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QUARK DISTRIBUTIONS AND QUARK JETS FROM THE
CERN BUBBLE CHAMBER NEUTRINO EXPERIMENTS

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ABSTRACT

The measurement of the nucleon structure functions in the BEBC and GGM neutrino experiments is reviewed. The data demonstrate the importance of quark quantum numbers and provide a direct experimental measurement of the QCD anomalous dimensions. Studies of properties of the hadronic jet in these experiments have also revealed evidence for quark quantum numbers. An increase in the mean transverse momentum of the hadronic secondaries at large q^2 has been observed in the BEBC experiment. Related investigations are discussed.

1. NEUTRINO STRUCTURE FUNCTIONS AND TESTS OF QUARK QUANTUM NUMBERS

In the quark parton model the differential cross section for high energy neutrino (antineutrino) scattering from an isoscalar target is given by

$$\frac{d^2\sigma}{dx dy} = \frac{G}{\pi} \frac{mE}{\pi} (Q + \bar{Q} (1-y)^2) \text{ for neutrinos,} \quad (1a)$$

$$\frac{d^2\sigma}{dx dy} = \frac{G}{\pi} \frac{mE}{\pi} (Q (1-y)^2 + \bar{Q}) \text{ for antineutrinos,} \quad (1b)$$

($x = q^2/2m\nu$ and $y = \nu/E$, where q^2 is the square of the four-momentum transfer and ν is the energy transfer to the hadrons in the lab. frame). The structure functions F_2 and xF_3 represent the sum and difference of the quark and antiquark contributions respectively

$$F_2 = Q + \bar{Q} \quad (2a)$$

$$xF_3 = Q - \bar{Q}. \quad (2b)$$

Q and $\bar{Q}$ are the momentum distributions (weighted by x) of quarks and antiquarks in the nucleon.

Fig. 1 shows the BEBC data [1] on Q and $\bar{Q}$ averaged over a large range of q^2 ($q^2 = 3-100 \text{ GeV}^2$). The quark contribution falls like $(1-x)^3$ approximately for $x > 0.3$ while the antiquark contribution is consistent with $(1-x)^5$.

To investigate the q^2 dependence we have plotted the structure functions F_2 versus q^2 for fixed x ranges as shown in fig. 2. The data from the GGM-PS experiment and the BEBC experiment fit together smoothly in q^2 , even though the neutrino energies in the two experiments differ by a large factor (~ 20) as is to be expected if the interaction has the assumed current-current form.

The data for F_2 are compared to the corresponding data from the SLAC electron [2] and FNAL muon [3] experiments on deuterium targets. If the quarks have the usual fractional charges than we expect $F_2^{\text{ed}} \sim 5/9 F_2^{\text{vN}}$.

The measure of agreement between the neutrino data and the electron/muon data ($1.8 F_2^{\text{ed}}$) is evidence for fractionally charged quarks [4].

Finally, we observe that the structure functions do show a significant q^2 dependence at fixed x . The structure functions increase with increasing q^2 at small x ($x < 0.2$) and decrease with increasing q^2 at large x ($x > 0.3$). This is qualitatively consistent with the predicted behaviour in field theories of the strong interactions in particular in QCD [5].

The corresponding data for xF_3 is similar to the data for F_2 except that the errors on xF_3 are about a factor of two larger. The data on xF_3 have been used to test the Gross Llewellyn Smith (GLS) sum rule [6]: the difference of the quark and antiquark populations integrated over x is the number of valence quarks equal to 3 in the usual quark model.

$$\int_0^1 F_3 dx = \int_0^1 \frac{Q - \bar{Q}}{x} dx = N_Q - N_{\bar{Q}} = 3. \quad (3)$$

The main problem which arises in testing the sum rule experimentally is that the largest contribution to the integral comes from the small x region. Since $x = q^2/2m\nu$, there is always a minimum value of x ($x = x_{\text{min}}$), for a given q^2 , which can be reached with presently available neutrino energies, and part of the integral necessarily remains unmeasured.

The best that can be done is to evaluate $\int_{x_{\text{min}}}^1 F_3 dx$ for the smallest possible x_{min} . If xF_3 remains positive for small x then the results can be regarded as lower limits for the complete integral. Table 1 shows the experimental value for this integral for various choices of x_{min} . Clearly the data suggest ~ 3 valence quarks per nucleon in excellent agreement with the prediction from the GLS sum rule.

Further studies involve the higher moments of the structure functions which depend more and more on the data at large x ($x \geq 0.6$) where the effects of smearing in x due, for example, to poorly measured hadron energies are particularly important. In this analysis we have attempted to correct for the effects of smearing by multiplying the observed event rates by correction factors

computed by a Monte-Carlo which simulates the effects of hadron energy resolution etc. in our experiment. Clearly, these correction factors cannot be computed in an entirely model independent way and for this reason (and also because of limited statistics) we restrict our consideration to moments $N \lesssim 7$. Notice that for the higher moments ($N \gtrsim 2$) the missing contribution at small x (which we discussed in connection with GLS sum rule) gets weighted by powers of x and can therefore be safely neglected.

In QCD the q^2 evolution of the structure functions is governed by the magnitude of the strong interaction coupling constant given by

$$\frac{\alpha_s(q^2)}{\pi} = \frac{12}{27 \ln q^2 / \Lambda^2} \quad (\text{for 3 quark flavours})^{(*)} \quad (4)$$

where Λ is the parameter which determines the magnitude of the coupling constant at fixed q^2 . The theory makes predictions for the q^2 dependence of moments of structure functions $M_i(N, q^2)$. For a non-singlet structure function (like $xF_3 = Q - \bar{Q}$) the leading order predictions take the simple form

$$M_{NS}(N, q^2) = C_N / [\ln q^2 / \Lambda^2]^{d_N^{NS}}, \quad (5)$$

where d_N^{NS} are the non-singlet anomalous dimensions given by

$$d_N^{NS} = \frac{4}{27} \left[1 - \frac{2}{N(N+1)} + 4 \sum_{j=2}^N \frac{1}{j} \right]. \quad (6)$$

We have computed the Nachtmann moments of the structure functions defined by

$$M_2(N, q^2) = \int_0^1 \frac{\xi^{N+1}}{x^3} \cdot F_2(x, q^2) \cdot \frac{[(N^2+2N+3) + 3(N+1)\sqrt{1+4M^2x^2/q^2} + N(N+2)\frac{4M^2x^2}{q^2}]}{(N+2)(N+3)} dx \quad (7)$$

$$M_3(N, q^2) = \int_0^1 \frac{\xi^{N+1}}{x^3} \cdot xF_3(x, q^2) \cdot \frac{[1+(N+1)\sqrt{1+4M^2x^2/q^2}]}{(N+2)} dx, \quad (8)$$

$$\text{where } \xi = \frac{2x}{(1 + \sqrt{1 + 4m^2x^2/q^2})}.$$

(*) We assume throughout that 3 is the relevant number of quark flavours in our q^2 range.

The Nachtmann moments are defined to take account of target mass effects which are important at small values of q^2 [7]. Fig. 3 shows the Nachtmann moments of F_2 and xF_3 plotted versus q^2 .

In order to test the prediction [5] we begin by eliminating the dependence on Λ by considering a pair of moments N and N' . If the moments are falling logarithmically with q^2 as predicted by eq. (5) then by taking logs we have

$$\ln M_3(N, q^2) = \ln C_N - d_N^{NS} \ln \ln q^2 / \Lambda^2$$

and
$$\ln M_3(N', q^2) = \ln C_{N'} - d_{N'}^{NS} \ln \ln q^2 / \Lambda^2.$$

thus a plot of $\ln M(N', q^2)$ versus $\ln M(N, q^2)$ should yield a straight line with a slope equal to the ratio of the relevant anomalous dimensions $d_{N'}^{NS}/d_N^{NS}$. The data are shown in fig. 4. The solid lines show the predicted slopes from QCD (vector gluons). Clearly the data are in very good agreement with the slopes predicted by QCD.

The data shown in fig. 3 for the moments of xF_3 have been fitted to the form (5) to obtain a value for Λ . The best value of Λ (in leading order) from the $N = 3, 5$ and 7 moments is $\Lambda = 0.74 \pm 0.05$ GeV.

Since QCD formulae evaluated to leading order provide such a remarkably accurate description of the moment data even at such small values of q^2 ($q^2 \sim 1$ GeV²) it is natural to ask to what extent would the higher order (in α_s) corrections be expected to improve or worsen the agreement between theory and experiment. This question has been studied recently by Bardeen et al., [8] combining the two loop calculations of the anomalous dimensions by Floratos et al., [9] with their own calculation of the α_s corrections to the coefficient function. In next to leading order the q^2 dependence of the xF_3 moment takes the form

$$M_3(N, q^2) = \frac{C_N}{[\ln q^2 / \Lambda^2]^{d_N^{NS}}} \left[1 + \frac{\alpha_s}{4\pi} [B_N + P_N + L_N \ln \ln (q^2 / \Lambda^2)] + \dots \right],$$

where B_N , P_N and L_N are constants specified by the theory.

Fig. 5 shows the $N = 3$ Nachtmann moments of xF_3 plotted versus q^2 . The solid curve shows the next to leading order QCD prediction normalised to the data assuming $\Lambda = 0.45$ GeV and the broken curve shows the corresponding leading order contribution. Clearly the contribution of the next to leading order (α_s^2) correction is not small: even for $q^2/\Lambda^2 \sim 10$ the magnitude of the correction to the $N = 3$ moment is of order 50%. One interesting observation, however, [8] is that the curve representing the leading order contribution can be made to coincide almost exactly (at least for $q^2 > 1$ GeV²) with the curve computed including the α_s^2 correction by transporting the leading order curve horizontally to the right. The same feature is also apparent for the higher moments, for example for $N = 5$. The significance of this is that the effect of the α_s^2 corrections on the analysis can be largely accounted for by a change in the value of Λ and that the remarkable agreement between experiment and leading order QCD (in particular the moment versus moment comparison (fig. 4) and the determination of the anomalous dimension ratios) is left essentially undisturbed.

The results of maximum likelihood fits for Λ in leading order only, and also including α_s^2 corrections for the $N = 3$ and $N = 5$ moments are given in table 2. The best value for Λ obtained from the $N = 3$ and $N = 5$ moments including α_s^2 corrections is ^(*): $\Lambda = 0.45 \pm 0.05$ GeV.

The final determination of Λ from the present data is further complicated by the possible effects of higher twist contributions which have not been considered in the forgoing analysis [11]. Fits to the data including a correction term proportional $1/q^2$ show that the value of Λ and the coefficient of the $1/q^2$ term are highly correlated.

At low q^2 a substantial contribution to the moments is expected to come from nucleon resonances (notice for example the important contribution of the elastic events to the $N = 3$ moment). In the GGM freon experiment the nucleon resonances are badly smeared and for this reason a detailed analysis of the low q^2 region would be extremely difficult.

(*) The value for Λ including second order corrections quoted in ref. [1], $\Lambda = 0.66 \pm 0.05$ GeV is incorrect. That fit was based on the formulae of ref. [10] which combined calculations of the coefficient function and of the anomalous dimensions performed in different renormalisation schemes.

2. PROPERTIES OF THE HADRONIC JET PRODUCED IN NEUTRINO REACTIONS

Lepton nucleon scattering experiments play a particularly important role in the study of hadronic jets. The reason for this is that at high q^2 the primary interaction occurs with a single quark and the form of the forward jet is largely determined by the radiation due to a single elementary constituent. Lepton scattering experiments have an advantage over e^+e^- annihilation in the study of quark jets in that the direction of the jet axis is essentially determined if the scattered lepton is detected and measured.

Neutrino and antineutrino scattering experiments have the particular advantage that the flavour of the quark which initiates the jet is usually known. Neutrinos scatter from d-quarks to produce u-quarks and antineutrinos scatter from u-quark to produce d-quarks.

$$\begin{array}{ccc} \nu d \rightarrow \mu^- u & , & \bar{\nu} u \rightarrow \mu^+ d \\ \downarrow \text{jet} & & \downarrow \text{jet} \end{array}$$

Neutrino and antineutrino experiments, performed using bubble chambers, are particularly suited to the study of quark jets. Charged hadrons are measured with a momentum resolution $\delta p/p \lesssim 5\%$ and in neon neutral hadrons (gamma rays from π^0 decays) are detected with efficiency $\sim 50\%$ although the momentum resolution is rather poor. Fig. 6 shows a picture of a neutrino event detected in the 275 GeV narrowband neon experiment in BEBC. There is a high energy muon making a large angle to the beam direction and a well defined hadronic jet comprising 5 charged hadrons and $\geq 1 \pi^0$. In this event the energy of the incident neutrino is 220 GeV and the q^2 is 190 GeV².

One fundamental question about jets which can be answered in neutrino and antineutrino experiments is to what extent the properties of the hadronic jet reflect the quantum numbers of the original quark. This question has already been investigated in some detail [12] by comparing the observed jet properties in neutrino and antineutrino events with the standard jet model due to Field and Feynman [13]. The magnitude and distribution of the charge of the jet is of particular importance.

The fig. 7 shows the distribution of the jet charge (positives minus negatives) for neutrino and antineutrino events as measured in the BEBC narrowband neon experiment plotted as a function of z . The variable z in this case is defined by $z = 2P_L/\sqrt{q^2}$ where P_L is the longitudinal momentum of a hadron along the direction of the current measured in the current Breit frame where the energy transfer to the interacting parton is zero. Thus, only those tracks which are forward in the Breit frame are considered as part of the jet. The charge of the jet is spread over several units in $\ln z$ very much as assumed in the standard jet model. The mean charge of the jet is 0.55 ± 0.06 for neutrinos and -0.12 ± 0.13 for antineutrinos consistent with the expected charge for $1/3$ integer charged quarks.

One can argue that at these relatively low energies and low multiplicities such distributions are largely determined by charge conservation constraints once the multiplicity and the transverse momenta are fixed. The selection of tracks which are forward in the Breit frame to define the jet charge is somewhat arbitrary, and one should also be aware that the precise value and distribution of the charge depends somewhat on the cuts applied (eg. $W > 4$ GeV, $x > 0.1$ etc.). Furthermore, in this experiment the interactions occur in neon nuclei and charge exchange processes may distort the initial distributions. None the less, the data rather naturally suggest fractional charges and in my opinion this type of plot constitutes some of the best evidence that the jets we see really do come from quarks.

In the first part of my lecture I discussed how the observed q^2 variation of the nucleon structure functions is accounted for in QCD. These effects are due to strong radiative corrections (processes like gluon bremsstrahlung) which modify the simple parton model picture. If QCD is right, then related energy dependent effects should be manifest in the properties of the quark jets and may already have been observed in existing experiments. Indeed one of the most important new results from the BEBC narrowband neon experiments at CERN is the observation that the transverse momentum of the hadronic secondaries relative to the jet axis is increasing with increasing q^2 .

The results for $\langle p_T^2 \rangle$ versus q^2 and versus W^2 for neutrino events are shown in fig. 8. To minimise any bias in the transverse momentum distributions arising from uncertainties in the neutrino energy, the transverse momenta are computed relative to the direction of the total measured hadron momentum vector in the laboratory. For $z > 0.2$ there is a significant increase in $\langle p_T^2 \rangle$ both as a function of W^2 and as a function of q^2 (z in this case is defined by $z = E_h/\nu$ where E_h is the energy of the hadron in the laboratory frame). As illustrated by the Longitudinal Phase Space curve (LPS) this effect is not accounted for by kinematic restrictions on the transverse momenta which are important at small values of W .

An increase in the mean transverse momentum of the hadronic secondaries with q^2 arising from processes like gluon bremsstrahlung is predicted in QCD [14]. More detailed tests are being formulated in order to find out whether the observed effect can be explained in this way. One promising approach [15] focuses on the energy distribution within the jet as function of angle measured from the jet axis. Motivated by the theoretical work of Sterman and Weinberg [16] for jets in e^+e^- annihilation one computes the fraction of events f with a fraction $1 - \epsilon$ or more of the energy of the jet inside a cone of $1/2$ angle δ . If the Sterman and Weinberg formula applies then $f = 1 - \alpha_s g(\epsilon, \delta)$ and a plot of $1/(1 - f)$ versus q^2 should show a logarithmic dependence with an intercept at $q^2 \sim \Lambda^2$. Fig. 9 shows the data from the BEBC narrow band neon experiment plotted versus q^2 . The jet in this case is defined as all particles which are forward in the c.m.s. frame and the jet axis is the vector sum of their momenta. The data are consistent with a logarithmic q^2 dependence (at least at large q^2). Notice however that while the data are plotted versus q^2 , the other independent variable W^2 (or equivalently x) is varying as the q^2 varies and more detailed investigations are required to separate the dependence on the two variables. Assuming this complication can be resolved, it is clear that quantitative comparisons with the full theoretical formulae will be very interesting.

TABLE 1

$$\int_{x_{\min}}^1 F_3 dx$$

$q^2 \text{GeV}^2$	$x_{\min} = 0.02$	$x_{\min} = 0.06$	$x_{\min} = 0.1$
0.3 - 1	2.94 ± 0.56	2.33 ± 0.16	2.01 ± 0.15
1 - 3	2.79 ± 0.51	2.04 ± 0.33	1.38 ± 0.11
3 - 10	2.69 ± 0.41	1.96 ± 0.26	1.20 ± 0.21
10 - 30	-	1.39 ± 0.17	1.07 ± 0.11
30 - 100	-	-	0.94 ± 0.17

TABLE 2

Maximum likelihood fits for Λ (Λ in GeV)

$N = 3 \ q^2 > 1 \text{ GeV}^2$	
Leading order only	$\Lambda = 0.70 \pm 0.08 \ (\chi^2 = 2.7/5)$
Including α_s^2 corrections	$\Lambda = 0.40 \pm 0.06 \ (\chi^2 = 2.6/5)$
$N = 5 \ q^2 > 1 \text{ GeV}$	
Leading order only	$\Lambda = 0.77 \pm 0.07 \ (\chi^2 = 3.8/5)$
Including α_s^2 corrections	$\Lambda = 0.50 \pm 0.06 \ (\chi^2 = 3.9/5)$

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FIGURE CAPTIONS

- Fig. 1 BEBC data for the quark Q and antiquark $\bar{Q}$ momentum distributions averaged over the q^2 range $q^2 = 3-100 \text{ GeV}^2$.
- Fig. 2 The structure function F_2 extracted from the BEBC-SPS and GGM-PS data plotted versus q^2 for fixed x ranges.
- Fig. 3 The Nachtmann moments of F_2 and xF_3 plotted versus q^2 .
- Fig. 4 $\ln M(N, q^2)$ versus $\ln M(N', q^2)$ for selected pairs of Nachtmann moments N and N' . In leading order QCD the data points fall on straight lines with the slopes indicated.
- Fig. 5 The $N = 3$ Nachtmann moment of xF_3 plotted versus q^2 . The solid curve is the next to leading order QCD prediction fitted to the data ($q^2 > 1 \text{ GeV}^2$) for $\Lambda = 0.45 \text{ GeV}$ and the broken curve shows the corresponding leading order contribution.
- Fig. 6 Picture of a neutrino event found in the BEBC narrowband neon (275 GeV) experiment. The energy of the incident neutrino is 220 GeV and the q^2 is 190 GeV^2 .
- Fig. 7 The distribution of the charge of the jet $dQ/d\ln z$ plotted versus $\ln z$ for neutrinos and antineutrinos in the BEBC NB Neon experiment. The curve is from Field and Feynman [13].
- Fig. 8 The mean transverse momentum squared $\langle p_T^2 \rangle$ relative to the jet axis for charged hadrons ($z > 0.2$) plotted versus q^2 and W^2 . The broken curve is a longitudinal phase-space (limited p_T) model.
- Fig. 9 The quantity $1/(1 - f)$ plotted versus q^2 (f is the fraction of events with a fraction $(1 - \epsilon)$ or more of the jet energy inside a cone of $\frac{1}{2}$ angle δ).

Fig. 1

$F_2^{VN}(x, q^2)$ assuming $2xF_1 = F_2$

Fig. 2

Fig. 3

Fig. 4

Fig. 5

Fig. 6

Fig. 7

Fig. 8

Fig. 9