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THE TRANSVERSE MOMENTUM BEHAVIOUR OF HADRONS,
MULTIHADRON SYSTEMS, AND LEPTON PAIRS ^{*)}

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A B S T R A C T

We discuss and compare the transverse momentum distributions of hadrons and resonant multihadron states, of uncorrelated multihadron systems and of lepton pair production in hadronic collisions.

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1. INTRODUCTION

The two most characteristic features of strong interactions are dominance of multiparticle production and jet structure in form of a (dimensional) transverse momentum bound. Recent years have seen exciting developments for those kinematic regions of hadron physics, where the weak coupling limit of constituent interactions is a good approximation. It must be remembered, however, that the two essential features mentioned above both come from the strong coupling region, and for this a workable dynamical theory is lacking as much as ever.

Model descriptions of the strong coupling region have often been of a statistical nature: the uncorrelated jet model (as scale-invariant gas) gives a rather good account of dominant production features in the longitudinal^{*)} direction, while a statistical approach to the dual resonance model or the statistical bootstrap model (as resonance gas) reproduce well the observed transverse momentum spectra up to about 1-2 GeV. If one takes dynamical theories such as QCD as starting point, this is perhaps not unreasonable: the weak coupling limit then gives the basic interaction dynamics, while the strong coupling region becomes a sort of thermodynamic limit always involving many degrees of freedom; it can therefore be expected to show some thermodynamic "universality", or insensitivity towards dynamical details.

In the present paper, we want to discuss the rather universal behaviour displayed by the transverse momentum distributions of hadrons, multihadron systems, and, as far as we can say to-day, also by the bulk of lepton pair production. We want to show how variations of p_T spectra with mass and energy of the secondary can be understood, how differences between resonant and non-resonant production can arise, and where we should expect hard scattering aspects of lepton pair production to come into play.

2. TRANSVERSE MOMENTUM SPECTRA OF HADRONS

On a qualitative level, it was predicted quite early¹⁾ that the transverse momenta of secondaries from hadronic collisions should on the average not exceed some finite value determined by the range of strong interactions.

^{*)} We refer here always to the axis of the collision which actually leads to production; this can be the beam axis, in small p_T hadron interactions, but also the quark-quark axis, as in e^+e^- annihilation.

Experimentally, it was found over the years that the transverse momentum distribution of a given type of secondary, say of a pion, is perhaps the most universal distribution in high energy hadron physics: p_T spectra up to 1-2 GeV/c are apparently independent of incident energy and of the type of projectile or target; they are the same in hadron-hadron collisions, lepton-hadron collisions, e^+e^- annihilation, and probably also for production initiated by hard scattering hadron interactions; they are the same whether measured inclusively or exclusively^{*)}. In fact, it became more fruitful to look for something that p_T spectra did not depend on.

It was found that transverse momentum distributions certainly depend on the mass of the secondary: thus a π has a steeper spectrum than a ρ , and a ρ has a steeper one than a ψ^2). When measured relative to the incident beam, p_T spectra from soft hadronic collisions and from lepton-hadron interactions seem to depend on the energy of the secondary: pions carrying a finite fraction of the incident energy ($x \neq 0$) have a larger $\bar{p}_T$ than those in the central region ($x = 0$): the "seagull effect"³⁾. Finally, it has been conjectured⁴⁾ that p_T spectra in lepton-hadron collisions should depend on q^2 , the squared four-momentum of the relevant current; so far, there is little experimental evidence for this.

The mass dependence of p_T spectra can, as is well known, be taken into account without loss of universality. Hadronic spectra in the central region, taken as function of the transverse kinetic energy

$$\epsilon_T \equiv \sqrt{p_T^2 + m^2} - m \quad (1)$$

with m denoting the mass of the secondary, fall essentially on one curve (up to $\epsilon_T \simeq 0.5 - 1$ GeV), which can be well parametrized by

$$F(p_T) = e^{-\lambda \epsilon_T} \quad (2)$$

with $\lambda \simeq 6 \text{ GeV}^{-1}$.

*) With the exception of some asymptotically vanishing processes, such as $p\bar{p} \rightarrow$ mesons, where baryon exchange leads to higher $\bar{p}_T$ values.

We have at present two ways of obtaining the distribution (2), and both are, as already indicated, of statistical nature. In the dual resonance model, the number $\rho(M)$ of resonances of a given mass M increases linearly exponential with M ⁵⁾

$$\rho(M) \sim e^{cM} \quad (3)$$

with c essentially determined by the universal slope α' of all Regge trajectories: $\alpha(M) = \alpha_0 + \alpha' M^2$. In the statistical bootstrap model⁶⁾, the hadronic mass spectrum $\rho(M)$ is assumed to approach asymptotically the level density of an ideal gas of hadrons and all possible hadronic resonances, confined in a volume V_0 determined by the range of nuclear forces. As a result, $\rho(M)$ again grows with M in the form (3), with c here determined essentially by V_0 ; $T_0 \equiv 1/c$ is interpreted in statistical hadron physics as the universal critical temperature of hadronic matter.

Now, if only the number of available states counts, i.e., if all possible states have equal weights, then the energy spectrum of a secondary from a system described by Eq. (3) is⁷⁾

$$F(p_0) = \rho((P-p)^2) / \rho(P^2) \simeq e^{-cp_0} \quad (4)$$

with $P^2 = M^2$ as squared CMS energy of the whole system. Applying Eq. (4) for $p_L = 0$ yields the distribution (2), with $\lambda = c$. The universal value of λ for all types of secondaries can thus be interpreted either thermodynamically, as reflection of a universal ultimate hadron temperature, or dynamically, as reflection of a universal Regge resonance pattern.

We should emphasize here one particular aspect of the statistical bootstrap picture. Consider (Fig. 1) a linear chain cascade⁸⁾ as simplest bootstrap model: a resonance of mass $M_i = \sqrt{P_i^2}$ decays into a pion of momentum q_i and a resonance of mass $M_{i+1} = \sqrt{P_{i+1}^2}$, etc. The bootstrap condition here is

$$\rho(M_i^2) = \delta(M_i^2 - \mu^2) + \kappa \int d^4 P_{i+1} \rho(P_{i+1}^2) \int \frac{d^3 q_i}{2q_{i0}} \delta(P_{i+1} + q_i - P_i) \quad (5)$$

where μ denotes the coupling constant or volume, and the first term allows for the case $M_1^2 < 4\mu^2$. The solution of Eq. (5) again has the form (3), for any M_1 sufficiently high along the chain. Using Eq. (4), we can then calculate the p_0 (or p_T) distribution anywhere along the chain. Eq.(4) provides a generation-independent p_T spectrum. In other words: the form (2/4) is just such that "direct" (i.e., first generation) pions and "decay" (i.e., nth generation) pions have essentially the same spectrum. This of course requires a broader p_T distribution for more massive secondaries, and the transverse kinetic energy in Eq. (2) provides just that. Using Eq. (2) we find⁹⁾

$$\bar{p}_T(m) = m \left(\frac{\lambda m}{\lambda m + 1} \right) K_2(\lambda m) e^{\lambda m} \quad (6)$$

for the average p_T of a secondary of mass m ; for large m this gives

$$\bar{p}_T(m) \approx \sqrt{\pi/2\lambda} m^{1/2} \quad (7)$$

as the growth of $\bar{p}_T$ with m . This broadening, which is in accord with all hadrons and hadron resonance data, thus assures that direct and decay pions have the same p_T spectra.

We close this section with a comment on the seagull effect¹⁰⁾. If such an effect persists at all energies, it is well accounted for by a two-step picture in which first a quark is scattered to some angle relative to the beam, then emits hadrons with a bounded p_T distribution relative to its line of flight. This automatically gives a larger $\bar{p}_T$ relative to the beam for particles with higher momenta. There are some high energy data¹¹⁾ confirming the seagull effect, but more would certainly be desirable. Moreover, $\bar{p}_T - x$ correlations in e^+e^- annihilation can provide a crucial test: the two-step explanation forbids a seagull effect here.

III. TRANSVERSE MOMENTUM SPECTRA OF MULTIHADRON SYSTEMS

In the last section, we obtained the p_T distribution of a resonant multihadron system; we now want to find the distribution of a completely uncorrelated multihadron system⁹⁾. Taking pions for simplicity, we consider the transverse momentum behaviour of the process $pp \rightarrow n\pi + \text{anything}$.

We shall assume the total energy of the n pion system to be small compared to the incident energy, and hence neglect overall energy-momentum conservation effects^{*)}. For independent particle emission, we then have

$$d^{2n}\sigma/dp_{iT}^2 \dots dp_{nT}^2 \sim e^{-\lambda \sum_i |\vec{p}_{iT}|}, \quad (8)$$

neglecting the pion mass in ϵ_T . From Eq. (8), the distribution of $\vec{P}_{nT} = \sum_i \vec{p}_{iT}$ is given by the random walk integral

$$d^2\sigma/dP_{nT}^2 \sim \int \prod_{i=1}^n \{d^2p_{iT} e^{-\lambda |\vec{p}_{iT}|}\} \delta^{(2)}(\sum_i \vec{p}_{iT} - \vec{P}_{nT}) \quad (9)$$

which becomes

$$d^2\sigma/dP_{nT}^2 \sim e^{-\vec{P}_{nT}^2/n\bar{p}_T^2} \quad (10)$$

in the limit of large N . Finite n corrections to the limiting form (10) can easily be calculated¹³⁾ and fall as $1/n$ in the manner of the Stirling approximation. From Eq. (8), the mean squared p_T of a single pion is

$$\bar{p}_T^2 = 6/\lambda^2 \quad (11)$$

and hence

$$d^2\sigma/dP_{nT}^2 \sim e^{-\lambda^2 \vec{P}_{nT}^2/6n}. \quad (12)$$

If the n observed pions were decay products from an isotopic cluster ("fireball"), then the average energy of a single pion would be

$$\bar{p} = 3/\lambda \quad (13)$$

*) Their inclusion is straightforward; see ref. 12) and section IV.

and hence

$$m = n\bar{p} = 3n/\lambda \quad (14)$$

would be the fireball mass. In this case, Eq. (12) becomes

$$d^2\sigma/dp_{nT}^2 \sim e^{-\lambda \vec{p}_{nT}^2/2m} \quad (15)$$

which is just the large mass limit of the hadron distribution (2). We thus find that uncorrelated multihadron systems, with an isotropic momentum cut-off $\exp - \lambda |\vec{p}|$ for each particle, leads to the same mass dependence of the transverse momentum distribution as obtained for massive resonances in the dual resonance or statistical bootstrap model. This fact illustrates once more the equivalence between "direct" and "resonance decay" pion production.

Returning now to general multihadron systems (i.e., with unspecified p_L dependence), we have from Eq. (10)

$$\bar{p}_{nT} = (n\pi \bar{p}_T^2/4)^{1/2} \quad (16)$$

This $\sqrt{n}$ growth of the average transverse momentum is indeed observed^{14,15)} as seen in Figs. 2 and 3. For low masses of the multihadron system, kinematic effects of course provide a suppression, as evident in Fig. 2. As a consistency check, Eq. (16) moreover allows a determination of $\bar{p}_T$ from the multiplicity distribution of $\bar{p}_{nT}$. With $\bar{p}_T \simeq 0.4$ GeV in Fig. 3 we find a reasonable value for the average over all charged particles.

For isotropic fireball decay, relation (14) connected mass and particle number of the multihadron system. More generally, we can replace n everywhere by $\bar{n}(m)$, leaving the functional dependence of the multiplicity on m to be determined empirically or through different models. For the multihadron systems usually under consideration, the multiplicity is expected to grow much weaker than linearly m - as reflection of jet structure. The general case

$$\bar{P}_T(m) \simeq \sqrt{3\pi/2\lambda^2} [\bar{n}(m)]^{1/2} \quad (17)$$

obtained from Eq. (11) and (16), thus leads to two extremes

$$\bar{P}_T(m) \simeq \begin{cases} \sqrt{\pi/2\lambda^2} m^{1/2} & \text{fireballs/resonances} \\ \sqrt{9\pi/\lambda^2} \ell_n^{1/2} m & \text{jets} \end{cases} \quad (18)$$

where we have taken $\bar{n}(m) = 3 \ell_n m^2$ as jet multiplicity. The two forms are compared in Fig. 4; only for masses of 10 GeV or higher does the difference become crucial. We shall return to this point in the next section.

IV. TRANSVERSE MOMENTUM SPECTRA OF LEPTON PAIRS

We now consider the process $pp \rightarrow \ell\bar{\ell} + \text{anything}$, where ℓ denotes either an electron or a muon. For lepton pairs of low mass ($\lesssim 1-2$ GeV) and low p_T ($\lesssim 1-2$ GeV), one expects electromagnetic decays of hadronic systems to provide the dominant contribution. In the complementary kinematic region, the situation is less clear. Since for $|\vec{p}_T| > 1-2$ GeV, we leave hadronic jet physics and come into the hard scattering region even for pion production, it seems very likely that the same happens for lepton pairs. The real question hence concerns the region $|\vec{p}_T| < 1-2$ GeV, for large lepton pair masses.

It has been conjectured¹⁶⁾ that even in this low p_T region, lepton pairs of sufficiently large mass ($M \gtrsim 4$ GeV) are dominated by hard scattering in form of pointlike $q\bar{q}$ annihilation (Fig. 5). The Drell-Yan model thus suggests that we can extend the hard scattering region (where a weak coupling approach is applicable) into the small p_T domain by choosing large lepton pair masses. This amounts to lepton pair p_T distributions which, in the limit of high incident energy, for large but finite pair mass M become independent of M : they are determined simply by the p_T distribution of quark and antiquark inside the protons.

The observation of rather large transverse momenta for massive pairs, together with the advent of QCD, has led to the consideration of additional

hard scattering contributions¹⁷⁾, such as shown in Fig. 6. The total observed transverse momentum would then be a combination of the "primordial" partonic p_T (via $q\bar{q}$ annihilation) and gluon jet contributions.

It still remains unclear, however, whether the original or the modified weak-coupling approach is ever applicable to pair production at small p_T . It therefore seems useful to approach the question in the same phenomenological way used to find the hard scattering region, e.g., for pion production: by looking for deviations from conventional hadronic behaviour¹⁸⁾. In the present context, the question thus is: for what values of mass and p_T do the lepton pair p_T distributions differ from the conventional hadronic form, discussed in sections II and III ?

A serious difficulty in answering this question is caused by the comparatively low incident energy ($\lesssim 27$ GeV) of the presently available data. It is hard to believe that the production, however it occurs, of pairs with a mass of a third to a half of the incident energy, is not subject to strong kinematic suppression. Let us sketch one way to estimate such a suppression¹²⁾. If the energy remaining after the production of a lepton pair with transverse momentum P_T ($\lesssim 1-2$ GeV) goes into a conventional multihadron state, then the recoil is [analogous to Eq. (10)] given by

$$e^{-P_T^2 / \sum_i p_{iT}^2} \quad (19)$$

where the summation runs over all final state hadrons. As their number diverges when the incident energy $\sqrt{s} \rightarrow \infty$, the correction factor (19) asymptotically becomes unity for all finite P_T . Any pair production mechanism which does not provide its own specific p_T balance (such as an away-side gluon jet) will be subject to a recoil damping of this form.

With this in mind, we now apply our forms for hadronic p_T spectra to lepton pair production. At infinite energy, the transverse momentum spectrum of the pair is then given by Eq. (2) or (12), depending on whether the pair is associated with a hadronic resonance or with an uncorrelated multihadron system. The corresponding average transverse momenta are given by Eq. (18) and compared in fig. 4. At finite energy and fixed rapidity, we have for pairs emitted from massive hadronic resonances the distribution

$$d^3\sigma/dy dP_T^2 \sim \exp - P_T^2 \left[\frac{3}{M} + \frac{1}{\sum_i p_{iT}^2} \right] \quad (20)$$

obtained from Eqs.(2) and (19). As the correction term is determined by hadronic production data¹⁹⁾, Eq. (20) provides a parameter free prediction for the transverse momentum distribution of lepton pairs. In Fig. 7 this is compared with data²⁰⁾ at $\sqrt{s} = 27.4$ GeV and seen to give excellent agreement for all pair masses, provided $|\vec{P}_T| \lesssim 1$ GeV. In Fig. 8 we leave this P_T range and find that for lepton pairs, just as for hadrons, we then come into a region of weaker P_T dependence, presumably due to hard scattering. In contrast to pion spectra, however, we have here only a reduction of one order of magnitude before the onset of large p_T physics; hence the latter will have an effect of about 10% on average transverse momenta. In Fig. 9 this is found to be the case^{20), 21)}. In the same figure we also see quite clearly that the weak mass dependence of $\vec{P}_T$ is, in the present approach, caused by the kinematic suppression (19).

We are thus led to the conclusion, that as far as P_T spectra are concerned, all data up to $|\vec{P}_T| \sim 1$ GeV are in accord with pair production through the electromagnetic decay of massive hadronic systems ("heavy vector mesons"?). It moreover appears that other features of lepton pair production (p_L distribution, M^2/s distributions, pion vs. proton beam ratios, π^+ vs. π^- beam ratios) are in accord with this¹⁸⁾. We must therefore go to the large P_T region, here as in hadron production, in order to come to a genuine hard scattering domain.

Note that our conclusion does not imply the absence of Drell-Yan contributions in the small p_T region. Both the hadronic decay spectrum and that from Fig. 5 will be of the form $\exp \sim b P_T^2$. However, asymptotically in s , we expect for 10 GeV pairs $b \simeq 0.3 \text{ GeV}^{-2}$ from hadronic decay, $b \simeq 3-6 \text{ GeV}^{-2}$ for that from the Drell-Yan term. Hence the latter, being much steeper, contributes only a small part of the P_T distribution.

We close with a comment on the nature of the hadronic system leading to pair production. At sufficiently high incident energies, the curves from resonant and from uncorrelated multihadron systems (fig. 4) differ enough to allow a distinction. At present energies ($\lesssim 27$ GeV), however, the suppression factor (19) brings the two very close together; we show

the result [Eq. (20) vs. Eqs. (12) and (19)] for $\sqrt{s} = 27$ GeV in Fig. 10. What happens as $s \rightarrow \infty$? If we use recent e^+e^- data²²⁾ as a basis for speculation, we might anticipate uncorrelated two-jet behaviour for the continuum, and multi-jet or isotropic resonance behaviour for all those quark-antiquark systems whose hadronic decay is forbidden by the Zweig rule (J/ψ , Υ , ...?). The resulting asymptotic prediction is illustrated in fig. 11.

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FIGURE CAPTIONS

- Fig. 1 : Linear chain decay.
- Fig. 2 : Average transverse momentum for uncorrelated n pion systems of mass M from ref. 14).
- Fig. 3 : Average transverse momentum for uncorrelated n pion systems, from ref.15); the slope 0.43 corresponds to $\bar{p}_T = 0.39$ GeV.
- Fig. 4 : Average transverse momenta of fireball or resonant (a) and uncorrelated (b) multipion systems, vs. the mass of the system.
- Fig. 5 : Drell-Yan model for lepton pair production.
- Fig. 6 : Gluon jet contribution to lepton pair production.
- Fig. 7 : Transverse momentum distributions of lepton pairs, from Ref. 19), compared to hadronic spectra with kinematic suppression ($\sqrt{s} = 27$ GeV).
- Fig. 8 : Transverse momentum distributions of lepton pairs, from Ref. 19), compared to hadronic spectra with ($\sqrt{s} = 27$ GeV) and without kinematic suppression.
- Fig. 9 : Average transverse momenta of lepton pairs, from Refs. 19) and 20) compared to hadronic $\bar{p}_T$ with and without kinematic suppression.
- Fig. 10 : Average transverse momenta for fireball (a) and uncorrelated (b) multipion systems, asymptotically and with kinematic suppression.
- Fig. 11 : Speculated asymptotic behaviour of $\bar{p}_T(M)$.

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