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HADRONIC WAVE FUNCTIONS AT SHORT DISTANCES AND THE
OPERATOR PRODUCT EXPANSION

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ABSTRACT

The operator product expansion, of appropriate products of quark fields, is used to yield the anomalous (logarithm) dimensions which control the short distance behaviour of hadronic wave functions. This behaviour in turn controls the high (momentum transfer) Q^2 limit of hadronic form factors.

In particular, we relate each anomalous dimension of the non-singlet structure functions to a corresponding logarithmic correction factor to the nominal $\alpha_s(Q^2)/Q^2$ fall off of meson form factors.

Unlike the case of deep inelastic lepton-hadron scattering, the operator product necessary here involves extra terms which do not contribute to forward matrix elements.

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In this paper we shall show how the operator product expansion links together two critical testing grounds of quantum chromodynamics (QCD): the evolution of the moments of the deep inelastic structure functions and the short distance structure of the hadronic wave functions which appear in large momentum transfer exclusive reactions. In particular, we shall be able to relate each anomalous (logarithmic) dimension γ_n of the non-singlet with operator that appears in the description of deep inelastic lepton-hadron scattering to a corresponding logarithmic correction factor $(\ln Q^2/\Lambda^2)^{-\gamma_n/2\beta_1}$ that multiplies the nominal $\alpha_s(Q^2)/Q^2$ fall off of (helicity zero) meson form factors. In fact, each anomalous dimension is associated with a specific Gegenbauer moment of the lowest $q\bar{q}$ Fock state component of the meson wave function (q stands for quark, $\bar{q}$ for anti-quark). Here β_1 is defined by $\beta(g) = -\beta_1 g^3 + O(g^5)$, and $\gamma_n g^2$ is the anomalous dimension of the n^{th} non-singlet operator appearing in the expansion of two currents (in the leading contribution to forward matrix elements). Λ is the renormalization group invariant scale, and $\alpha_s(Q^2) = 1/[4\pi\beta_1 \log(Q^2/\Lambda^2)]$ with $\beta_1 = 1/16\pi^2(11 - 2/3n_f)$ (n_f is the number of quark flavours).

The general result in QCD for the electromagnetic form factors of hadrons at large momentum transfer is ($Q^2 = -q^2 = -t$),

$$F(Q^2) = \left[\frac{\alpha_s(Q^2)}{Q^2} \right]^{(n-1)} \left[\sum_{k=0}^{\infty} a_k \left(\ln \frac{Q^2}{\Lambda^2} \right)^{-\tilde{\gamma}_k/2\beta_1} \right]^2 \cdot \left[1 + O\left(\alpha_s(Q^2), \frac{m^2}{Q^2}\right) \right] \quad (1)$$

where $n = 2$ and 3 for mesons and baryons, respectively. For mesons, $\tilde{\gamma}_k$ are equal to the γ_k that appear in the moments of the non-singlet structure functions. m represents mass effects (target mass, transverse motion, etc.). This result holds for all elastic and transition form factors where (the orbital angular momentum of the wave function) $L_z = 0$, the hadronic helicity is conserved and is less than one (zero for mesons, one half for baryons); otherwise the form factor is suppressed by additional powers of Q^2 (with new powers of $\ln Q^2/\Lambda^2$). In the case of mesons, the leading $k = 0$ term in Eq. (1) is with $\tilde{\gamma}_0 = \gamma_0 = 0$ and is normalized to the $M \rightarrow \ell\bar{\ell}$ electromagnetic or weak decay amplitudes ($\pi^+ \rightarrow \mu^+ \nu$ for π^+ form factor and $\rho^0 \rightarrow e^+ e^-$ for ρ^+ form factor). In the case of baryons the leading $k = 0$ terms have $\tilde{\gamma}_0 = 1/16\pi^2(4/3)$, and the ratios of the asymptotic form factors are given by the SU(3) of colour and the flavour group symmetry, e.g., $F_n(Q^2)/F_p(Q^2) \xrightarrow{Q^2 \rightarrow \infty} -2/3$ (n stands for neutron, p for proton).

The essential structure of the form factor calculation is represented in the Figure. Since we only consider contributions to leading order in $\alpha_s(Q^2)$, T_H is the sum of connected Born diagrams including vertex, propagator and external leg ("root" of propagator) corrections (indicated by $\cdot$ and $\times$ in Fig. 1, respectively). The dominant momentum transfer occurs in T_H , and the form factor has the form^{1),2)}.

$$F(Q^2) \approx \int_0^1 [dx_i] \int_0^1 [dy_j] \phi^+(x_i, Q^2) T_H(x_i, y_j, Q^2) \phi(y_j, Q^2) \quad (2)$$

where

$$T_H = \left[\frac{\alpha_s(Q^2)}{Q^2} \right]^{n-1} f(x_i, y_j) \quad (3)$$

and the symbol for integration over the constituent's longitudinal momenta is

$$[dx_i] \equiv dx_1 dx_2 dx_3 \delta(1 - \sum_{i=1}^n x_i) \quad (4)$$

with

$$x_i = x_i^{(4)} = \frac{k_i^0 + k_i^3}{p^0 + p^3} \quad (5)$$

k_i^ν is the ν component of the momentum of the i^{th} constituent (quark or antiquark). We shall use the standard light cone frame with

$$\begin{aligned} k^\mu &= (k^+, k^-, \vec{k}_\perp) \\ p^\mu &= (p^+, \frac{m^2}{p^+}, \vec{0}) \\ q^\mu &= (0, \frac{2p \cdot q}{p^+}, \vec{q}_\perp) \end{aligned} \quad (6)$$

where $p^\pm = p^0 \pm p^3$ and $\vec{q}_\perp^2 = Q^2$. First we discuss the results in the light cone gauge $A^+ = 0$. The effective wave function $\phi(x_i, Q^2)$ is then the n - quark Fock state meson or baryon wave function integrated over transverse momenta $(\vec{k}_T^{(i)})^2 < Q^2$,

$$\phi(x_i, Q^2) = \left(\log \frac{Q^2}{\Lambda^2} \right)^{-\frac{\gamma_F}{\beta_1}} \int \left(\prod_{i=1}^n d^2 \vec{k}_i^{(i)} \right) \delta^{(4)} \left(\sum \vec{k}_i^{(i)} \right) \cdot \tilde{\phi}(x_i, \vec{k}_i^{(i)}) \quad (7)$$

$\gamma_F = 1/16\pi^2 \times C_F = 1/16\pi^2 \times 4/3$ is the anomalous dimension of a quark in the $A^+ = 0$ gauge. $\tilde{\phi}(x_i, k_T^{(\ell)})$ is the positive energy projection of the gauge invariant Bethe-Salpeter wave function on the null plane,

$$\tilde{\phi}_M(x_i, \vec{k}_i^{(i)}) \sim \text{F.T.} \left[\langle 0 | T \psi(z_1) \bar{\psi}(z_2) | M \rangle \right]_{z_1^+ = z_2^+} \quad (8)$$

$$\tilde{\phi}_B(x_i, \vec{k}_i^{(i)}) \sim \text{F.T.} \left[\langle 0 | T \psi(z_1) \psi(z_2) \psi(z_3) | B \rangle \right]_{z_1^+ = z_2^+ = z_3^+}$$

F.T. stands for Fourier transform and ψ are the quark fields. For gauge invariance, we need path ordered exponentials of the gauge potentials in the operator product. Thus we should have, for mesons, matrix elements of the form

$$P \langle 0 | T \psi(z_1) e^{ig \int_{z_1}^{z_2} dz^\mu A_\mu(z)} \bar{\psi}(z_2) | M \rangle$$

where P stands for path ordering of the exponential of the line integral, g is the QCD coupling and $A_\mu = 1/2 A_\mu^a \lambda^a$, where A_μ^a are the gluon field operators and $\frac{1}{2} \lambda^a$ the matrices of colour $SU(3)$. Analogous factors appear in the baryon case (extra factors involving the colour conjugation matrix, since we are now dealing with three quarks). In the case of a covariant gauge, the wave function includes contributions from any number of longitudinally polarized $\epsilon^\mu = (\epsilon^+, 0, \vec{0}_T)$ gluons. In the gauge $A^+ = 0$ these do not appear. Since we work near $z_1^+ = z_2^+$, we are left with only A_T in the line integral. This amounts to looking at components of the wave functions which also include extra transverse gluons. This contribution is down by powers of Q^2 (one for each extra transverse gluon). Thus the line integral can be ignored. Despite the fact that the factor $f(x_i, y_j)$ in T_H diverges linearly as $x_i \rightarrow 0$ or $y_j \rightarrow 0$, the wave function $\phi(x_i, Q^2)$ vanishes linearly for sufficiently high Q^2 . This is so since for composite systems $\phi(x_i, \lambda^2) \sim x_i^\epsilon$ with $\epsilon > 0$ for all λ^2 . If $\epsilon < 1$ for some given λ^2 , then $\phi(x_i, Q^2)$ will vanish linearly for all $Q^2 > Q_c^2$ with³⁾

$$\epsilon + \frac{16}{3(11 - \frac{2}{3}n_f)} \left[\ln \ln \frac{Q_c^2}{\Lambda^2} - \ln \ln \frac{\lambda^2}{\Lambda^2} \right] = 1 \quad (9)$$

If $\epsilon = 0.1$ or 0.5 for $\lambda^2/\Lambda^2 = 3$, then (with $n_f = 3$) we get $Q_c^2/\Lambda^2 = 151$ or 13 . $\phi(x_i, Q^2)$ vanishes like x_i^δ for $\lambda^2 < Q^2 < Q_c^2$, with

$$\epsilon + \frac{16}{3(11 - \frac{2}{3}n_f)} \left[\ln \ln \frac{Q^2}{\Lambda^2} - \ln \ln \frac{\lambda^2}{\Lambda^2} \right] = \delta \quad (10)$$

Thus the form factor in QCD is not dominated by the end point region in the x_i integration, and the short distance domain of the operator products of $\psi\bar{\psi}$ (for mesons) and $\psi\psi\psi$ (for baryons) controls the asymptotic behaviour of the form factors^{4),5)}. In particular, the same singularities that appear in the operator product expansion which controls the twist-two contributions to the non-singlet structure function moments at large Q^2 , also determine the behaviour of the meson wave function at short distances, and this in turn determines the behaviour of the form factor of helicity zero mesons at large momentum transfer. The distinguishing characteristic between the moment and wave function calculations is essentially just the difference between "forward" $\langle p | 0_{(n)} | p \rangle$ and "non-forward" $\langle 0 | 0_{(n)} | p \rangle$ matrix elements, respectively, $0_{(n)}$ are the local operators that appear in the short distance expansions (to be specified below). As we shall see, there are new terms in the operator product expansion in the "non-forward" case, that do not contribute in the former case. To be more precise: in the case of deep inelastic (non-singlet) structure functions we will require the leading twist fermion operators in the operator product expansion of two currents

$$J_\mu(\frac{x}{2}) J_\nu(-\frac{x}{2}) \sim \sum_n \left(\frac{1}{x^2 - i\epsilon x_0} \right) C_n(x^2 - i\epsilon x_0) \quad (11)$$

$$\cdot \Gamma_{\mu\nu\alpha\beta}(i) \bar{z}_{\alpha_1} \dots \bar{z}_{\alpha_n} \bar{z}_\beta O_{(n)}^{\alpha_1 \dots \alpha_n \alpha}(i) (0)$$

where

$$\Gamma_{\mu\nu\alpha\beta}(1) = g_{\nu\alpha} g_{\mu\beta} + g_{\nu\beta} g_{\mu\alpha} - g_{\mu\nu} g_{\alpha\beta}$$

$$\Gamma_{\mu\nu\alpha\beta}(2) = \varepsilon_{\mu\nu\alpha\beta}$$

and (for the non-singlet part)

$$O_{(1\pm 2)}^{\alpha_1 \dots \alpha_n \beta} = \bar{\psi} \gamma^\beta \overleftrightarrow{D}^{\alpha_1} \dots \overleftrightarrow{D}^{\alpha_n} \frac{1 \pm \gamma_5}{2} \frac{1}{6} (\lambda^3 + \frac{1}{\sqrt{3}} \lambda^8) \psi$$

In the case of the meson $q\bar{q}$ wave function we will require the leading twist fermion operators in the operator product expansion of two fermion fields

$$\psi(\frac{z}{2}) \bar{\psi}(-\frac{z}{2}) = \sum_n \tilde{C}_n(z^2 - i\epsilon z_0) \Gamma_i z_{\alpha_1} \dots z_{\alpha_n} \tilde{O}_{(i)}^{\alpha_1 \dots \alpha_n}(0) \quad (12)$$

where the Γ_i run over the 16 matrices $1, \gamma_5, \gamma_\mu, \gamma_\mu \gamma_5, \sigma_{\mu\nu}$ multiplying 1 or $\frac{1}{2}\lambda^a$, and $O_{(i)}^{\alpha_1 \dots \alpha_n}$ involve combinations of $z^{\alpha_{n+1}} \dots z^{\alpha_m} \partial_{\alpha_{k+1}} \dots \partial_{\alpha_m} (\bar{\psi} \Gamma_i \overleftrightarrow{D}_{\alpha_1} \dots \overleftrightarrow{D}_{\alpha_n} \psi)$ for all $m \geq n \geq k$ (to be specified below). That $m \geq n$ follows from the fact that all operators appearing in the n^{th} combination with well-defined dimensions have at least n factors of z 's. That $n \geq k$ follows from the fact that a spin n operator mixes with operators of lower spin only. The factor of $(1/z^2)^2$ in Eq. (11) is related to the canonical dimensions of the currents and does not appear here. The singular functions $\tilde{C}_n(z^2)$ differ from the $C_n(z^2)$ only by the anomalous dimensions of the Fermi field (in the light cone gauge), which will mean an over-all factor of $(\ln Q^2/\Lambda^2)^{\gamma_F/\beta_1}$ in the appropriate Fourier transform. This cancels the factor $(\ln Q^2/\Lambda^2)^{-\gamma_F/\beta_1}$ in Eq. (7) (we now discuss mesons $n = 2$). This cancellation is expected since anomalous dimensions of internal legs never appear in the final answer for an amplitude. (They appear in $\tilde{C}_n$ since we deal with an operator product of two Fermi fields.) The above connection between C_n and $\tilde{C}_n$ follows from the fact that the operators $O_{(n)}$ that occur in $J_\mu(z/2) J_\nu(-z/2)$ are the same as the operator $m = n = k$ (lowest in number of z 's, namely $m = n$, and highest in spin, namely $k = n$) that occurs in $\bar{\psi}(z/2) \psi(-z/2)$. In the case of the moments of the structure functions F_1, F_2, F_3 only the helicity non-flip forward matrix elements of $O_{(n)}$ are required. In the case of the meson wave functions we require the non-forward matrix elements for helicity non-flip and helicity flip operators, depending on whether the meson has helicity zero or one, respectively.

As an illustration, let us first consider the operator product expansion for two scalar fields. The general form is

$$\phi^+(z_1) \phi(z_2) \sim \sum_n \bar{C}_n (\Delta^2 - i\epsilon \Delta_0) \Delta^{\alpha_1} \dots \Delta^{\alpha_m} \cdot \sum_{k=0}^n d_{mnk} \partial_{\alpha_{k+1}} \dots \partial_{\alpha_m} [\phi^+(z) \overleftrightarrow{D}_{\alpha_1} \dots \overleftrightarrow{D}_{\alpha_n} \phi(z)] \quad (13)$$

where $d_{mnk} = 0$ unless $m \geq n \geq k$ (as explained before). Here $\Delta = z_1 - z_2$ and $z = \frac{1}{2}(z_1 + z_2)$. The "outside" derivatives $\partial_{\alpha_{k+1}} \dots \partial_{\alpha_m}$ are always required in the operator product expansion. When computing the anomalous dimensions (that determine the $\bar{C}_n$'s) to one loop level, one can determine the d_{mnk} by using conformal invariance. Namely, one first finds the operator product expansion in a conformally invariant theory⁽⁶⁾, and then extracts from that expansion the coefficients d_{mnk} computed for canonical dimensions (zero coupling). To one loop level in β and γ_n , the breaking of conformal invariance is only in the singular functions $\bar{C}_n$ and not in the d_{mnk} ^(7),8),9).

In the case of the forward matrix element $\langle p | \phi^+(z_1) \phi(z_2) | p \rangle$, the "outside" derivatives vanish so that $m = k$, and hence only $m = n = k$ appear. Taking the appropriate Fourier transform, we get for the Q^2 behaviour $(\ln Q^2/\Lambda^2)^{-(\bar{\gamma}_n - 2\gamma_F)/2\beta_1}$, where $\bar{\gamma}_n$ is the anomalous dimension of the n^{th} operator in Eq. (13) and γ_ϕ the anomalous dimension of ϕ . When we deal with the expansion of the spinors $\psi\bar{\psi}$ as in Eq. (12), we will get $(\ln Q^2/\Lambda^2)^{-(\gamma_n - 2\gamma_F)/2\beta_1}$, where γ_n are the same anomalous dimensions as the ones that determine the Q^2 behaviour of the moments of the structure functions. As discussed before, the powers involving γ_F cancel, and we thus get that the powers of $(\ln Q^2/\Lambda^2)$ appearing in the expression for meson form factors are identical to those in deep inelastic scattering. In the case of the non-forward matrix element, the "outside" derivatives do contribute, and we get

$$\langle 0 | \phi^+(z_1) \phi(z_2) | p \rangle \sim \sum_n C_n (\Delta^2 - i\epsilon \Delta_0) \cdot \sum_{m=n}^{\infty} (\Delta \cdot p)^m a_{mn} \quad (14)$$

with

$$a_{mn} = \sum_{k=0}^n d_{mnk} C_k$$

and C_k is the normalization factor in

$$\langle 0 | \phi^+ \overleftrightarrow{D}_{\alpha_1} \dots \overleftrightarrow{D}_{\alpha_n} \phi | p \rangle = C_k p_{\alpha_1} \dots p_{\alpha_n}$$

The required null-plane ($\Delta_+ = 0$) wave function is

$$\tilde{\phi}_S(x, k_\perp) = \int d^2 \Delta_\perp e^{-i \vec{k}_\perp \cdot \vec{\Delta}_\perp} \int d\Delta_- e^{\frac{i}{2} x \Delta_- P_+}$$

$$\left[\langle 0 | \phi^+(\frac{\Delta}{2}) \phi(-\frac{\Delta}{2}) | P \rangle \right]_{\substack{\Delta_+ = 0 \\ \vec{P}_\perp = 0}} \quad (15)$$

$$\sim \sum_n \bar{C}_n^{(F)}(\vec{k}_\perp^2) \sum_{m=n}^{\infty} a_{mn} \int d\Delta_- (P_+ \Delta_-)^m e^{-\frac{i}{2} x \Delta_- P_+}$$

with $x = 1/p^+(k_1^+ - k_2^+) = x_1 - x_2$. We now take ϕ^3 in six dimensions as an example. Using conformal invariance, we get^{(6), (10)}

$$\sum_{m=n}^{\infty} a_{mn} (P_+ \Delta_-)^m = \bar{a}_n (P_+ \Delta_-)^n e^{-\frac{i}{2} P_+ \Delta_-} \int_0^1 du [u(1-u)]^{n+1} e^{i u P_+ \Delta_-} \quad (16)$$

Combining Eqs (15) and (16) we get

$$\tilde{\phi}_S(x, k_\perp^2) \sim \sum_n a_n^{(s)} \bar{C}_n^{(F)}(\vec{k}_\perp^2) \left\{ \frac{\partial^n}{\partial x^n} [(1-x^2)^{n+1}] \right\} \quad (17)$$

Here

$$\frac{\partial^n}{\partial x^n} [(1-x^2)^{n+1}] = b_n (1-x^2) C_n^{3/2}(x) \quad (18)$$

where $C_n^{3/2}(x)$ are the Gegenbauer polynomials. Going over to $\phi_S(x, Q^2)$ as in Eq. (7), we obtain

$$\phi_S(x, Q^2) \sim \sum_n b_n^{(s)} (1-x^2) C_n^{3/2}(x) \left(\ln \frac{Q^2}{\Lambda^2} \right)^{-\frac{\bar{\gamma}_n}{2\beta_1}} \quad (19)$$

for our example with scalar fields.

Let us now come back to the case of the quark anti-quark wave function, including the full effects of spin. When discussing the form factor of the pion, the leading set of operators will be the one built on $\gamma_\alpha \bar{\psi} \gamma^\alpha \gamma_5 (\frac{1}{2} \tau^+) \psi$ as the lowest operator ($\frac{1}{2} \tau_k$ are isospin matrices), in a similar way as in Eq. (13), namely with extra inner and outer derivatives. The wave functions obtained are again $(1-x^2) C_n^{3/2}(x)$. The lowest operator has zero anomalous dimension, thus giving the leading result in Eq. (1). The other anomalous dimensions come from the operators $\bar{\psi} \gamma^\alpha \gamma_5 \overleftrightarrow{D}^{\alpha_1} \dots \overleftrightarrow{D}^{\alpha_n} (\frac{1}{2} \tau^+) \psi$, namely the non-singlet anomalous dimensions

of deep inelastic scattering, as stated before. Since $\langle 0 | \bar{\psi} \gamma_\mu \gamma_5 (\frac{1}{2} \tau^+) \psi | \pi^+ \rangle = f_\pi p_\mu$, the normalization of the leading term in the pion form factor is known from π decay. In the case of the ρ meson, the helicity zero form factor will be given by the tower built on $\gamma_\alpha \bar{\psi} \gamma_\alpha (\frac{1}{2} \tau^+) \psi$. Here again the leading term has zero anomalous dimensions, and the other anomalous dimensions are as in the pion case. Here also the normalization of the leading term is known, since $\langle 0 | \bar{\psi} \gamma_\alpha (\frac{1}{2} \tau^+) \psi | \rho^+ \rangle = \epsilon_\alpha g_\rho m_\rho$, and g_ρ is measured in $\rho^0 \rightarrow \ell \bar{\ell}$ decay. In fact, $g_\rho \approx f_\pi$ from the KSFR relation¹¹⁾.

For the transverse ρ case (helicity = ± 1), the tower has $\sigma_{\alpha\beta} \bar{\psi} \sigma_{\alpha\beta} (\frac{1}{2} \tau^+) \psi$ as its lowest operator. The higher operators are related in the same way as the helicity zero case. Since the anomalous dimensions of $\bar{\psi} \sigma_{\alpha\beta} (\frac{1}{2} \tau^+) \psi$ is $2\gamma_F$, a factor $(\ln Q^2/\mu^2)^{-\gamma_F/\beta_1}$ will appear in the form factor. Also, due to the $(\sigma_{\alpha\beta})$ coefficient, we will get, after convoluting with T_H , an extra m/Q in the form factor. The asymptotic behaviour of the baryon form factors can similarly be calculated in terms of the anomalous dimensions of towers of operators based on three quark operators. Again, asymptotically it is the operator with the least number of derivatives which has the lowest anomalous dimension, in this case it is γ_F/β_1 for the helicity $\frac{1}{2}$ and $3\gamma_F/\beta_1$ for the helicity $3/2$ baryons. This agrees with the results previously obtained using diagrammatic methods¹⁾.

In this letter we have shown that the results obtained previously¹⁾ for the form factors of hadrons, can be quite naturally understood in terms of the operator product expansion. In particular, we see that the exponents which appear in Eq. (1), which originally were obtained by solving the bound state equations explicitly, are just the anomalous dimensions of familiar operators.

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FIGURE CAPTION

Structure of form factor calculations.

1a. Meson form factor.

2b. Baryon form factor (+...stands for all other connected Born graphs).

(a)

(b)

$$T_H^\mu =$$

The equation shows T_H^μ is equal to the sum of two diagrams. The first diagram consists of a horizontal line with two vertices (marked with 'x'). A wavy line enters the left vertex. The second diagram is identical but the wavy line enters the right vertex.

$$T_H^\beta =$$

The equation shows T_H^β is equal to a diagram with a horizontal line and three vertices. A wavy line enters the left vertex. This is followed by a plus sign and another similar diagram.

FIG.1