$ GeV ⁻²	$\alpha=4$ GeV ⁻²	$\alpha=6$ GeV ⁻²	$\alpha=2$ GeV ⁻²	$\alpha=4$ GeV ⁻²	$\alpha=6$ GeV ⁻²	$\alpha=2$ GeV ⁻²	$\alpha=4$ GeV ⁻²	$\alpha=6$ GeV ⁻²
$D \rightarrow \pi \ell \nu_\ell$	8.5	4.75	3.4	3.0	1.5	1.1	1.25	0.6	0.4
$D \rightarrow K \ell \nu_\ell$	5.6	2.7	1.7	1.7	0.7	0.4	0.6	0.2	0.12
$D \rightarrow K^* \ell \nu_\ell$	3.2	1.3	0.75	0.8	0.27	0.16	0.25	0.08	0.04
$b \rightarrow c \ell \nu_\ell$ (V - A)	34.0	32.3	31.6	11.2	10.0	9.25	1.6	1.0	0.8
$b \rightarrow c \ell \nu_\ell$ (V + A)	28.0	26.5	26.0	7.5	6.5	6.0	0.8	0.5	0.35
$B \rightarrow \rho \ell \nu_\ell$	40.0	38.5	37.8	17.2	16.0	15.0	5.0	4.0	3.7

Table VI: The fraction of events surviving a transverse lepton momentum cut of 1.0, 1.5 and 2.0 GeV for the various choices of α , the slope of the hadron transverse momentum. The entries for the mode $b \rightarrow c \ell \nu_\ell$ correspond to the choice $m_b = m_B = 5.1$ GeV and $m_c = 2.0$ GeV. The entries for $B \rightarrow \rho \ell \nu_\ell$ are also applicable to the mode $T \rightarrow \rho \ell \nu_\ell$ with $m_T = m_B$, and to a very good approximation also to the mode $T \rightarrow K^* \ell \nu_\ell$.

$E_{\text{beam}} = 12.0 \text{ GeV}$

Decay Mode	$E_{\ell_1, \ell_2} \geq E_c = 0 \text{ GeV}$		$E_{\ell_1, \ell_2} \geq E_c = 0.5 \text{ GeV}$		$E_{\ell_1, \ell_2} \geq E_c = 1.0 \text{ GeV}$	
	$\langle \cos \theta_{\text{coll.}} \rangle$	$\langle \cos^2 \theta_{\text{coll.}} \rangle$	$\langle \cos \theta_{\text{coll.}} \rangle$	$\langle \cos^2 \theta_{\text{coll.}} \rangle$	$\langle \cos \theta_{\text{coll.}} \rangle$	$\langle \cos^2 \theta_{\text{coll.}} \rangle$
$\tau^{\pm} \rightarrow \ell^{\pm} \nu_{\ell} \nu_{\tau}$	0.73	0.66	0.86	0.76	0.91	0.83
$c \rightarrow s \ell \nu_{\ell}$	0.36	0.43	0.62	0.55	0.88	0.8
$b \rightarrow c \ell \nu_{\ell}$	0.138	0.34	0.155	0.34	0.25	0.35
$b \rightarrow u \ell \nu_{\ell}$ (or $t \rightarrow d \ell \nu_{\ell}$)	0.14	0.35	0.145	0.345	0.18	0.34
$t \rightarrow s \ell \nu_{\ell}$	0.14	0.35	0.145	0.345	0.2	0.35
$b \rightarrow c \ell \nu_{\ell}$ $\hookrightarrow s \ell \nu_{\ell}$	0.08	0.31	0.049	0.32	-	-

Table VII: $\langle \cos \theta_{\text{coll.}} \rangle$ and $\langle \cos^2 \theta_{\text{coll.}} \rangle$ for the leptons produced in the $e^+ e^-$ colliding beams from the production and decay of a $\tau^+ \tau^-$, $c\bar{c}$, $b\bar{b}$ and $t\bar{t}$ pairs at $\sqrt{s} = 12.0 \text{ GeV}$. For the choice of the various parameter see text (Section IV).

$E_{\text{beam}} = 20.0 \text{ GeV}$

Decay Mode	$E_{\ell_1, \ell_2} \geq E_c = 0 \text{ GeV}$		$E_{\ell_1, \ell_2} \geq E_c = 0.5 \text{ GeV}$		$E_{\ell_1, \ell_2} \geq E_c = 1.0 \text{ GeV}$	
	$\langle \cos \theta_{\text{coll.}} \rangle$	$\langle \cos^2 \theta_{\text{coll.}} \rangle$	$\langle \cos \theta_{\text{coll.}} \rangle$	$\langle \cos^2 \theta_{\text{coll.}} \rangle$	$\langle \cos \theta_{\text{coll.}} \rangle$	$\langle \cos^2 \theta_{\text{coll.}} \rangle$
$\tau^{\pm} \rightarrow \ell^{\pm} \nu_{\ell} \nu_{\tau}$	0.87	0.81	0.935	0.88	0.95	0.91
$c \rightarrow s \ell \nu_{\ell}$	0.52	0.54	0.75	0.67	0.92	0.85
$b \rightarrow c \ell \nu_{\ell}$	0.187	0.36	0.21	0.36	0.31	0.38
$b \rightarrow u \ell \nu_{\ell}$ (or $t \rightarrow d \ell \nu_{\ell}$)	0.188	0.36	0.195	0.36	0.25	0.37
$t \rightarrow s \ell \nu_{\ell}$	0.188	0.36	0.195	0.36	0.26	0.37
$b \rightarrow c \ell \nu_{\ell}$	-0.026	0.33	-0.048	0.33	-	-

Table VIII: The same as Table VII for $\sqrt{s} = 20 \text{ GeV}$.

APPENDIX A

Based on the free quark decay model, we estimate below the various single particle contributions to the $b(t)$ quark decays. The derivation is analogous to the $\tau^\pm$ decay. The inclusive rate for the $b(t)$ decay is calculated from Fig. 1(a) and is given for the decay $b \rightarrow u (d\bar{u})$ {or $t \rightarrow d(\bar{d}u)$, $t \rightarrow s(\bar{d}u)$ } by:

$$\Gamma(b \rightarrow u + (d\bar{u})) \simeq \frac{r^2 G_F^2 m_b^5}{26 \pi^3} \quad (A.1)$$

where $r = g_{buw}^- / g_{duw}^-$. Note the factor 3 due to colour. The relevant single particle decay rates are given by

$$\Gamma(b \rightarrow u \rho^-) = \frac{r^2 G_F^2 m_b^3 g_\rho^2}{16 \pi m_\rho^2} G(x_\rho)$$

$$\Gamma(b \rightarrow u A_1^-) = \frac{r^2 G_F^2 m_b^3 g_{A_1}^2}{16 \pi m_{A_1}^2} G(x_{A_1}) \quad (A.2)$$

$$\Gamma(b \rightarrow u \pi^-) = \frac{r^2 G_F^2 m_b^3}{16 \pi} f_\pi^2$$

where $G(x_i) = (1 - x_i^2)(1 + 2x_i^2)$, $x_i = \frac{m_i}{m_b}$

and we have used the definitions:

$$\begin{aligned} \langle 0 | V_\lambda | \rho \rangle &= g_\rho \epsilon_\lambda^\rho \\ \langle 0 | A_\lambda | A_1 \rangle &= g_{A_1} \epsilon_\lambda^{A_1} \\ \langle 0 | A_\lambda | \pi \rangle &= f_\pi p_{\pi\lambda} \end{aligned} \quad (A.3)$$

(A.1) and (A.2) lead to the following estimates:

$$\begin{aligned} \frac{\Gamma(b \rightarrow u \rho^-)}{\Gamma(b \rightarrow u + (d\bar{u}))} &\simeq \frac{4\pi^2 g_\rho^2}{m_\rho^2 m_b^2} \simeq \pi \frac{m_\rho^2}{m_b^2} \simeq 0.06 \\ \frac{\Gamma(b \rightarrow u A_1^-)}{\Gamma(b \rightarrow u + (d\bar{u}))} &\simeq \frac{4\pi^2 g_{A_1}^2}{m_{A_1}^2 m_b^2} \simeq \frac{\pi}{2} \frac{m_\rho^2}{m_b^2} \simeq 0.03 \\ \frac{\Gamma(b \rightarrow u \pi^-)}{\Gamma(b \rightarrow u + (d\bar{u}))} &\simeq \frac{4\pi^2 f_\pi^2}{m_b^2} \simeq 0.04 \end{aligned} \quad (A.4)$$

In the above estimates we have used $g_\rho^2 = 1/4\pi m_\rho^4$ (from ρ universality), the Weinberg sum rule

$$g_\rho \simeq g_{A_1}$$

$$m_{A_1} \simeq \sqrt{2} m_\rho$$

$f_\pi = m_\pi$ and $m_b = 5.1$ GeV. The suppression factors $(m_\rho/m_b)^2$, $(m_\pi/m_b)^2$, etc. show that the individual decay modes become increasingly unimportant as the mass of the quark increases. The generalization of the above results to the decays $b \rightarrow u(\bar{c}s)$, $b \rightarrow c(\bar{c}s)$, etc. is straightforward. One now has

$$\frac{\Gamma(b \rightarrow c F^{\pm*})}{\Gamma(b \rightarrow c + (\bar{c}s))} = \pi \left(\frac{m_\rho^2}{m_b m_{F^*}} \right)^2 \left(\frac{g_{F^*}}{g_\rho} \right)^2 \times \text{phase space factor}$$

$$\frac{\Gamma(b \rightarrow c F_A^\pm)}{\Gamma(b \rightarrow c + (\bar{c}s))} = \pi \left(\frac{m_\rho^2}{m_b m_{F_A}} \right)^2 \left(\frac{g_{F_A}}{g_\rho} \right)^2 \times \text{phase space factor}$$

$$\frac{\Gamma(b \rightarrow c F^\pm)}{\Gamma(b \rightarrow c + (\bar{c}s))} = 4\pi^2 \frac{f_F^2}{m_b^2} \times \text{phase space factor} \quad (\text{A.5})$$

There are varying estimates for the quantities g_{F^*} and f_F in the literature²². Assuming the empirical result $m_i g_{V_i}^2 = \text{constant}$ ($i = \rho, J/\Psi$) and taking $m_\pi \leq f_F \leq 2m_\pi$, we estimate

$$\frac{\Gamma(b \rightarrow c F^{\pm*})}{\Gamma(b \rightarrow c + (\bar{c}s))} \simeq \frac{\Gamma(b \rightarrow c F_A^\pm)}{\Gamma(b \rightarrow c + (\bar{c}s))} \simeq 0.05$$

$$\frac{\Gamma(b \rightarrow c F^\pm)}{\Gamma(b \rightarrow c + (\bar{c}s))} \simeq 0.08 - 0.3 \quad (\text{A.6})$$

which shows that the most important single particle states may together account for 30 - 40% of the inclusive rate.

APPENDIX B

We first write the analytic expressions for the various decay mechanisms discussed in Section II, in the rest frame of the decaying particle. The general decay process is represented as $x \rightarrow y \ell \bar{\nu}_\ell$.

Lepton energy distribution

The normalized lepton energy distribution is symbolically written as

$$\frac{1}{\Gamma} \frac{d\Gamma}{dE_\ell} = C_1 F(E_\ell) \quad (B.1)$$

The expressions corresponding to the various models are:

Model I: Free quark decay

(V - A):

$$C_1 = \frac{96}{m_x^6 f(\gamma)} \quad (B.2)$$

$$F(E_\ell) = \frac{E_\ell^2 (m_x^2 - m_Y^2 - 2m_x E_\ell)^2}{(m_x - 2E_\ell)}$$

$$f(\gamma) = 1 - 8\gamma^2 + 8\gamma^6 - \gamma^8 - 24\gamma^2 \ln \gamma$$

$$\gamma = m_Y / m_x$$

(V + A):

$$C_1 = \frac{16}{m_x^6 f(\gamma)} \quad (B.3)$$

$$F(E_\ell) = E_\ell^2 (m_x^2 - 2m_x E_\ell - m_Y^2) \frac{1}{m_\theta (m_\theta - 2E_\ell)^3} \times \left[(m_x - 2E_\ell) (m_x^2 - 2m_x E_\ell - m_Y^2) + 2(m_x - E_\ell) \times (m_x^2 - 2m_x E_\ell + 2m_Y^2) \right]$$

Model II: x and y both 0^- mesons

The lepton energy distribution is given by (B.2).

Model III: $x = 0^-$, $y = 1^-$ mesons

$$C_1 = \frac{96}{m_x^5 f^*(\gamma)}$$

$$F(E_\ell) = \frac{E_\ell^2 (m_x^2 - m_Y^2 - 2m_x E_\ell)^2 (2m_x^2 + m_Y^2 - 2m_x E_\ell)}{(m_x^2 - 2m_x E_\ell)^2}$$

$$f^*(\gamma) = 1 + 72\gamma^4 - 64\gamma^6 - 9\gamma^8 + (36\gamma^4 + 48\gamma^6) \ln \gamma^2 \quad (B.4)$$

Hadron energy distribution

The normalized hadron energy distribution is expressed as

$$\frac{1}{\Gamma} \frac{d\Gamma}{dE_Y} = C_2 G(E_Y) \quad (B.5)$$

The expressions corresponding to the various decay models are

Model I:

(V ± A):

$$C_2 = \frac{16}{m_x^6 f(\gamma)}$$

$$G(E_Y) = (E_Y^2 - m_Y^2) \left[3m_x E_Y (m_x^2 + m_Y^2) - 2m_x^2 m_Y^2 - 4m_x^2 E_Y^2 \right] \quad (B.6)$$

Model II:

$$C_2 = \frac{64}{m_x^4 f(\gamma)}$$

$$G(E_Y) = (E_Y^2 - m_Y^2)^{3/2} \quad (B.7)$$

Model III:

$$C_2 = \frac{128}{m_X^6 f^*(\theta)}$$

$$G(E_Y) = (E_Y^2 - m_Y^2)^{1/2} \left[2m_Y^2 (m_X^2 + m_Y^2 - 2m_X E_B + (m_X E_Y - m_Y^2)^2) \right] \quad (B.8)$$

The expression for the decay in flight can be written down in a straightforward way¹⁷⁾:

$$\begin{aligned} \frac{1}{\Gamma} \frac{d\Gamma(E_X, E_\ell)}{dE_\ell} &\equiv H(E_X, E_\ell) \\ &= C_1 \int_{E_\ell(E_X - |p_X|)}^{\Lambda_1} d(p_X \cdot p_\ell) m_X \frac{1}{2 |p_X| (p_X \cdot p_\ell)} F(p_X \cdot p_\ell / m_X) \end{aligned} \quad (B.9)$$

where

$$\Lambda_1 = \min. \left[E_\ell (E_X + |p_X|), \frac{m_X^2 - m_Y^2}{2} \right]$$

The corresponding expression for the hadron energy distribution for the decay in flight is given by

$$\begin{aligned} \frac{1}{\Gamma} \frac{d\Gamma(E_X, E_Y)}{dE_Y} &\equiv J(E_X, E_Y) \\ &= C_2 \int_{(E_X E_Y - |p_X| |p_Y|)}^{\Lambda_2} d(p_X \cdot p_Y) \frac{m_X G(p_X \cdot p_Y / m_X)}{[2 |p_X| \{ (p_X \cdot p_Y)^2 - m_X^4 \}^{1/2}]} \end{aligned} \quad (B.10)$$

where

$$\Lambda_2 = \min. \left[E_X E_Y + |p_X| |p_Y|, \frac{m_X^2 + m_Y^2}{2} \right]$$

The normalized lepton energy distribution for any hadron x which has fragmented from a quark Q is obtained simply by folding $H(E_x, E_\ell)$ with the fragmentation function. Defining

$$z \equiv \frac{|p_x|}{|p_Q|} = \frac{|p_x|}{(s/4 - m_x^2)^{1/2}}$$

the normalized lepton energy distribution is given by

$$\begin{aligned} \frac{1}{\Gamma} \frac{d\Gamma}{dE_\ell} &\equiv K(E_Q, E_\ell) \\ &= \int_{z_{\min}}^1 dz H(E_x, E_\ell) D(z) \end{aligned} \quad (B.11)$$

where

$$z_{\min} = \max. \left[0, \frac{4m_x^2 E_\ell^2 - (m_x^2 - m_Y^2)^2}{4|p_Q|(m_x^2 - m_Y^2)} \right]$$

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FIGURE CAPTIONS

- Fig. 1: a) Lowest order diagram for the bottom (top) decays.
b) Cascade decays of the bottom quark.
- Fig. 2: a) Kinematic configuration corresponding to the production of a $b\bar{b}$ quark pair in the e^+e^- colliding beam experiments, fragmentation to the hadron B and subsequent semileptonic decay.
b) Production of a $b\bar{b}$ pair in the e^+e^- colliding beam experiments and the subsequent fragmentation-cascade decay via the weak current of Eq. (1).
- Fig. 3: Lepton energy spectra for the rest frame decays of B meson from the semileptonic modes $B \rightarrow D\ell\nu_\ell$, $B \rightarrow D^*\ell\nu_\ell$. The spectrum for $B \rightarrow D\ell\nu_\ell$ also applies to the decay mechanism $b \rightarrow c\ell\nu_\ell$ with $m_b = m_B$ and $m_c = m_D$. We have assumed $m_B = 5.1$ GeV.
- Fig. 4: The effect of varying the mass of the B meson, the final state hadron mass, and the chirality of the current on the rest frame lepton energy spectrum from the decay $b \rightarrow c\ell\nu_\ell$.
- Fig. 5: Lepton energy spectra for the leptons from the $\tau^+\tau^-$, $c\bar{c}$, $b\bar{b}$ and $t\bar{t}$ decays for $\sqrt{s} = 12$ GeV, $m_B(m_T) = 5.1$ GeV and $(V - A)$ interaction. For the charm fragmentation we have assumed $D(Z_c) = \text{constant}$, and $m_s = \frac{m_K + m_{K^*}}{2}$.
- Fig. 6: Comparison of the lepton spectra for $\tau^+\tau^-$, $c\bar{c}$, bottom primary and bottom secondary leptons at $\sqrt{s} = 20$ GeV. We have assumed $m_B = 5.1$ GeV, $(V - A)$ interaction, $m_c = 2$ GeV and $D(Z_b) = D(Z_c) = \text{constant}$.
- Fig. 7: Effect of the fragmentation function on the lepton energy spectra for bottom primary and bottom secondary leptons. The curves correspond to the energy $\sqrt{s} = 20$ GeV, $m_B = 5.1$ GeV, $m_c = 2.0$ GeV, $m_s = 0.7$ GeV, $(V - A)$ interaction.
- Fig. 8: Transverse lepton momentum distributions for the decays $D \rightarrow \pi\ell\nu_\ell$, $K\ell\nu_\ell$, $K^*\ell\nu_\ell$, $b \rightarrow c\ell\nu_\ell$ and $B \rightarrow \rho\ell\nu_\ell$. The curve for $B \rightarrow \rho\ell\nu_\ell$ also represents $T \rightarrow \rho\ell\nu_\ell$, and $T \rightarrow K^*\ell\nu_\ell$ as well. The distributions assume $m_B(m_T) = 5.1$ GeV and $\alpha = 2 \text{ GeV}^{-2}$.
- Fig. 9: Effect of the chirality of the interaction on the transverse lepton energy distribution from the decay mode $b \rightarrow c\ell\nu_\ell$, for $m_b = 5.1$ GeV and $m_c = 2.0$ GeV.

- Fig. 10: Effect of varying the final hadron mass in the decay $b \rightarrow c \ell \bar{\nu}_\ell$ on the transverse lepton momentum distribution, for $m_b = 5.1$ GeV, $m_c = 2.0, 2.2, 2.4$ GeV and (V - A) interaction.
- Fig. 11: Collinearity angle distributions for the leptons from the production and decay of heavy leptons ($\tau^\pm$), charm, bottom primaries and bottom cascade (both the leptons coming from a single bottom quark). The distributions correspond to $\sqrt{s} = 12.0$ GeV, $m_B = 5.1$ GeV, $m_c = 2.0$ GeV, $D(Z_c) = \text{constant}$ and (V - A) interaction.
- Fig. 12: Effect of the lepton energy cut on the collinearity angle distributions for the heavy leptons, charm, bottom primary and bottom cascade decay leptons. The distributions are calculated for $\sqrt{s} = 12$ GeV, $m_B = 5.1$ GeV, $D(Z_c) = \text{constant}$ and a lepton energy cut of 0.5 GeV.
- Fig. 13: Energy dependence of the collinearity angle distributions for the charm, bottom primary and bottom cascade decay leptons. The distributions are calculated for $\sqrt{s} = 12.0$ GeV and 20.0 GeV, $m_B = 5.1$ GeV, $D(Z_c) = D(Z_b) = \text{constant}$, and no cuts.
- Fig. 14: Effect of the fragmentation function on the collinearity angle distributions for the charm, bottom primary and bottom cascade leptons for $\sqrt{s} = 20$ GeV and $m_B = 5.1$ GeV.
- Fig. 15: $\langle \cos \theta_{\text{coll.}} \rangle$ for the leptons from heavy leptons, charm, bottom primary and bottom cascade decay leptons as a function of the beam energy, $m_B = 5.1$ GeV.
- Fig. 16: $\langle \cos^2 \theta_{\text{coll.}} \rangle$ for the leptons from heavy leptons, charm, bottom primary and bottom cascade decays as a function of the beam energy, $m_B = 5.1$ GeV.
- Fig. 17: Invariant mass squared for the charged leptons from the cascade decay of the bottom quark through the chain $b \rightarrow c \ell \bar{\nu}_\ell$. We have assumed $m_b = 5.1$ GeV, $m_c = 2.0$ GeV, $D(Z_c) = \text{constant}$ and (V - A) interaction.

FIG.1

FIG. 2

Fig. 3

FIG.4

FIG. 5

FIG.6

FIG.7

FIG.9

FIG.10

C.M.S. Energy = 12 GeV

FIG.11

FIG. 12

FIG.13

FIG.14

FIG. 15

FIG.16

Fig.17