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SUMMING LEADING LOGS IN THRUST DISTRIBUTIONS

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ABSTRACT

We stress the importance of summing leading logs in thrust distributions at presently available energies and give results for e^+e^- annihilation and deep inelastic processes.

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The jet analysis of e^+e^- annihilation is usually interpreted as being a success of perturbative QCD. However, when one looks at the currently available theoretical predictions, one becomes aware of the fact that a substantial part thereof is a non-perturbative guess that has nothing to do with perturbative QCD. In this paper we stress the fact that the summation of leading logs in jet distributions provides a first step towards the understanding of the so-called "non-perturbative" background.

As an example, we take the thrust defined as¹⁾

$$T = \max \frac{\sum_i |\vec{p}_{Ti}|}{\sum_i |\vec{p}_i|} \quad (1)$$

where Σ runs over all observed particles, $\tilde{\Sigma}$ over all particles emitted in a hemisphere chosen to maximize T , and the p_{Ti} 's are transverse to the plane defining the hemisphere (or parallel to its normal): $T = 1$ for a two-jet event. The thrust distribution as computed to the first order in α_s ²⁾ contains a doubly logarithmic term:

$$\frac{\sigma(T_0)}{\sigma_0} = 1 - \frac{4\alpha_s}{3n} \left[\ln^2(1-T_0) + \dots \right] \quad (2)$$

where $\sigma(T_0)$ is the cross-section integrated over the range $T_0 \leq T \leq 1$ and σ_0 is the total cross-section. The reason for the existence of such terms is well known: when $T \rightarrow 1$, the emitted gluon becomes soft or collinear, thus giving rise to infra-red divergences in the cross-section. Therefore for $T \approx 1$, these terms spoil the perturbative expansion.

In the following we intend to sum these leading terms. The interest is two-fold:

- i) we cure the unwanted behaviour when T goes to 1;
- ii) by studying the higher orders, we dress the two emitted quarks with gluons and quark pairs, thus going one step further towards the non-perturbative reshuffling.

Let us consider the order α_s^n contribution. As usual, at the leading log level it is given by a ladder diagram, as shown in Fig. 1. Because the quark-gluon transition is hard and because we start with quarks at the electromagnetic vertex, the ladder blocks are all of the same type. We choose to work in a light-like axial gauge: $A \cdot p_2 = 0$ where p_2 is the momentum of one of the two

outgoing quarks. As is well known³⁾, this condition eliminates the radiation of gluons from the p_2 leg. We use the Sudakov variables⁴⁾ to parametrize the gluon momenta:

$$\begin{aligned} k^i &= u^i p_1 + v^i p_2 + k_\perp^i \\ k_\perp^i \cdot p_1 &= k_\perp^i \cdot p_2 = 0 \end{aligned} \quad (3)$$

The on-shell condition ($p_1^2 = p_2^2 = 0$) reads

$$2u^i v^i p_1 \cdot p_2 = -k_\perp^2 \quad (4)$$

Since we are interested in longitudinal momenta, we shall keep u^i, v^i as independent variables (thus differing from the standard procedure that uses k_T^2).

We are now in a position to compute the ladder diagram of Fig. 1. To give it in a simple form, we use the following trick: we add to the diagram all the diagrams obtained by permuting gluon vertices (none of them gives any contribution to the leading logs); we can then permute the momenta of integration k^i ($i = 1, \dots, n$), add each contribution thus obtained and divide by $n!$ without changing the original quantity. Thus

$$\frac{\sigma^{(n)}(T_0)}{\sigma_0} = \left(\frac{4\alpha_s(Q^2)}{3n} \right)^n \int \frac{du_1}{u_1} \int \frac{dv_1}{v_1} \dots \int \frac{du_n}{u_n} \int \frac{dv_n}{v_n} \quad (5)$$

We then have to find the phase space limits. To avoid infra-red divergences, we make the usual trick⁵⁾ of going to the complementary region of phase space, that is $\frac{1}{2} < T < T_0$. We just need a factor $(-)^n$ to recover $\sigma^{(n)}(T_0)$. Now, let us first suppose that all gluons are emitted quasi-parallel to p_2 :

$$(V_i) \quad u_i \ll v_i \quad (6)$$

Then $p_1^0 = T(Q/2)$, (where $Q^2 = q^2$) and writing $p_2^2 = (q - p_1 - \sum_{i=1}^n k_i)^2 = 0$ gives

$$\sum_{i=1}^n u_i = 1 - T > 1 - T_0 \quad (7)$$

To the leading log approximation, this is equivalent to

$$(\forall i) \quad u_i > 1-T_0 \quad (8)$$

Now if we switch k_{i_0} , for instance, and put it parallel to p_1 ($v_{i_0} \ll u_{i_0}$), we recover the above boundaries by exchanging $u_{i_0} \leftrightarrow v_{i_0}$. We then just have to multiply by 2^n the total contribution. We finally obtain

$$\begin{aligned} \frac{\sigma^{(n)}(T_0)}{\sigma_0} &= \left[- \frac{4\alpha_s(Q^2)}{3n} 2 \int_{1-T_0}^1 \frac{du}{u} \int_u^1 \frac{dv}{v} \right]^n \\ &= \left[- \frac{4\alpha_s(Q^2)}{3n} \ln^2(1-T_0) \right]^n \end{aligned} \quad (9)$$

The summation is straightforward:

$$\frac{\sigma(T_0)}{\sigma_0} = e^{-\frac{4\alpha_s(Q^2)}{3n} \ln^2(1-T_0)} \quad (10)$$

which gives a thrust distribution:

$$\frac{1}{\sigma_0} \frac{d\sigma}{dT} = - \frac{8\alpha_s(Q^2)}{3n} \frac{\ln(1-T)}{1-T} e^{-\frac{4\alpha_s(Q^2)}{3n} \ln^2(1-T)} \quad (11)$$

We have checked explicitly this formula to the two-loop level^{*)}. An explicit calculation requires special attention to infra-red divergences. The light-like gauge that we use introduces spurious divergences into the calculation; we have regularized them in the usual way⁶⁾ by taking the principal value method:

$$\frac{1}{k.n} \rightarrow \frac{1}{2} \left[\frac{1}{k.n+i\delta} + \frac{1}{k.n-i\delta} \right]$$

*) I wish to thank G. Girardi who took part in this verification.

We have regularized the other infra-red divergences either by giving a mass to the gluon or by dimensional regularization (both methods are equivalent at the leading log level).

Formula (11) requires several remarks. First, we point out that we have used a coupling depending on Q^2 in formula (5). The conventional wisdom in ladder diagram calculations would require a dependence on the transverse momentum k_T^{i2} carried by the gluon at each rung. Of course, within our leading log approximation, it is perfectly allowable to replace it by Q^2 , but correcting this would be a way of including some of the next-to-leading contributions. This point is discussed in a forthcoming paper⁷⁾. We only draw the following consequence from it: a typical dependence of α_s on $Q^2(1-T)$ arises in formula (11) from such considerations (of course, the situation is much more complicated and (11) does not hold any longer); when $1-T \rightarrow 0$, $\alpha_s(Q^2(1-T))$ blows up; this just reflects the fact that non-perturbative effects come in to turn quarks and gluons into hadrons. But this also allows us to define a typical non-perturbative width ΔT_{np} which expresses the consistency of the model; we require

$$\alpha_s(Q^2 \Delta T_{np}) \sim 1 \quad (12)$$

or, with four flavours

$$\Delta T_{np} \sim e^{\frac{25}{12n}} \frac{\Lambda^2}{Q^2} \quad (13)$$

In Fig. 2 we compare the leading log distribution with experiment⁸⁾. The non-perturbative smearing according to (13) is negligible. The agreement with experiment looks encouraging. Of course, this is not the end of the story. We have not yet included the next-to-leading contribution⁷⁾, which certainly plays a role for thrust not too close to one. The so-called perturbative QCD tail²⁾ has not been taken into account.

Finally, we come to the leptonproduction case. It is easy to check (for example, on the first order⁹⁾) that the summation given in (11) holds also in this case. This gives, we think, a clue to the so far unexplained similarity^{10),11)} between thrust distributions in e^+e^- and νp scattering: the first order calculations¹²⁾ found important differences between the two. We compare the prediction with experiment¹⁰⁾ in Fig. 3: the situation looks as good as in the e^+e^- case (we take $\alpha_s(W^2)$ in formula (11) as we can do in our leading-log approximation).

In conclusion, we think that the summation of leading logs in thrust distributions provides clean predictions, largely independent of the unknown non-perturbative effects: theoretical curves in Figs 2 and 3 do not depend on any parameter except Λ (via α_s). This position seems supported by experiment, especially by the experimental agreement of e^+e^- with νp thrust distributions. Of course, the study is not complete and one still has to pay attention to next-to-leading logs and constant terms.

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Note added: After completion of this work, Dr G. Schierholz has drawn our attention to the fact that he had already mentioned the result of Eq. (11) in a talk at the SLAC Summer Institute¹³⁾.

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Figure captions

- Fig. 1 : Ladder diagram giving rise to the n^{th} order contribution to $\sigma(T_0)$ in the light-like gauge: $A \cdot n = 0$.
- Fig. 2 : Thrust distribution at the leading log level (Eq. (11)) for e^+e^- annihilation at $Q = 17$ GeV and $Q = 27.6$ GeV. The experimental points are from Ref. 8).
- Fig. 3 : Thrust distribution at the leading log level for νp scattering. The experimental points are from Ref. 10). The dashed line is the first order QCD result¹²⁾.

FIG. 1

FIG. 2

FIG. 3