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e^+e^- JETS : CONES OR TUBES ?

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A B S T R A C T

The predictions of quantum chromodynamics (QCD) for narrow angular jets in e^+e^- annihilation are compared with the expectations coming from the hadron phenomenology (fixed p_T jets).

It is shown that the hadronization effects are responsible for surprisingly strong signals in the calorimeter measurements at present energies and, when superimposed on QCD jets, modify drastically the naïve perturbative predictions. The transition energy above which the QCD effects start to dominate over the non-perturbative background is found to lie in the LEP energy region.

A new method of analyzing the calorimeter data is proposed, providing the quantities which are expected to be insensitive to the hadronic effects also at present energies.

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INTRODUCTION

The jet calorimetry, discussed recently by many authors [1-8] provides an attractive possibility of testing the predictions of the perturbative quantum chromodynamics (QCD) in the final states of deep inelastic processes, in spite of our present ignorance concerning the confinement problem.

According to widespread belief, the confining forces are soft in the sense that their only effect in hard processes is the conversion of high-energy quarks and gluons into fixed p_T hadronic jets (with $\langle p_T \rangle$ of the order of a few hundred MeV).

Consequently, the angular spread of the non-perturbative effects, when measured relative to the jet axis should decrease with the jet momentum P ($P \sim \sqrt{Q^2}$) like $\Delta\theta \sim \langle p_T \rangle / P$. Therefore, a measurement performed by means of a fixed angle (δ) calorimeter should become asymptotically insensitive to the details of the hadronization process and can provide some information about the missing perturbative component of the jet, extending over a much broader angular region.

Thus, the inclusive calorimetric method is in a sense a complementary way of detecting perturbative effects, as compared with the explicit study of multi-jet events [2].

The idea sketched above and the lowest order QCD calculations for e^+e^- jets have been presented two years ago by Sterman and Weinberg [1].

Following this scheme, the jet calorimetry was then investigated by many people [3]. In particular, it has been shown that one can go beyond the lowest order and resume the whole perturbative series in the leading $\log \delta$ approximation, either by explicit summation of the relevant set of Feynman diagrams [4,5] or by means of the probabilistic method of Parisi and Altarelli [6] or, finally, by using the formalism of coherent states [7].

The limit of small angular size δ of the calorimeter, for which the leading $\log \delta$ formulae are relevant, is of particular practical importance since the QCD corrections get stronger for small values of δ .

The calculations existing in the literature are asymptotic in the sense that no detailed discussion of the influence of the non-perturbative effects on the experimental signal has been performed. Apparently, one might expect this point to be irrelevant in view of the forthcoming generation of high-energy machines where the true asymptotic regime should be reached.

However, the first measurements done at PETRA do not reveal any definite QCD signal and therefore a more precise analysis of the relative importance of the QCD and fixed p_T effects at present energies is needed.

The present paper is an attempt in this direction. Following the folklore, we replace the high-energy quark produced, for example in e^+e^- annihilation, by a hadronic jet and analyse the combined result of perturbative and non-perturbative emissions. The hadronic component is described by the simple independent emission model with the longitudinal momentum phase space, whereas for the QCD part we use the leading $\log \delta$ formulae derived in Refs. [4-7].

For definiteness, we will mainly discuss the hadronic effects in the Sterman-Weinberg cross-section; however, the same method can be used for any other calorimeter measurement.

The paper is organized as follows: In Section 1, we present the QCD jet formulae using the probabilistic language of Parisi and Altarelli. In Section 2, we analyse the fixed p_T jets in the context of calorimeter measurements. Both contributions are merged in Section 3, where we derive the final formula for the S-W cross-section with the hadronic corrections included. Section 4 is devoted to numerical estimates; the general conclusion of this section is that the hadronic "corrections" are very strong at present energies. In particular, it is shown that for any values of ϵ and δ the dominant contribution to the calorimeter signal in PETRA comes from the fixed p_T component, the QCD part being only a very small correction. In LEP both components are of the same order of magnitude. Consequently, a more refined approach is needed in order to look for perturbative effects in the calorimeter data available in the near future. We discuss this problem in

Section 5, where some "hadronization-free" calorimeter quantities are proposed. A rough estimate shows that the QCD effect, extracted in this way, should be measurable in PETRA.

1. QCD JETS (CONES)

We begin this section with the precise definition of the calorimeter measurement we have in mind.

Let us consider some high-energy collision like e^+e^- , ep, pp, etc., in the over-all c.m. frame and place the calorimeter with the small opening angle Ω at some fixed angle $\theta \neq 0$ relative to the beam (Fig. 1).

The calorimeter should have the angular resolving power δ , i.e. an arbitrary set of particles with the total energy E (or with the fraction $x = E/P$ of the total available c.m. energy $P = \frac{1}{2}\sqrt{s}$), and with relative opening angles less than 2δ should give the same signal in the detector. The simplest way to realize it, in practice, is to divide the cone Ω into the set of small sectors, each one with the solid angle $\pi\delta^2$ and to count independently the signals in different sectors.

The basic quantity $d\sigma/dxd\Omega$, which can be measured by means of this calorimeter and, on the other hand, can be easily derived in QCD, will be called in the following the single-jet inclusive cross-section and is defined as (see also Ref. 8 for similar considerations):

$$\frac{d\sigma}{dxd\Omega} = \frac{\text{number of signals with energy } xP, (x + dx)P}{\Omega \cdot \text{flux}}.$$

Imagine now that the single parton with momentum P enters the calorimeter and define $J(P, x, \delta)$ as

$$J(P, x, \delta) = \frac{\text{number of signals with energy } xP, (x + dx)P}{\Omega}.$$

In the parton model we have

$$\frac{d\sigma}{dxd\Omega} = \int_0^1 dy \frac{d\sigma_0}{dy d\Omega} J(yP, x, \delta) \delta(x - yz) dz \quad (1.1)$$

where $d\sigma_0/dy d\Omega$ is the cross-section for producing the wide-angle parton with fraction y of the total available c.m. energy. The quantity $J(x)$ will be called in the following the jet fragmentation function. For jets initiated by quarks and gluons, respectively, we will use the notation $J_Q(x)$, $J_G(x)$.

Using the functions $J(x)$ one can easily write the formulae for arbitrary multi-jet inclusive distributions as well, e.g.

$$\frac{d\sigma}{dx_1 dx_2 d\Omega_1 d\Omega_2} = \int_0^1 dy_1 dy_2 \frac{d\sigma_0}{dy_1 dy_2 d\Omega_1 d\Omega_2} J(z_1) J(z_2) \delta(x_1 - y_1 z_1) \delta(x_2 - y_2 z_2) dz_1 dz_2. \quad (1.2)$$

In particular, the Serman-Weinberg (SW) cross-section $\sigma(P, \epsilon, \delta)$ may be written as

$$\sigma_{SW}(P, \epsilon, \delta) = \sigma_0(P) f_{SW}(P, \epsilon, \delta) \quad (1.3)$$

where

$$f_{SW}(P, \epsilon, \delta) = \int_0^1 dx_1 \int_0^1 dx_2 J_Q(x_1) J_Q(x_2) \Theta(-2(1-\epsilon) + x_1 + x_2)$$

and $\sigma_0(P)$ is the Born cross-section for $e^+e^- \rightarrow q\bar{q}$.

At the leading $\log \delta$ level in QCD the parton model formulae (1.1)-(1.3) remain valid and the functions $J(z)$ can be easily derived by using the method of Parisi and Altarelli [8]. (A more rigorous covariant derivation of analogous quantities can be found in Refs. 4 and 5).

In our case the "master equation" will describe the change of the number of signals in the detector when one varies the resolving power δ [6].

Let us assume for the moment that the detector can distinguish between the signals caused by quarks and gluons and let us denote the corresponding numbers of signals, measured with some resolving power δ by F, G . (i.e. $J(x) = \sum_F \{ [F(x) + \bar{F}(x)] + G(x) \}$).

If we decrease δ by some amount $\Delta\delta$ (e.g. by shifting the calorimeter) the number of signals will change, since we resolve now in more detail the bremsstrahlung cloud

of the parton, responsible for one signal when probed with δ . Denoting by dP_{AB} the probability that the parton A will emit the parton B with the momentum fraction z and with the opening angle θ , we may write

$$\frac{dP_{AB}}{dzd\theta} = \frac{\alpha(P_0)}{\pi} P_{AB}(z) \frac{2}{\theta}, \quad (1.4)$$

where $P_{AB}(z)$ are the well-known Parisi-Altarelli functions:

$$\begin{aligned} P_{FF}(z) &= C_F \frac{1+z^2}{1-z}, \\ P_{FG}(z) &= C_F \frac{1+(1-z)^2}{z}, \\ P_{GF}(z) &= T_F [z^2 + (1-z)^2], \\ P_{GG}(z) &= C_G \left[\frac{1-z}{z} + \frac{z}{1-z} + z(1-z) \right]. \end{aligned} \quad (1.5)$$

Let us consider, for example, the $q \rightarrow q + G$ transition described by dP_{FF} . The emission with the opening angle $\delta - \Delta\delta < \theta < \delta$ was "unseen" with the resolving power δ , but for $\delta' = \delta - \Delta\delta$, instead of one signal (quark) with fraction x we will see either two separate signals (quark and gluon) with fractions xz , $x(1-z)$ and with probability

$$P_2 = \int_{\delta'}^{\delta} d\theta \frac{dP_{FF}}{dzd\theta}$$

or one signal (quark) with probability

$$P_1 = 1 - \int_0^1 dz \int_{\delta'}^{\delta} d\theta \frac{dP_{FF}}{dzd\theta}.$$

Finally, since the number of signals with δ' is proportional to the number of signals with δ times the appropriate probability P_1 or P_2 , we may write:

$$\begin{aligned}
 F(x, \delta - \Delta\delta) &= \int_0^1 dy F(y, \delta) \left[\int_0^1 dz \int_{\delta - \Delta\delta}^{\delta} d\theta \frac{dP_{FF}}{dzd\theta} \delta(x - yz) + \left(1 - \int_0^1 dz \int_{\delta - \Delta\delta}^{\delta} d\theta \frac{dP_{FF}}{dzd\theta}\right) \delta(x - y) \right] + \\
 &\quad + \int_0^1 dy G(y, \delta) \left[\int_0^1 dz \int_{\delta - \Delta\delta}^{\delta} d\theta \frac{dP_{GF}}{dzd\theta} \delta(x - yz) \right], \\
 G(x, \delta - \Delta\delta) &= \int_0^1 dy G(y, \delta) \left[\int_0^1 dz \int_{\delta - \Delta\delta}^{\delta} d\theta \frac{dP_{GG}}{dzd\theta} \{ \delta(x - yz) + \delta(x - (1-z)y) \} + \right. \\
 &\quad + \left. \left(1 - \int_0^1 dz \int_{\delta - \Delta\delta}^{\delta} d\theta \left\{ \frac{dP_{GG}}{dzd\theta} + \sum_F \frac{dP_{GF}}{dzd\theta} \right\} \right) \delta(x - y) \right] + \\
 &\quad + \sum_F \int_0^1 dy \{ F(y, \delta) + \bar{F}(y, \delta) \} \int_0^1 dz \int_{\delta - \Delta\delta}^{\delta} d\theta \frac{dP_{FG}}{dzd\theta} \delta(x - yz).
 \end{aligned} \tag{1.6}$$

In the limit $\Delta\delta \rightarrow 0$ we recover the Parisi-Altarelli master equations with the resolving power δ instead of Q^2 . Denoting $t = \ln(1/\delta)$ and taking moments we get

$$\begin{aligned}
 \frac{d}{dt} F_s(t) &= \frac{\alpha(P\delta)}{\pi} [F_s(t) A_{FF}^{(s)} + G_s(t) A_{GF}^{(s)}], \\
 \frac{d}{dt} G_s(t) &= \frac{\alpha(P\delta)}{\pi} [G_s(t) A_{GG}^{(s)} + \sum_F (F_s(t) + \bar{F}_s(t)) A_{FG}^{(s)}]
 \end{aligned} \tag{1.7}$$

with the boundary condition $F_s(t_0) = 1$, $G_s(t_0) = 0$ for the quark parent and $F_s(t_0) = 0$, $G_s(t_0) = 1$ for the gluon parent, where $t_0 \approx 0$. The "anomalous dimension" factors $A^{(s)}$ are defined as follows:

$$\begin{aligned}
 A_{FF}^{(s)} &= \int_0^1 dz (z^{s-1} - 1) P_{FF}(z) = C_F \left[-2(\psi(s+1) + \gamma) + \frac{3}{2} + \frac{1}{s(s+1)} \right], \\
 A_{FG}^{(s)} &= \int_0^1 dz z^{s-1} P_{FG}(z) = C_F \frac{s^2 + s + 2}{s(s^2 - 1)}, \\
 A_{GF}^{(s)} &= \int_0^1 dz z^{s-1} P_{GF}(z) = T_F \frac{s^2 + s + 2}{s(s+1)(s+2)}, \\
 A_{GG}^{(s)} &= \int_0^1 dz \left[\left[z^{s-1} + (1-z)^{s-1} \right] - 1 \right] P_{GG}(z) - N_F \int_0^1 dz z^{s-1} P_{GF}(z) = \\
 &= C_G \left[-2(\psi(s+1) + \gamma) + \frac{11}{6} + \frac{2}{s(s-1)} + \frac{2}{(s+1)(s+2)} \right] - \frac{2}{3} N_F T_F.
 \end{aligned} \tag{1.8}$$

(Note that the resulting integrals for $A^{(s)}$ are already well defined in the limit $z \rightarrow 1$ without the necessity of imposing the momentum sum rules as in Ref. 9, since the diagonal terms $(1 - \int dP_{AB})$ in Eqs. (1.6) take care of the conservation of probability during the evolution).

As seen from Eq. (1.3), in order to derive the Sterman-Weinberg cross-section, we need the function $J_Q(x) = \sum_F [F(x) + \bar{F}(x)] + G(x)$ in the limit $x \sim 1 - \epsilon \rightarrow 1$. The explicit formulae can be found by inverting the Mellin transforms. A systematic method of inverting moments in the limit $x \rightarrow 1$ is discussed in the Appendix. Here we present only the leading asymptotic terms:

$$J_Q(x) = \frac{1}{\Gamma(2C_F\xi)} e^{a_F\xi} (1-x)^{2C_F\xi-1} [1 + o(1-x)], \quad (1.9a)$$

$$J_G(x) = \frac{1}{\Gamma(2C_G\xi)} e^{a_G\xi} (1-x)^{2C_G\xi-1} [1 + o(1-x)], \quad (1.9b)$$

where

$$\xi = \int_{\delta}^1 \frac{d\alpha}{\alpha} \frac{\alpha(P_0)}{\pi} \approx \frac{12}{2(33-2N_F)} \ln\left(\frac{\ln \frac{P}{\Lambda}}{\ln \frac{\delta P}{\Lambda}}\right), \quad (1.10)$$

$$a_F = C_F\left(\frac{3}{2} - 2\gamma\right),$$

$$a_G = C_G\left(\frac{11}{6} - 2\gamma\right) - \frac{2}{3}N_F T_F,$$

$$C_F = 4/3, C_G = 3, T_F = 1/2, \gamma = 0.577\ldots \text{ (Euler constant)}, N_F = f = \text{number of flavours.}$$

The Sterman-Weinberg cross-section can be easily derived using Eqs. (1.3) and (1.9a). We get

$$f_{SW}(P, \epsilon, \delta) = e^{2a_F\xi} \frac{(2\epsilon)^{4C_F\xi}}{\Gamma(1+4C_F\xi)} [1 + o(\epsilon)].$$

2. FIXED p_T JETS (TUBES)

The QCD formula for the SW cross-section would be the end of the story in the world of free quarks and gluons. In the real world, however, the parent parton, after some perturbative radiation resulting in the energy loss described by the formulae (1.9) will evolve into the fixed p_T hadronic jet. As seen from Fig. 2, there is always some part of the hadronic tube lying outside the δ cone. In other words, the hadronization process will result in some additional energy loss, modifying the perturbative formulae (1.9).

For the time being, we forget about the perturbative emission processes and consider only the hadronic effects.

Let us pose the following question: what is the probability $dP/dx \equiv J_N(x)$ that the quark with momentum P entering the calorimeter will produce a signal with momentum xP or, in other words, that the fraction x of the hadronic jet lies in the cone with the opening angle 2δ surrounding the jet? We will study this problem in the simplified independent emission model ("Uncorrelated Jet Model") [10] with exponential p_T cut-off and flat rapidity spectrum.

More explicitly, we write the probability distribution in the jet as follows

$$dP = c \sum_m \frac{\lambda^m}{m!} P(x_L) dx_L \prod_{i=1}^m \frac{d^3 p_i}{E_i} f(p_{Ti}) \delta\left(1 - x_L - \sum_{i=1}^m x_i\right), \quad (2.1)$$

where x_1, x_L are the longitudinal momentum fractions, $P(x_L)$ stands for the leading particle spectrum, $f(p_T) = (A^2/2\pi) e^{-Ap_T}$, and the normalization constant should be derived from $\int dP = 1$.

In order to carry out the explicit analytic calculations up to the end we introduce some simplifications relative to the orthodox uncorrelated jet model, considered in hadron physics. Firstly, because of the strong damping in p_T we neglect the transverse momentum conservation. Next, since we probe the fixed p_T jet by the fixed angle calorimeter we may neglect the masses of hadrons or, equivalently, we replace rapidity y by the pseudorapidity $\eta = -\ln \tg(\theta/2)$.

The single particle distribution in the considered model is given by

$$\frac{dN}{dy d^2p_T} = \lambda (1-x)^\lambda \frac{A^2}{2\pi} e^{-A p_T}, \quad \left(y = \ln \frac{E+p_L}{m_T} \right),$$

so that the parameter λ is fixed by the height of the hadronic rapidity plateau, whereas A is related to the mean transverse momentum according to $\langle p_T \rangle = 2/A$.

For the leading particle spectrum we take $P(x_L) \approx \text{const.}$, as motivated by hadron phenomenology. Also, we neglect the transverse momentum of the leading particle as an inessential correction for small values of ϵ and δ .

Finally, a comment should be made concerning the clustering effects. In order to do successful phenomenology in terms of the independent emission, one has to work with clusters instead of single hadrons in the formula (2.1) [11]. Certainly, it affects the numerical values of λ and $\langle p_T \rangle$, since, roughly speaking,

$$\lambda_{\text{CLUSTER}} \sim \frac{1}{N} \lambda_{\text{HADRON}}, \quad \langle p_T \rangle_{\text{CLUSTER}} \sim N \langle p_T \rangle_{\text{HADRON}},$$

where N is the mean multiplicity of hadrons per cluster. However, as will be seen in the following, our final formula for the SW cross-section will be (approximately) only a function of $\lambda \langle p_T \rangle$, so in a sense it does not matter if there are clusters of hadrons in the model. The physical origin of this effect is very clear: the result of the calorimeter measurement should be weakly dependent on the clustering effects.

The probability dP/dx [or, by analogy with the QCD calculation, the non-perturbative jet fragmentation function $J_N(x)$] defined above is given by the expression:

$$J_N(x) = c \int_0^1 dx_L \sum_{n=0}^{\infty} \frac{\lambda^n}{n!} \sum_{k=0}^n \binom{n}{k} \prod_{i=1}^k \int \frac{d^3 p_i}{E_i} f(p_{Ti}) \prod_{i=k+1}^n \int \frac{d^3 p_i}{E_i} f(p_{Ti}) \times$$

$$\times \delta\left(x - x_L - \sum_{i=1}^k x_i\right) \delta\left(1 - x_L - \sum_{i=1}^n x_i\right), \quad (2.2)$$

where the subscripts in (out) denote that the corresponding momenta should be integrated inside (outside) the δ cone (i.e. with $p_{T_i} > x_i P \delta$, $p_{T_i} < x_i P \delta$, respectively).

The formula (2.2) can be integrated by using the technique of the double Laplace transforms. Replacing 1 in the last δ function by x_0 , denoting the resulting formula by $J_N(x, x_0)$, and transforming in x and x_0 , we get

$$\begin{aligned} \varrho(s, s_0) &\equiv \int_0^\infty dx e^{-sx} \int_0^\infty dx_0 e^{-s_0 x_0} J_N(x, x_0) = \frac{c}{s+s_0} e^{\lambda [\xi_{IN}(s, s_0) + \xi_{OUT}(s_0)]}, \\ \xi_{IN}(s, s_0) &= \int_{in} \frac{d^3 p}{E} f(p_T) e^{-(xs + x s_0)}, \\ \xi_{OUT}(s_0) &= \int_{out} \frac{d^3 p}{E} f(p_T) e^{-x s_0}. \end{aligned} \quad (2.3)$$

The integrals (2.3) can be easily performed in the limit $\langle p_T \rangle / P \rightarrow 0$ and the final result can be expressed in terms of the inverse Laplace transforms:

$$J_N(x) = F_0(x) F_1(x),$$

where

$$F_0(x) = T(1+\lambda) \int_{-\infty}^{\infty} \frac{ds}{2\pi i} e^{xs} \frac{1}{s} \left(\frac{1+\tau s}{s} \right)^\lambda e^{\frac{\lambda s \tau}{1+\tau s}}, \quad (2.4a)$$

$$F_1(x) = \int_{-\infty}^{\infty} \frac{ds}{2\pi i} e^{(1-x)s} \left(\frac{1}{1+\tau s} \right)^\lambda e^{-\frac{\lambda \tau s}{1+\tau s}} \quad (2.4b)$$

and

$$\tau \equiv \frac{\langle p_T \rangle}{2P\delta}.$$

As seen from Eqs. (2.4), in the limit $P \rightarrow \infty$, we get

$$F_0(x) \rightarrow x^\lambda,$$

$$F_1(x) \rightarrow \delta(1-x),$$

so that

$$J_N(x) \rightarrow \delta(1-x)$$

as expected (full fixed p_T jet inside the δ cone). For finite energies, the corrections to the formula (2.4b) are controlled by $\tau/(1-x)$ and those to formula (2.4a) by τ/x . Since the limit relevant for the Sherman-Weinberg cross-section is: $\delta \gg \langle p_T \rangle / P$, i.e. τ small but also $1-x \sim \epsilon$ small, we will put simply $F_0(x) = 1$, but calculate explicitly $F_1(x)$ [more formally, we take the limit: $\tau \rightarrow 0$, $x \rightarrow 1$, $\tau/(1-x)$ fixed]. The integral (2.4b) can be easily derived and we get finally

$$J_N(x) = \frac{1}{\tau} e^{-\lambda} e^{-\frac{z}{\tau}} \left(\frac{z}{\lambda \tau} \right)^{\frac{\lambda-1}{2}} I_{\lambda-1} \left(2 \sqrt{\frac{\lambda z}{\tau}} \right), \quad (2.5)$$

where $I_{\lambda-1}(z)$ is the modified Bessel function and $z = 1-x$. Inserting the fragmentation function $J_N(x)$ into Eq. (1.3) we get after some algebra the following formula for the Sterman-Weinberg cross-section:

$$f_{SW}(P, \epsilon, \delta) \equiv f_N(P, \epsilon, \delta) = \sum_{n=0}^{\infty} \frac{(2\lambda)^n}{n!} e^{-2\lambda} \frac{\gamma(n+2\lambda; \frac{2\epsilon}{\tau})}{\Gamma(n+2\lambda)}, \quad (2.6)$$

where $\gamma(\alpha; x)$ is the incomplete gamma function. Since $\gamma(\alpha; x) \rightarrow \Gamma(\alpha)$ for $x \gg \alpha$ and the essential contribution to the sum in Eq. (2.6) comes from $n \sim 2\lambda$ (mean value of the Poisson distribution) we conclude that the non-perturbative effects vanish (i.e. $f_{SW} \rightarrow 1$) when

$$\tau_N \equiv \frac{\lambda \langle p_T \rangle}{P \epsilon \delta} \ll 1. \quad (2.7)$$

A nice feature of the function $f_N(P, \epsilon, \delta)$ is that, to a reasonable approximation, it is only a function of τ_N , i.e. $f_N(P, \epsilon, \delta) \approx f_N(\tau_N)$. It can be seen in Fig. 3, where we plot $f_N(P, \epsilon, \delta)$ versus τ_N for some values of λ . [Thus, as discussed before, the formula (2.6) is only weakly dependent on the clustering effects].

The relevance of the parameter τ_N for the calorimeter signal produced by the fixed p_T jet can be illustrated as follows: the mean fraction of the jet energy, lying outside the δ cone is given approximately by

$$\langle x \rangle_{\text{out}} = \frac{1}{P} \int_{\text{out}} \frac{dN}{dy d^2p_T} p_T \ln y dy d^2p_T \approx \frac{\langle p_T \rangle}{P} \int_0^{-\ln \frac{\delta}{2}} \frac{dN}{d\eta} \ln \eta d\eta \approx \frac{\lambda \langle p_T \rangle}{P \delta}.$$

On the other hand, one can observe the fraction $1 - \varepsilon$ inside the cone without biasing the jet only if $\langle x \rangle_{\text{out}} \sim \varepsilon$. When $\varepsilon < \langle x \rangle_{\text{out}}$, i.e. $\tau_N > 1$, we get damping since we force the unfavoured momentum configuration.

3. REALISTIC JETS

In this section we combine the perturbative and non-perturbative effects in order to derive the full calorimeter signal produced by the realistic jet.

The simplest possible ansatz for the full fragmentation function, being an explicit realization of our pictorial understanding of the hadronization process, reads:

$$J(z) = \int_0^1 dz_1 J_P(z_1) \int_0^1 dz_2 J_N(z_2) \delta(z - z_1 z_2), \quad (3.1)$$

where the perturbative and non-perturbative fragmentation functions were derived in preceding sections [formulae (1.8), (2.5)]^{*}.

In other words, we treat both components as acting independently: the high-energy quark "first" reduces the energy by perturbative radiation and "then" converts into hadrons, leaving a part of its hadronization component outside the detector according to the function $J_N(z)$.

^{*}) Strictly speaking, the formula (3.1) is a little bit more complicated as far as the z_1 dependence is concerned, since the function $J_N(z_2)$ depends also on the total jet energy $P' = z_1 P$. However, the perturbative contribution is peaked near $z_1 \approx 1$, so to a reasonable approximation we may neglect this complication putting simply $P' \approx P$.

Certainly, the perturbative and non-perturbative contributions can be distinguished as in Eq. (3.1) only if

$$\delta \gg \frac{\langle p_T \rangle}{P} \quad (3.2)$$

since otherwise the perturbative effects are out of control (formally, the effective coupling constant diverges). However, as will be seen in the next section, even if the condition (3.2) is fulfilled, we get typically $\tau_N > 1$ at present energies. Thus, being apparently in the perturbative region, we get essential corrections from the fixed p_T effects.

Inserting the formula (3.1) into (1.3) we get after some algebra the following result (up to terms of order ϵ) for the Stermann-Weinberg cross-section:

$$f_{SW}(P, \epsilon, \delta) = f_N(P, \epsilon, \delta) f_P(P, \epsilon, \delta) [1 + O(\tau_N \xi)] \quad (3.3)$$

Thus, apart from the deep non-perturbative region $\tau_N \xi \gg 1$ [where, as discussed above, the ansatz (3.1) is meaningless] the QCD and fixed p_T effects factorize in the final formula.

4. NUMERICAL ESTIMATES

In this section we present the results of the numerical analysis of the formula (3.3).

Let us start with the observation that for the present energies the asymptotic condition

$$\tau_N = \frac{\lambda \langle p_T \rangle}{P \epsilon \delta} \ll 1 \quad (4.1)$$

defining the region when the fixed p_T corrections can be neglected, is badly violated. Taking

$$\langle p_T \rangle = 400 \text{ GeV}$$

$$\lambda = 3 \text{ (typical value known from hadron phenomenology)}$$

$$P = 15 \text{ GeV (PETRA)}$$

$$\varepsilon = 0.3$$

$$\delta = 0.1$$

we get $\tau_N = 2.66$, $f_P = 0.65$, $f_N = 0.015$. Thus the dominant contribution comes from the fixed p_T effects. In other words, the angle $\delta = 0.1$ is definitely too small in PETRA to see perturbative signal, since almost all energy of the fixed p_T jet lies outside the δ cone. On the other hand, the naive estimation of the allowed values of δ

$$0.1 = \delta \gg \frac{\langle p_T \rangle}{P} = 0.0266 \quad (4.2)$$

does not indicate any trouble.

Comparing the two estimates (4.1) and (4.2) we see that the hadronic effect, going down with energy like $\langle p_T \rangle / P$ is numerically enhanced at finite energies by the relatively large value of density of hadrons in rapidity as well as by $1/\varepsilon$. The latter effect is due to the flat rapidity distribution: the substantial part of the jet is concentrated at relatively low momenta where the p_T cut-off is less efficient in damping the large-angle emission. Thus we see that the trouble is caused by simple geometrical mismatch between the cone and the tube; there is always a part of the tube, lying outside the cone. Since the energy distribution in the tube is highly non-uniform, a substantial part of it misses the cone.

The situation does not change essentially if we increase δ or ε , since then both perturbative and fixed p_T effects decrease. It can be seen in Fig. 4, where we plot the functions f_P (dashed line) and f_N (solid lines) for some values of λ versus δ in the PETRA energy region. More generally, there is no choice of ε and δ for which the perturbative effect could be observed without essential fixed p_T background in PETRA.

There is only one way out -- to increase energy. In other words, there is some "threshold" energy, below which no clean perturbative signal can be seen.

To illustrate this effect, let us note that, as seen from Eq. (1.9a), the effective perturbative parameter can be written as

$$\tau_P = 4C_F \xi \ln \frac{1}{2\epsilon} ,$$

so that $f_P \sim e^{-\tau_P}$. Taking for simplicity $\Lambda \sim \langle p_T \rangle \sim 0.4$ GeV and denoting $\gamma = \Lambda/P$ we can write

$$\tau_P \approx \frac{24}{25} C_F \frac{(\ln \frac{1}{\delta})(\ln \frac{1}{2\epsilon})}{(\ln \frac{1}{\gamma})} , \quad \tau_N = 2\lambda \frac{(\frac{1}{\delta})(\frac{1}{2\epsilon})}{(\frac{1}{\gamma})}$$

The QCD effects start to dominate over the non-perturbative background when $\tau_P > \tau_N$, which gives:

$$\gamma \ln \frac{1}{\gamma} < \frac{12}{25} \frac{C_F}{\lambda} (\delta \ln \frac{1}{\delta}) (2\epsilon \ln \frac{1}{2\epsilon}) . \quad (4.3)$$

For given energy, the inequality (4.3) defines a closed contour in the (ϵ, δ) plane, inside which the QCD signal dominates. However, since the quantity $x \ln(1/x)$ is bounded by $1/e$, the solution of Eq. (4.3) exists only if

$$\gamma \ln \frac{1}{\gamma} < \frac{12}{50} \frac{2C_F}{\lambda} \left(\frac{1}{e}\right)^2$$

which gives the absolute lower bound $P_{\min}$ for energy. For $\lambda = 1, 2, 3$ we get, respectively, $P_{\min} = 22$ GeV, 50 GeV, 91 GeV^{*}). Thus, if $\lambda \sim 3$, as suggested by the hadronic phenomenology, we should not expect any clean QCD signal below the LEP energy region.

^{*}) Certainly, these estimates should be taken with caution, since we have neglected constant terms relative to $\ln(1/\delta)$, $\ln(1/\epsilon)$. Consequently, the perturbative effect does not vanish exactly for $\delta, \epsilon \rightarrow 1$, but tends to some small constant of order of $\alpha(P)$ which may affect slightly the numerical values of $P_{\min}$.

The situation in LEP is presented in Fig. 5. We see that even here the perturbative effects are visible without any corrections only in a very restricted interval of δ .

A very transparent picture of relative importance of perturbative and non-perturbative effects in the Sterman-Weinberg cross-section emerges when one looks at the energy dependence of the perturbative angle $\delta_p(P, f, \epsilon)$, defined as the solution of the equation

$$f_{SW}(P, \epsilon, \delta_p) = f.$$

Since f_p (f_N) depends on P and δ through the variable $\ln \delta / \ln P$ ($P\delta$), in the perturbative case we get $\delta_p \sim (\Lambda/P)^{c(f, \epsilon)}$ ($c < 1$), whereas for the fixed p_T jet $\delta_p \sim c'(f, \epsilon)(\langle p_T \rangle / P)$.

Therefore on the $[\ln(1/\delta), \ln(P/\Lambda)]$ plot we expect to see the set of straight lines parametrized by f and ϵ .

By changing f and ϵ the perturbative lines change slopes (all less than 1), whereas the fixed p_T lines (all with slope 1) are shifted.

In Figs. 6a-c we plot the perturbative angle δ_p versus energy for some values of λ . The dashed (dotted) lines correspond to QCD (fixed p_T) jets; the solid line is the result for the realistic jet.

We see that the genuine asymptotics is reached for extremely high energies, especially when we want to see a sizeable QCD signal and that the weak perturbative effects become visible in the LEP energy region.

5. "HADRONIZATION-FREE" QUANTITIES

The results of the numerical analysis presented in the preceding section indicate that at present energies the QCD effects are still a very small correction to the non-perturbative background. Consequently, the quantities like f_{SW} or perturbative angle, proposed for QCD tests in calorimeter experiments, are useless in the available energy region.

In the meantime, before the next generation of high-energy machines start to operate, it would be useful to invent more refined methods of analysing the calorimeter data less sensitive to the hadronic effects. We devote this section to the discussion of some possibilities of this type.

Our idea is very simple and follows directly from the formula (3.2); since the non-perturbative background (approximately) factorizes and depends on P and δ only through the product $P\delta$, the ratios

$$r_\epsilon = \frac{f_{SW}(P_1, \epsilon, \delta_1)}{f_{SW}(P_2, \epsilon, \delta_2)} \Big|_{P_1 \delta_1 = P_2 \delta_2} \quad (5.1)$$

should be free of hadronic effects even at present energies. Note that for pure fixed p_T jets we have $r_\epsilon \equiv 1$, so that any energy dependence of r_ϵ provides direct evidence for the perturbative component.

To the end of this section we want to propose another calorimeter measurement, conceptually even simpler than the one proposed by Sterman and Weinberg and such that the corresponding "hadronization-free" quantity is less sensitive to various approximations done in the calculations presented before.

Let us extract two general properties of the relative interplay of QCD and fixed p_T effects, following from previous discussion:

- a) the jet fragmentation function is the convolution of the corresponding quantities $J_P(x)$, $J_N(x)$ for QCD and fixed p_T components [e.g. Eq. (3.3)];
- b) the perturbative function $J_P(x)$ depends both on P and δP [e.g. Eq. (1.10)], whereas $J_N(x)$ is only a function of $P\delta$.

The latter fact has a very clear physical interpretation: the loss of energy inside the detector is related to the emission going outside and therefore for both types of jets the effect is controlled by the integral

$$\int_{\delta P}^P dp_T F(p_T) \quad (5.2)$$

where $F(p_T)$ describes the transverse momentum distribution to the jet axis. For the fixed p_T jet the integral (5.2) is insensitive to the upper limit because of the strong damping of large transverse momenta. Consequently the function $J_N(x)$ depends only on δP in any reasonable model for the fixed p_T component. In the perturbative case, the corresponding integral (5.2) diverges at large momenta [for QCD we have $F(p_T) \sim \alpha(p_T)/p_T$] and therefore the function $J_P(x)$ depends both on P and δP [see, for example, expression (1.10)].

The "hadronization-free" quantity we propose is defined as follows:

- i) Measure the single jet cross-section $d\sigma/dx d\Omega$.
- ii) Take moments ^{*)}

$$\rho_H(P, \delta) = \frac{1}{\sigma_0(P)} \int d\Omega \int_0^1 dx x^{M-1} \frac{d\sigma}{dx d\Omega}.$$

- iii) Calculate the ratio

$$r_H(P, \delta, \eta) = \frac{\rho_H(P, \delta)}{\rho_H(\eta P, \delta/\eta)}.$$

As follows from (a) and (b), the quantity r_M has the following properties:

- a) $r_M(P, \delta, \eta) \equiv 1$ if there is no perturbative component in the jet;
- b) consequently, $r_M(P, \delta, \eta)$ is free of fixed p_T corrections and gives direct information about the perturbative content of the jet. By taking moments, the convolution (3.1) factorizes and we get

$$r_H(P, \delta, \eta) = \frac{J_Q^{(n)}(P, \delta)}{J_Q^{(M)}(\eta P, \delta/\eta)}$$

where $J_Q^{(M)}(P, \delta) = \int dx x^{M-1} J_Q(P, \delta, x)$ and J_Q is the QCD jet fragmentation function;

*) In order to minimize the error made by approximation mentioned in the first footnote to this paper, one should take relatively large values of M .

c) for small δ and large M one can use the approximate formula given by Eq. (1.9a).

The QCD prediction for $r_M(P, \delta, \eta)$ reads in this case:

$$r_M(P, \delta, \eta) \cong \eta^{2C_F \frac{\alpha(P)}{\pi} \ln M} \quad (5.3)$$

Note that in this particular limit r_M does not depend on δ at all. It follows simply from the fact that in the leading log approximation the integral (5.2) exponentiates and the dependence on the lower limit, kept constant by rescaling $P, \delta \rightarrow \eta P, \delta/\eta$, cancels in the ratio r_M . Consequently, the integral (5.2) is tested only in the momentum region $(\eta P, P)$. (It suggests that the quantity r_M is weakly sensitive to the low momentum part of the function $F(p_T)$ and, hopefully, also to the non-leading corrections, even for moderate values of δ and M).

The variation of r_M with energy should be visible already in the PETRA energy region. In Fig. 7 we plot r_M versus $Q = 2P\eta$ for $P = 15$ GeV, $\delta = 0.1$ and η in the interval $(0.5, 1)$. As seen in Fig. 7, a 30% effect is expected for reasonable values of M .

6. CONCLUSIONS

We have analysed the relative importance of the QCD and fixed p_T jets in the calorimeter measurements at present energies. Our conclusions can be summarized as follows:

- a) The hadronic corrections are very strong in the PETRA region and non-negligible even in the LEP region;
- b) We have estimated an absolute lower bound $P_{\min}$ (with respect to ϵ, δ) for the energies $P = \frac{1}{2}\sqrt{s}$, where the QCD effects start to be greater than the fixed p_T background, to be of the order of 90 GeV. Below this energy region many of the proposed tests of QCD in calorimeter experiments are useless;

- c) We have proposed a method of extracting the perturbative signal from the single-arm calorimeter data at present energies. By changing simultaneously the c.m. energy and the opening angle δ of the detector so that δP remains fixed, one can construct quantities (ratios of x moments) which are insensitive to the hadronic effects even at present energies. A 30% QCD effect is expected in PETRA.

APPENDIX

We present here the analytic solution of the master equations (1.6) in the region $x \rightarrow 1$ in the form of the series expansion.

The Eqs. (1.7) for moments can be solved in the standard way and we get (omitting the subscript s for simplicity):

$$S(\tau) = E_{FF}(\tau) S(0) + E_{GF}(\tau) G(0), \quad (A.1)$$

$$G(\tau) = E_{FG}(\tau) S(0) + E_{GG}(\tau) G(0),$$

where

$$S(\tau) = \sum_F \{ F(\tau) + \bar{F}(\tau) \},$$

$$E_{FF} = \left(1 - \frac{D}{A}\right) e^{\lambda_F \xi} + \frac{D}{A} e^{\lambda_G \xi},$$

$$E_{FG} = \frac{A_{FG}}{A} (e^{\lambda_F \xi} - e^{\lambda_G \xi}),$$

$$E_{GF} = \frac{2f A_{GF}}{A} (e^{\lambda_F \xi} - e^{\lambda_G \xi}),$$

$$E_{GG} = \frac{D}{A} e^{\lambda_F \xi} + \left(1 - \frac{D}{A}\right) e^{\lambda_G \xi},$$

$$A = \sqrt{(A_{GG} - A_{FF})^2 + 8f A_{FG} A_{GF}},$$

$$D = \frac{1}{2} [A_{GG} - A_{GF} + A],$$

$$\lambda_F = A_{FF} + D,$$

$$\lambda_G = A_{GG} - D,$$

$$\xi = \int_0^1 \frac{d\theta}{\theta} \frac{\alpha(\theta)}{\pi},$$

$$f = \text{number of flavours},$$

the boundary conditions are: $S(0) = 1$, $G(0) = 0$ for the quark jet, $S(0) = 0$, $G(0) = 1$ for the gluon jet.

The main difficulty with the analytic inverting of Eqs. (A.1) is due to the fact that the position of the branch point produced by the square root $\sqrt{(A_{GG} - A_{FF})^2 + 8f A_{FG} A_{GF}}$ cannot be found analytically. However, since for $x \rightarrow 1$ large values of s are relevant and

$$A_{FF} - A_{GG} = a \ln s + b + o\left(\frac{1}{s}\right)$$

where

$$a = 2(c_G - c_F), \quad b = a_F - a_G$$

whereas $A_{FG} A_{GF} \sim O(1/s^2)$, we may expand the square root in powers of $1/u$, where $u = a \ln s + b$.

Let us denote by $E_{AB}^{(J)}$ ($A, B, J = F, G$) the part of the function E_{AB} in Eq. (A.1), proportional to $e^{\lambda_J \xi}$ ($J = F, G$), so that $E_{AB} = E_{AB}^{(F)} + E_{AB}^{(G)}$. One can easily verify that, after expanding the square-root, each of the functions $E_{AB}^{(J)}$ can be written as follows:

$$E_{AB}^{(J)}(s) = \left(\frac{1}{s}\right)^{2c_J \xi} e^{a_J \xi} \sum_{n=0}^{\infty} \sum_{m=0}^n E_{AB}^{(J)}(n, m; \xi) \left(\frac{1}{s}\right)^n \left(\frac{1}{n!}\right)^m \quad (A.2)$$

where $E_{AB}^{(J)}(n, m; \xi)$ are the polynomials of order n in the variable ξ .

The formula (A.2) can be easily inverted into the x space. Denoting

$$y = \ln \frac{1}{x}, \quad z = \frac{1}{\ln \frac{1}{y} + \frac{b}{a}}$$

we get

$$E_{AB}^{(J)}(x) = \int_{-\infty}^{\infty} \frac{ds}{2\pi i} e^{ys} E_{AB}^{(J)}(s) = \frac{y^{2c_J \xi - 1}}{\Gamma(2c_J \xi)} \sum_{n=0}^{\infty} y^n E_{AB}^{(J)}(n; z) \quad (A.3)$$

where

$$E_{AB}^{(j)}(m; z) = \frac{\Gamma(2C_J \xi)}{\Gamma(2C_J \xi + m)} \sum_{n=0}^m E_{AB}^{(j)}(m, n; \xi) \left(\frac{z}{a}\right)^m L_m(2C_J \xi + n; z). \quad (A.4)$$

The functions $L_m(\alpha, z)$ are defined as follows:

$$L_0(\alpha, z) \equiv 1, \\ L_m(\alpha, z) = \int_0^\infty d\lambda \lambda^{m-1} e^{-\lambda} \frac{\Gamma(\alpha)}{\Gamma(m) \Gamma(\alpha + \lambda z)}$$

and obey the recurrence relation

$$L_{m+1}(\alpha+1, z) = \frac{\alpha}{z m} \left[L_m(\alpha, z) - L_m(\alpha+1, z) \right]. \quad (A.5)$$

Thus, as seen from Eq. (A.3), after extracting the leading term $y^{2C_J \xi - 1} / \Gamma(2C_J \xi) e^{aJ\xi}$, the remaining part can be expanded in the power series in y : the coefficients of this series have still some weak (logarithmic) y dependence and can be expressed [owing to relation (A.5)] by one transcendent function

$$L_1(\alpha, z) = \int_0^\infty dx e^{-x} \frac{\Gamma(\alpha)}{\Gamma(\alpha + x z)}.$$

The polynomials $E_{AB}^{(j)}(n, m; \xi)$ for $n \leq 2$ are presented in Table 1.

Let us note, at the end of this technical remark, that if only the coefficients $E(n, m; \xi)$ do not blow up exponentially with n , the resulting series (A.3) is likely to be rapidly convergent even for finite values of $x (x \neq 0)$, because of the converging factor $\Gamma(2C_J \xi) / \Gamma(2C_J \xi + n)$ in front of expression (A.4).

Finally, since the master equations (1.7) are formally identical with the corresponding equations for the Q^2 dependence of the hadronic structure functions the method presented above can be used without any changes in order to derive exactly the analogous QCD predictions in the deep inelastic leptonproduction, once the input at some Q_0^2 is given.

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REFERENCES

- [1] G. Sterman and S. Weinberg, Phys. Rev. Lett. 39 (1977) 1436.
- [2] A. De Rújula, J. Ellis, E.G. Floratos and M.K. Gaillard, Nucl. Phys. B138 (1978) 387.
- [3] For lowest order calculations, see for example,
W. Furmański, Phys. Lett. 77B (1978) 312 (large p_T jets and probabilistic method);
P.M. Stevenson, Phys. Rev. Lett. 41 (1978) 787 [$O(\epsilon)$ corrections];
K. Shizuya and S.-H. Tye, Fermilab Pub. 78/54 THS (1978);
M.B. Einhorn and B.G. Weeks - SLAC-PUB-2164 (1978) (gluon jets);
P. Binetruy and G. Girardi, preprint CERN TH.2617 (exact lowest order result).
- [4] R.K. Ellis and R. Petronzio, Phys. Lett. 80B (1979) 249.
- [5] K. Konishi, A. Ukawa and G. Veneziano, preprint CERN TH.2577 (1978).
- [6] W. Furmański, talk presented at Symposium on Jets in High Energy Collisions, Copenhagen 10-14 July, 1978.
- [7] G. Curci and M. Greco, Phys. Lett. 79B (1978) 406.
- [8] G. Parisi and R. Petronzio, preprint CERN TH.2603 (1978).
- [9] G. Parisi, *in* Proc. 11^{ème} Rencontre de Moriond (1978), (ed. J. Tran Thanh Van);
G. Altarelli and G. Parisi, Nucl. Phys. B126 (1977) 298.
- [10] L. Van Hove, Nuovo Cimento 28 (1963) 788 and Rev. Mod. Phys. 36 655 (1964).
- [11] S. Pokorski and L. Van Hove, Acta Phys. Pol. B5 (1974) 229.

Table 1

Polynomials $E_{XY}^{(Z)}(n,m;\xi)$ for $n \leq 2$

$E_{XY}^{(Z)}$ n,m	0,0	1,0	1,1	2,0	2,1	2,2
$E_{FF}^{(F)}$	1	$\frac{4\xi}{3}$	0	$\frac{7\xi}{6} + \frac{8\xi^2}{9}$	$\frac{4f\xi}{3}$	$-\frac{4f}{3}$
$E_{FF}^{(G)}$	0	0	0	0	0	$\frac{4f}{3}$
$E_{FG}^{(F)}$	0	0	$\frac{4}{3}$	0	$\frac{4}{3} - \frac{16\xi}{9}$	$-\frac{20}{9}$
$E_{FG}^{(G)}$	0	0	$-\frac{4}{3}$	0	$-\frac{4}{3} + 4\xi$	$\frac{20}{9}$
$E_{GF}^{(F)}$	0	0	f	0	$-2f - \frac{4\xi}{3}$	$\frac{5f}{3}$
$E_{GG}^{(G)}$	0	0	-f	0	$2f + 3\xi$	$-\frac{5f}{3}$
$E_{GG}^{(F)}$	0	0	0	0	0	$\frac{4f}{3}$
$E_{GG}^{(G)}$	1	-3ξ	0	$\frac{25\xi}{2} + \frac{9\xi^2}{2}$	$-\frac{4f\xi}{3}$	$-\frac{4f}{3}$

Figure captions

- Fig. 1 : Experimental set-up for the single-arm calorimeter measurement in e^+e^- annihilation. (The final state, presented in the figure should give two signals in the detector.)
- Fig. 2 : Geometrical illustration of the origin of large non-perturbative corrections at non-asymptotic energies.
- Fig. 3 : Function $f_{SW}(P, \epsilon, \delta)$ for the fixed p_T jet versus τ_N for some values of λ .
- Fig. 4 : Comparison of perturbative and fixed p_T effects in PETRA for some values of λ .
- Fig. 5 : Comparison of perturbative and fixed p_T effects in IEP for some values of λ .
- Fig. 6 : Energy dependence of the perturbative angle δ_p for some values of λ . Dotted (dashed) lines correspond to fixed p_T (QCD) jets; the solid line is the solution for the realistic jet.
- Fig. 7 : Energy (η) dependence of the ratio $r_M(P, \delta, \eta)$ in the PETRA energy region: $P = 15$, $\delta = 0.1$, $0.5 < \eta < 1$, $M = 4, 8, 12, 16, 20$.

Fig. 1

Fig. 2

Fig. 3

Fig. 4

Fig. 5

Fig. 6 a)

Fig. 6 b)

Fig. 6 c)

Fig. 7