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MEASURING THE P ODD ASYMMETRY IN ELECTROPRODUCTION
OF THE $\Delta(\frac{3}{2}, \frac{3}{2})$ RESONANCE AT LOW ENERGIES

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A B S T R A C T

We show that measurements of the P odd asymmetry in the reaction $e^- + N \rightarrow e^- + \Delta(\frac{3}{2}, \frac{3}{2})$ at low energies might determine the combination $(\tilde{\alpha} + (1.0 \div 1.2)\tilde{\beta})$ of the isovector phenomenological couplings, characterizing the parity non-conserving electron quark neutral current interaction at presently accessible energies. They may provide also a very accurate determination of the parameter $\sin^2\theta_W$ of the standard electro-weak gauge theory.

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Almost two years have passed since the existence of a weak e^- -N interaction was established in the polarization experiments performed at Novosibirsk ¹⁾ and, on a different energy scale, at SLAC ²⁾. Additional information about the electron quark neutral current couplings which are believed to generate effectively the observed interaction was obtained in this period by studying the y dependence of the inelastic e^- -D polarization asymmetry ³⁾. A positive effect has also been observed recently in the polarization experiments with thallium atoms ⁴⁾ and a more accurate result has been reported by the Novosibirsk group ⁵⁾. The experimental observations are in good agreement with the predictions of the standard electroweak gauge theory ⁶⁾.

There still exists, however, the phenomenological problem of a gauge model independent determination of the four parameters $(\tilde{\alpha}, \tilde{\beta}, \tilde{\gamma}, \tilde{\delta})$ characterizing the parity violation neutral current phenomena in e^- -N interactions at presently accessible energies. These parameters are usually defined as coupling constants in the effective electron-quark interaction Lagrangian :

$$\begin{aligned} \mathcal{L}_{e-q}^{PV} = & - \frac{G}{\sqrt{2}} \left\{ \bar{e} \gamma_\mu \gamma_5 e \left[\frac{1}{2} \tilde{\alpha} (\bar{u} \gamma_\mu u - \bar{d} \gamma_\mu d) + \right. \right. \\ & + \frac{1}{2} \tilde{\gamma} (\bar{u} \gamma_\mu u + \bar{d} \gamma_\mu d) \left. \right] + \bar{e} \gamma_\mu e \left[\frac{1}{2} \tilde{\beta} (\bar{u} \gamma_\mu \gamma_5 u - \bar{d} \gamma_\mu \gamma_5 d) \right. \\ & \left. \left. + \frac{1}{2} \tilde{\delta} (\bar{u} \gamma_\mu \gamma_5 u + \bar{d} \gamma_\mu \gamma_5 d) + \dots \right] \right\} \end{aligned} \quad (1)$$

where the dots denote isoscalar axial heavy quark (s, c, ...) currents.

As is well known, the model independent approach has been successfully applied in the study of ν_μ -q (q=u, d) and ν_μ -e neutral current couplings which are known today with relatively good accuracy, and minor ambiguities ⁷⁾.

Using the data on the parity non-conserving transitions in bismuth ¹⁾ and thallium ⁴⁾ together with the experimental results obtained at SLAC ^{2), 3)}, it is possible to deduce ⁸⁾ :

$$\tilde{\alpha} = -0.72 \pm 0.25 \quad (2)$$

$$\tilde{g} = 0.36 \pm 0.28 \quad (3)$$

$$\tilde{\beta} + \frac{1}{3}\tilde{\delta} = 0.31 \pm 0.51 \quad (4)$$

So, a more accurate determination of $\tilde{\alpha}$ and $\tilde{\gamma}$ is desirable ^{*)} while $\tilde{\beta}$ and $\tilde{\delta}$ are practically unconstrained at present. Fixing $\tilde{\beta}$ and $\tilde{\delta}$ experimentally would allow one to test, e.g., the factorization relations between ν_{μ} -e, ν_{μ} -q and e-q couplings which take place if the neutral current interactions are mediated by a single Z boson ⁹⁾. First evidence in favour of this possibility has been found ⁸⁾ on the basis of the existing data.

Additional information about the parameters in $\mathcal{L}_{e-q}^{PV}$ may be obtained, in particular, by measuring the P odd asymmetries arising in the scattering of longitudinally polarized low energy electrons ($0.2 \text{ GeV} \lesssim E \lesssim 0.8 \text{ GeV}$) on unpolarized nucleons ^{**)} . The case of elastic scattering has been analyzed from this point of view recently ¹²⁾. In this note we present an analogous analysis for the case of excitation of the $\Delta(\frac{3}{2}, \frac{3}{2})$ resonance :

$$e^- + N \rightarrow e^- + \Delta \quad (5)$$

The leading contribution to the P odd asymmetries gives the interference between the weak parity non-conserving and the electromagnetic amplitudes. Since only the isovector part of the Lagrangian (1) contributes to the amplitude of the process (5), measurements of the P odd asymmetry in this process might give information about the parameters $\tilde{\alpha}$ and/or $\tilde{\beta}$. Our calculations show that at low energies they might determine the combination $(\tilde{\alpha} + (1.0 \pm 1.2)\tilde{\beta})$ and therefore $\tilde{\beta}$. Experiments of this kind, performed at high energies ($E \gtrsim 5 \text{ GeV}$) and low momentum transferred ($q^2 \ll 2ME$, M is the nucleon mass), would be sensitive only to the parameter $\tilde{\alpha}$ ¹³⁾.

^{*)} A better determination of $\tilde{\alpha}$ and $\tilde{\gamma}$ is possible if only the Novosibirsk and SLAC data are used as input.

^{**)} High precision experiments in which elastic scattering asymmetries $\sim 10^{-6}$ are planned to be detected are in preparation at present ^{10), 11)}.

The present analysis is based primarily on the existing data on Δ production by photons, electrons and neutrinos and some classical theoretical assumptions (e.g., the validity of PCAC). We use also the specific predictions of the Adler model ¹⁴⁾ for the form of the axial vector N - Δ transition form factors at low q^2 , the validity of which has been widely checked experimentally ¹⁵⁾⁻¹⁷⁾.

Before going into detailed discussion let us recall that in the standard $SU(2) \otimes U(1)$ gauge theory which successfully describes the neutral current data ($\sin^2 \theta_W \simeq 0.23$) we have:

$$\begin{aligned} \tilde{\alpha}^{ST} &= -(1 - 2 \sin^2 \theta_W), \quad \tilde{\gamma}^{ST} = \frac{2}{3} \sin^2 \theta_W, \\ \tilde{\beta}^{ST} &= -(1 - 4 \sin^2 \theta_W), \quad \tilde{\delta}^{ST} = 0. \end{aligned}$$

We shall consider the production of the Δ resonance as a production of spin $\frac{3}{2}$ elementary particle described by Rarita-Schwinger vector-spinor field ^{18),19)}. The matrix elements of the weak isovector vector V_λ^3 and axial vector A_λ^3 neutral currents between the nucleon and Δ resonance states are each characterized in this case by four real form factors ^{*)}:

$$\begin{aligned} \langle \Delta(p') | V_\lambda^3 | N(p) \rangle &= i N_0 \bar{U}_\nu(p') \left\{ \delta_{\nu\lambda} \left(\frac{M_\Delta + M}{M} C_3^V - \right. \right. \\ &- \frac{(p' \cdot q)}{M^2} C_4^V - \frac{(p q)_\lambda}{M^2} C_5^V + C_6^V \Big) + i p_\nu \gamma_\lambda \left(- \frac{C_3^V}{M} \right) + p_\nu (p + p')_\lambda \left(- \frac{C_4^V + C_5^V}{2M^2} \right) - \\ &\left. \left. - p_\nu q_\lambda \left(\frac{C_4^V - C_5^V}{2M^2} \right) \right\} \gamma_5 u(p), \end{aligned} \quad (6)$$

$$\begin{aligned} \langle \Delta(p') | A_\lambda^3 | N(p) \rangle &= i N_0 \bar{U}_\nu(p') \left\{ \delta_{\nu\lambda} \left(- \frac{M_\Delta - M}{M} C_3^A + \right. \right. \\ &+ \frac{(p' \cdot q)}{M^2} C_4^A - C_5^A \Big) + i p_\nu \gamma_\lambda \frac{C_3^A}{M} + p_\nu (p + p')_\lambda \frac{C_4^A}{2M^2} - p_\nu q_\lambda \left(- \frac{C_4^A + 2C_6^A}{2M^2} \right) \Big\} u(p) \end{aligned} \quad (7)$$

^{*)} We assume time-reversal invariance.

Here p and p' are the momenta of the nucleon and Δ resonance, respectively, $q = p' - p = k - k'$ (k and k' are the initial and final electron momenta), $\bar{U}_\nu(p')$ is the field describing the Δ resonance in the final state, M_Δ is the Δ resonance mass, N_0 is a standard normalization factor and finally, the form factors $C_i^{A,V}$ ($i=3, \dots, 6$) are functions of q^2 *). The matrix elements are written in a form ^{19), 20)} convenient for the present calculations. We use form factors ²¹⁾ in terms of which the existing data are usually interpreted.

The isotriplet vector current (CVC) hypothesis implies that the vector form factors coincide with the electromagnetic $N-\Delta$ transition form factors. It has been established experimentally ^{22), 23)} that the electromagnetic excitation of Δ is dominated by the magnetic dipole $N-\Delta$ coupling and that the electric quadrupole and the longitudinal quadrupole couplings are considerably weaker and can be neglected. Interpreted in terms of the form factors $C_i^V(q^2)$, these results mean :

$$C_5^V(q^2) \approx 0, \quad C_4^V(q^2) \approx -\frac{M}{M_\Delta} C_3^V(q^2) \quad (8)$$

which is in accord, in particular, with the quark model picture of the process ^{15), 21)}. Using Eqs. (8) and the fact that $C_6^V(q^2) = 0$ as a consequence of the conservation of the vector current, it was possible to extract from the data the q^2 dependence of $C_3^V(q^2)$ ²²⁾. For $q^2 \lesssim 0.5 \text{ GeV}^2$ it can be written in the form :

$$|C_3^V(q^2)| \approx 2.05 \left(1 + \frac{q^2}{0.54 \text{ GeV}^2}\right)^{-2} \quad (9)$$

The axial form factors in Eq. (7) are related through an isospin transformation to the axial form factors, describing the neutrino production of Δ **):

*) The following properties of the Rarita-Schwinger field are used to find the independent structures in the considered matrix elements :

$$(p' - iM_\Delta)U_\nu(p') = 0, \quad p'_\nu U_\nu(p') = 0, \quad \gamma_\nu U_\nu(p') = 0.$$

**) Note that in the analogous relations given in Refs. 15), 21), the negative signs have been omitted.

$$C_i^A(e^-p \rightarrow e^- \Delta^+) = C_i^A(e^-n \rightarrow e^- \Delta^0) = -C_i^A(\nu_\mu n \rightarrow \mu^- \Delta^+) = -\frac{1}{\sqrt{3}} C_i^A(\nu_\mu p \rightarrow \mu^- \Delta^{++}) \quad (10)$$

Among the models of neutrino production of Δ the most successful is the one ¹⁴⁾ proposed by Adler and we shall use it in our calculations ^{*)}. A convenient parametrization of the Adler model specific predictions for the axial form factors $C_i^A(e^-N \rightarrow e^- \Delta)$ was given for $q^2 \lesssim 0.5 \text{ GeV}^2$ in ¹⁵⁾:

$$C_i^A(q^2) = C_i^A(0) \frac{1 + a_i q^2 / (b_i + q^2)}{(1 + q^2 / M_A^2)^2}, \quad i = 3, 4, 5 \quad (11)$$

where

$$C_3^A(0) \simeq 0, \quad C_4^A(0) \simeq -0.3, \quad C_5^A(0) = 1.2, \quad (12)$$

$$a_i = -1.2 \text{ GeV}^{-2}, \quad b_i = 2.0 \text{ GeV}^2 \quad i = 4, 5. \quad (13)$$

and M_A is the mass parameter in the axial isovector nucleon form factor for which a standard dipole q^2 dependence has been assumed.

The prediction $|C_5^A(0)| \simeq 1.2$ follows from the PCAC hypothesis and is analogous to the Goldberger-Treiman prediction for the nucleon axial coupling constant. Approximately the same result can be obtained in the quark model which in addition predicts that $C_5^A(0)$ is positive. We also have in the quark model: $C_3^A(0) \simeq 0, C_4^A(0) \simeq 0$.

The form factor $C_6^A(q^2)$ is usually determined by assuming that the one-pion exchange contribution to the reaction (5) is dominant at low q^2 . It is not smaller, in general, than $C_5^A(q^2)$ for $q^2 \lesssim 0.5 \text{ GeV}^2$ but contributes to the asymmetry with a factor $\sim m_e/M$, where m_e is the electron mass, and therefore has been neglected.

^{*)} It is out of the scope of this note to discuss the existing models of Δ production and their experimental status. See, e.g., Refs. 15), 21), which contain extended bibliographies, and Ref. 16) for the new data analyses.

We would like to note also that the Adler model predictions for the vector form factors are in good agreement for $q^2 \lesssim 0.5 \text{ GeV}^2$ with the experimental observations, as expressed by Eqs. (8) and (9).

The standard definition of the P odd asymmetry in the case of excitation of Δ by electrons with longitudinal polarization λ and energy E is given by :

$$A(q^2, s) = \frac{\left(\frac{d\sigma(e^-N \rightarrow e^- \Delta)}{dq^2} \right)_{\lambda=1} - \left(\frac{d\sigma(e^-N \rightarrow e^- \Delta)}{dq^2} \right)_{\lambda=-1}}{\left(\frac{d\sigma(e^-N \rightarrow e^- \Delta)}{dq^2} \right)_{\lambda=1} + \left(\frac{d\sigma(e^-N \rightarrow e^- \Delta)}{dq^2} \right)_{\lambda=-1}} \quad (14)$$

where

$$s = -(p+k)^2 = M^2 + 2ME$$

and

$$\left(\frac{d\sigma(e^-N \rightarrow e^- \Delta)}{dq^2} \right)_{\lambda}$$

denotes the differential cross-section of the reaction (5) with unpolarized nucleons. The calculation of the asymmetry is straightforward. Using Eqs. (8), (11) and (12) we obtain the following expression :

$$A(q^2, s) = \frac{G}{\sqrt{2}} \frac{q^2}{2\pi\alpha} \left\{ \tilde{\alpha} + \tilde{\beta} F(q^2, s) \right\} \quad (15)$$

Here

$$F(q^2, s) = \frac{C_5^A(q^2)}{C_3^V(q^2)} \left[1 + \frac{M_\Delta^2 - M^2 - q^2}{2M^2} \frac{C_4^A(q^2)}{C_5^A(q^2)} \right] \frac{P^A(q^2, s)}{P^V(q^2, s)}, \quad (16)$$

where

$$P^A(q^2, s) = MM_\Delta [(s-M^2) + (s-M_\Delta^2) - q^2], \quad (17)$$

$$P^V(q^2, s) = \frac{1}{2} [q^2 + (M_\Delta + M)^2] [q^2 + (M_\Delta - M)^2] + (s-M^2)(s-M_\Delta^2) - q^2 s \quad (18)$$

are polynomials of q^2 and s .

The relative sign of $C_5^A(0)$ and $C_3^V(0)$ is fixed by the low energy neutrino data on Δ production ¹⁵⁾ which strongly favours the quark and the Adler model prediction

$$C_5^A(0) C_3^V(0) > 0$$

The q^2 dependence of the function $F(q^2, E)$ for four values of the initial electron energy, namely 0.5 GeV, 0.6 GeV, 0.7 GeV and 0.8 GeV and for $M_A^2 = 0.90 \text{ GeV}^2$ is shown in the Figure. The curves are cut at the kinematically allowed maximal values of q^2 (which for $M_\Delta = 1.22 \text{ GeV}$ ¹⁷⁾ are 0.17, 0.29, 0.43 and 0.57 GeV^2 , respectively). As we see, the function $F(q^2, E)$ takes values slightly higher than one and is quite smooth for $E \gtrsim 0.6 \text{ GeV}$. If M_A^2 is larger than 0.9 GeV^2 , as seems to be indicated by the new neutrino data on Δ production and quasi-elastic reactions ^{17), 24)} $F(q^2, E)$ would also be somewhat larger which, in principle, would lead to a better determination of $\tilde{\beta}$.

Since a particular model has been used to find $F(q^2, s)$ it is worth while to discuss at least qualitatively to what extent our results are model-dependent. Choosing a parametrization of the type (11) with $a_1 \sim -1 \text{ GeV}^{-2}$ and $b_1 \sim 1 \text{ GeV}^2$ is a possible way to take into account the experimental observation that the $N\Delta$ transition axial form factors decrease faster with q^2 than the nucleon axial isovector form factor. Further, in the Adler model $C_3^A(0) = 0$, but there are no experimental indications that $C_3^A(0)$ is not small. If $C_3^A(0) \simeq 0$ and if $C_4^A(q^2)$ and $C_5^A(q^2)$ have approximately the same q^2 dependence then the possibility $|C_4^A(0)| \gtrsim |C_5^A(0)|$ is not favoured ^{15), 16)} by the observed features of the neutrino production of Δ . In the range of the q^2 values relevant to the discussed experiments $[q^2 \sim (0.2-0.5) \text{ GeV}^2]$ this implies that $C_4^A(q^2)$ gives a non-leading contribution to $F(q^2, s)$. Therefore our conclusion that at low energies $F(q^2, s) \sim 1$, is based first of all on the validity of the PCAC prediction for $C_5^A(0)$ and on the assumed q^2 dependence of the axial isovector nucleon form factor. That the hadron axial vector current contribution to the asymmetry is not suppressed kinematically in comparison to the vector current contribution can be easily established using the general structure of the corresponding terms.

Our general conclusion is that the measurements of the P odd asymmetries in the reactions $e^- + N \rightarrow e^- + N$ and $e^- + N \rightarrow e^- + \Delta$ might complete the experimental determination of the phenomenological parameters, characterizing the low energy parity non-conserving electron-quark neutral current interaction. In principle, these experiments might also be used to study the q^2 dependence of certain form factors. To realize such a programme, however, an accuracy $\sim 10^{-7}$ in the asymmetry measurements is, probably, required.

Finally, we would like to note that the standard model prediction for Δ production asymmetry $A(q^2, s)$ strongly depends on the value of $\sin^2 \theta_W$. Therefore the asymmetry measurements might lead to a very accurate determination of $\sin^2 \theta_W$ which may have important implications for the theories unifying weak, electromagnetic and strong interactions ²⁵⁾.

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FIGURE CAPTION

The q^2 dependence of the function $F(q^2, E)$ for four values of the initial electron energy : $E = 0.5, 0.6, 0.7$ and 0.8 GeV ($M_A^2 = 0.90 \text{ GeV}^2$, $M_\Delta = 1.22 \text{ GeV}^2$).

FIG.1