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PION PHOTOPRODUCTION AND THE "ANOMALOUS" TRANSITION $^{14}_{\text{N}}_{\text{gs}} \rightarrow ^{14}_{\text{C}}_{\text{gs}}$

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A B S T R A C T

We study the reaction $^{14}_{\text{N}}(\gamma, \pi^+)^{14}_{\text{C}}_{\text{gs}}$ near threshold. The vanishingly small Gamow-Teller matrix element allows us to probe corrections to the normally dominant $\vec{\sigma} \cdot \vec{\epsilon}$ (Kroll-Ruderman) term. We find that, while the cross-section for this process is strongly sensitive to nuclear structure and pion distortion inputs, the pseudoscalar terms reduce the cross-section substantially from the Kroll-Ruderman value, irrespective of these uncertainties. Significant improvement is found in the comparison with experiment when the empirical nuclear wave functions, obtained from other weak and electromagnetic processes, are used, rather than the shell model wave functions.

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1. INTRODUCTION

The beta transition $^{14}\text{C}_{\text{gs}} (J^{\pi} T = 0^{+} 1) \xrightarrow{\beta^{-}} ^{14}\text{N}_{\text{gs}} (1^{+} 0)$, well known for its practical application in the radiocarbon dating technique, is also very interesting from the nuclear physics point of view. Its "anomalously" long lifetime ($\log ft \sim 9$ rather than the "superaligned" value of about 3) remained a source of puzzle in the theory of nuclear structure until the late sixties^{*}). It is now agreed¹⁾ that the inclusion of a tensor component in the two-body interaction within the 1p shell is *essential* in explaining this fantastic retardation of the β transition strength, while admixing a small fraction of the two-particle excitation out of the 1p shell to the 2s-1d shell improves the agreement with experiments even further. These cause the Gamow-Teller matrix element to become vanishingly small, or the β lifetime very large, compared with the superallowed value.

The near vanishing of the Gamow-Teller matrix element has important consequences for other weak and electromagnetic processes as well. For example, the muon capture reaction $^{14}\text{N}(\mu^{-}, \nu_{\mu}) ^{14}\text{C}_{\text{gs}}$ is dominated²⁾ by the nuclear multipole operator³⁾ $[\vec{\sigma} \otimes Y^2(\hat{r})]^{(2)}$; the rate of this reaction receives a large contribution from the induced pseudoscalar form factor, half of its magnitude coming from the interference of the pseudoscalar term with the axial-vector one. In the Kroll-Ruderman (KR) limit⁴⁾, cross-sections for the 1s radiative pion capture and charged pion photoproduction at threshold should be rather small in this instance as, indeed, is found to be the case experimentally^{5,6)}. Of course, many corrections to the KR term^{**)} are expected to be important and must be carefully investigated along with the corrections to the impulse approximation itself⁸⁾.

The photoproduction reaction

is the subject of theoretical investigation in this paper^{***)}. This process has certain obvious experimental advantages over the related muon and radiative pion capture reactions. The reactions $^{14}\text{N} + \mu^{-}(\pi^{-}) \rightarrow ^{14}\text{C} + \nu_{\mu}(\gamma)$ populate the ^{14}C ground state very weakly²⁾ compared with much stronger excitations of the other states. This makes any measurement of the $^{14}\text{N}(\mu^{-}, \nu_{\mu}) ^{14}\text{C}_{\text{gs}}$ reaction practically impossible and that of $^{14}\text{N}(\pi^{-}, \gamma) ^{14}\text{C}_{\text{gs}}$ very difficult. The study of the reaction (1), on the other hand, benefits from the fact that the next excited state

*) See, for an excellent review on the earlier works and references, the paper by Rose et al.¹⁾.

**) By this we mean the $\vec{\sigma} \cdot \vec{\epsilon}$ term, dominant near threshold⁷⁾, $\vec{\sigma}$ being the Pauli operator for the nucleon and $\vec{\epsilon}$ the photon polarization.

***) We have presented a preliminary account of this work at the Pittsburgh Conference⁹⁾.

of ^{14}C is 6.1 MeV above the ground state. Thus, using the high resolution bremsstrahlung photon beams, *exclusive excitation* of ^{14}C in the ground state by the reaction (1) becomes possible in a clean fashion, since the photon energy can be kept low enough so as to produce pions with ^{14}C only in its ground state.

Our main goal in this paper is to study how important are the corrections to the *normally dominant* $\vec{\sigma} \cdot \vec{E}$ term in the process (1). We also attempt to discern the sensitivity of the reaction cross-section to the nuclear structure inputs as well as to the distortion effects of the outgoing pion wave. We utilize nuclear wave functions^{1,10)} obtained from the many-particle shell model studies, as well as those¹¹⁾ extracted from the fits to the related weak and electromagnetic processes. The nuclear Coulomb and strong interaction effects on the pion are treated in the conventional manner. We find that the contribution of the "pseudoscalar" terms is essential, in addition to the KR one, in describing the experimental cross-section of (1) at threshold⁶⁾. We also note that the shell-model wave functions^{1,10)}, even the best ones, tend to overestimate the cross-section, while the "wave functions" extracted from the weak and electromagnetic data fits¹¹⁾ do much better.

The plan of this paper is the following: in Section 2 we discuss the effective Hamiltonian for the process (1) starting from the dispersion theory of Chew et al.¹²⁾, giving expressions of the cross-section in "exact" form as well as in the KR limit. This section, as well as the Appendix in which some technical remarks on the effective Hamiltonian and its use in the calculation of the cross-section are given, draws heavily from an unpublished work by Delorme et al.¹³⁾. In Section 3, we present our results and examine, in particular, their sensitivity to the nuclear wave functions and to the treatment of the pion distortion by the residual nucleus. Section 4 summarizes our conclusions regarding the importance of the corrections to the KR limit and their sensitivity to different inputs.

2. EFFECTIVE HAMILTONIAN

In the first part of this section we shall briefly describe the derivation¹³⁾ of an effective Hamiltonian for photoproduction of pions in nuclei. By construction, this operator will include contributions from s, p, and d pion partial waves and will take into account the nucleon recoil and Fermi motion. It will further satisfy the requirements of Galilean and gauge invariance principles. The development of this Hamiltonian is very similar in spirit to that recently done for radiative pion capture¹⁴⁾.

In the pion-nucleon centre-of-mass frame, the differential cross-section for the pion production by the photon incident on a nucleon is given by

$$\frac{d\sigma}{d\Omega} = \frac{|\vec{q}|}{|\vec{k}|} |\langle f | F | i \rangle|^2, \quad (2)$$

where i and f are Pauli spinors, $\vec{k}$ and $\vec{q}$ are the momenta of the incoming photon and the outgoing pion. F has the following general expression¹²⁾:

$$F = i \vec{\sigma} \cdot \vec{\epsilon} f_1 + \frac{\vec{\sigma} \cdot \vec{q} \vec{\sigma} \cdot (\vec{k} \times \vec{\epsilon})}{qk} f_2 + i \frac{\vec{\sigma} \cdot \vec{k} \vec{q} \cdot \vec{\epsilon}}{qk} f_3 + i \frac{\vec{\sigma} \cdot \vec{q} \vec{q} \cdot \vec{\epsilon}}{q^2} f_4. \quad (3)$$

Here, the quantities $f_1, \dots, f_4$ are as defined in Ref. 12, where they are given as expansions in Legendre polynomials with the cosine of the pion emittance angle in the pion-nucleon c.m. frame as the argument. Retaining terms up to order two in this expansion, the amplitude F takes the form:

$$F = i \vec{\sigma} \cdot \vec{\epsilon} A_1 + i \vec{\sigma} \cdot \vec{\epsilon} \vec{q} \cdot \vec{k} A_2 + i \vec{\sigma} \cdot \vec{\epsilon} (\vec{q} \cdot \vec{k})^2 A_3 + (\vec{q} \cdot \vec{k}, \vec{\epsilon}) A_4 + (\vec{q} \cdot \vec{k}, \vec{\epsilon}) \vec{q} \cdot \vec{k} A_5 + i \vec{\sigma} \cdot \vec{k} \vec{q} \cdot \vec{\epsilon} A_6 + i \vec{\sigma} \cdot \vec{k} \vec{q} \cdot \vec{\epsilon} \vec{q} \cdot \vec{k} A_7 + i \vec{\sigma} \cdot \vec{q} \vec{q} \cdot \vec{\epsilon} A_8, \quad (4)$$

where A_i 's are given in terms of the appropriate electric and magnetic multipoles (we do not write the isospin indices, with the understanding that the values appropriate to π^+ photoproduction are used):

$$\begin{aligned} A_1 &= E_{0+} + 3M_{2-} + E_{2-} - 3M_{2+} - \frac{3}{2} E_{2+} \\ A_2 &= \frac{3}{qk} (M_{1+} + E_{1+}) - \frac{1}{qk} (2M_{1+} + M_{1-}) \\ A_3 &= \frac{15}{2q^2k^2} (2M_{2+} + E_{2+}) - \frac{3}{q^2k^2} (3M_{2+} + 2M_{2-}) \\ A_4 &= \frac{1}{qk} (2M_{1+} + M_{1-}) \\ A_5 &= \frac{3}{q^2k^2} (3M_{2+} + 2M_{2-}) \\ A_6 &= \frac{1}{qk} (2M_{1+} + M_{1-}) + \frac{1}{qk} (3E_{2+} - 3M_{1+}) \end{aligned} \quad (5)$$

$$A_7 = \frac{3}{q^2 p^2} (3 M_{2+} + 2 M_{2-}) + \frac{15}{q^2 p^2} (E_{2+} - M_{2+}) \quad (5 \text{ cont'd})$$

$$A_8 = \frac{3}{q^2} (M_{2+} - E_{2+} - E_{2-} - M_{2-}) .$$

All numerical information on these multipoles will be taken from the phenomenological analysis of Ref. 15. Note, in particular, that the cross-section on hydrogen at threshold given by this set of inputs is 10% higher than the experimental value. As a consequence, when we compare our results with the experimental value in nitrogen, we will divide our cross-section by that obtained for hydrogen with the same input (see Table 1).

In order to proceed further in treating the problem of photoproduction by nuclei *heavier* than protons, one must make a transformation of the quantities, expressed in the pion-nucleon centre of mass, to those in the pion-nucleus centre of mass (indicated by the index N). Thus,

$$\begin{aligned} \vec{k} &= \vec{k}^N - \frac{p^N}{w} (\vec{k}^N + \vec{p}^N) , \\ \vec{q} &= \vec{q}^N - \frac{q_0}{w} (\vec{k}^N + \vec{p}^N) , \\ \vec{\epsilon} &= \vec{\epsilon}^N + \frac{\vec{p}^N \cdot \vec{\epsilon}^N}{k w} \vec{k}^N , \end{aligned} \quad (6)$$

where w is the total pion-nucleon energy, q_0 and p^N , respectively, the pion energy and nucleon momentum in the pion-nucleus centre-of-mass frame of reference. The momentum p^N will be considered as an operator on nucleonic variables and as such it is best to express it in the nucleus system:

$$\vec{p}^N = \frac{\vec{p}}{m} - \frac{\vec{k}^N}{W} ,$$

where W is the total pion-nucleus energy and m the nucleon mass. Hereafter, superscripts will be omitted and it will be understood that all variables except the nucleonic ones will be evaluated in the centre-of-mass system N.

For pion photoproduction near threshold, the multipoles (m) can be Taylor-expanded around the photon energy k_0 for the threshold pion production. One can thus write

$$m = m_0 + (k - k_0) \left[\frac{\partial}{\partial k} m \right]_0 . \quad (7)$$

Now retaining terms up to $O(q^2)$ and $O(1/m)$, one obtains an effective Hamiltonian H_{eff} for the process $\gamma + (Z, A) \rightarrow \pi^+ + (Z-1, A)$:

$$H_{\text{eff}} = 4\pi \sum_{\alpha j} \left[C_{\alpha} O_{\alpha} e^{i\vec{k} \cdot \vec{r}} \Phi_{\alpha}^*(\vec{r}) \delta(\vec{r} - \vec{r}_j) \tau_j^- \right], \quad (8)$$

where the sum over α runs from 1 to 19, the total number of terms being 19 in the above Hamiltonian, C_{α} and O_{α} being given in the Appendix^{*}). The sum j in Eq. (8) runs over the nucleons, indicating our recourse to the impulse approximation at this point. The wave function of the outgoing pion is $\Phi_{\alpha}(\vec{r})$; τ_j^- converts the j th proton to a neutron.

2.1 The Kroll-Ruderman limit

In the KR limit ($k \rightarrow 0$), all terms other than the $\vec{\sigma} \cdot \vec{E}$ one in Eq. (3) are absent. In effect, this amounts to neglecting all terms in Eq. (8) except that involving the $C_1 O_1$ operator. In this limit, the cross-section σ for the process (1) is given by

$$\sigma = A \left| C_1 (\omega(0) I_1^{0,0} + \omega(2) I_1^{2,0}) \right|^2, \quad (9)$$

where $A = 8.40 \times 10^4$ is a kinematic factor; C_1 is the coefficient of $\vec{\sigma} \cdot \vec{E}$ in H_{eff} [$C_1 \approx (1 + \mu/m) E_{0+}$, μ being the pion mass]; $I_1^{0,0}$ and $I_1^{2,0}$ are two radial integrals given by

$$I_1^{\ell,\ell} = \int_0^{\infty} j_{\ell}(kr) |\psi(r)|^2 \Phi_{\ell}(r) r^2 dr, \quad (10)$$

where $\psi(r)$ is the nuclear radial wave function, $\Phi_{\ell}(r)$ the pion radial function with $\ell = 0, 1, 2$ [in Eq. (9) we have kept only the contributions of s-wave pions]. The quantities ω are given in terms of the amplitudes of nuclear wave functions in the LS representation¹⁾:

$$\omega(0) = \frac{1}{9\sqrt{2}} \left[C_p^i C_p^{\ell} - \sqrt{3} C_s^i C_s^{\ell} \right], \quad (11a)$$

$$\omega(2) = -\frac{1}{18\sqrt{2}} C_p^i C_p^{\ell} - \frac{1}{3\sqrt{30}} C_d^i C_s^{\ell} - \frac{1}{4\sqrt{35}} C_d^i C_p^{\ell}; \quad (11b)$$

^{*}) The corrections for nuclear centre-of-mass motion are also described in this Appendix.

here the superscripts i and f correspond respectively to the ^{14}N and ^{14}C ground states; the subscripts s , p , and d represent the $L = 0, 1$, and 2 combinations of the $(1p)^{-2}$ hole configurations, with the quantum numbers appropriate for the nuclear states involved, L being the resultant orbital angular momentum of the two holes.

As we show below, the KR limit, which describes well the "strong" Gamow-Teller photoproduction transitions [like $^6\text{Li}(\gamma, \pi^+) ^6\text{He}_{gs}$, $^{12}\text{C}(\gamma, \pi^+) ^{12}\text{B}_{gs}$, etc.], is a poor approximation for the process (1). The "pseudoscalar" terms [numbered 8, 11, and 14 in Eq. (8)] are quite important in the present case, and *reduce substantially* the cross-section given by the KR limit.

2.2 The prediction of the "elementary particle" approach

Soft pion theorems predict that the process (1) is essentially given¹⁶⁾ by the axial-vector form factor $F_A(q_1^2)$, where q_1 is the momentum transfer in the reaction ($q_1 \approx m_\pi$). Writing the cross-section as

$$\frac{d\sigma}{d\Omega} = \frac{S}{4\pi} \frac{|\vec{q}|}{|\vec{k}|} a_{^{14}\text{N}}, \quad (12)$$

where $S = 2\pi\gamma/(e^{2\pi\gamma}-1)$ and $\gamma = e^2/\hbar v$ describe the Coulomb repulsion between the pion (of velocity v) and the nucleus $^{14}\text{C}_{gs}$ (with point-like charge), the prediction for $a_{^{14}\text{N}}$ is then:

$$a_{^{14}\text{N}} = (1+x)^2 c_\pi \frac{\alpha}{3 f_\pi^2} [\sqrt{2} F_A(q_1^2)]^2, \quad (13a)$$

where α is the fine structure constant, f_π the pion decay constant, and c_π a factor representing the distortion of the pion wave due to the strong pion-nucleus interaction; $(1+x)^2$ describes corrections to the soft-pion theorem¹⁶⁾ (in the limit of validity of this theorem, $x \rightarrow 0$). For comparison, the corresponding expression for the hydrogen target is:

$$a_H = (1+x')^2 \frac{\alpha}{f_\pi^2}, \quad (13b)$$

where $(1+x')^2 \approx 1.13$; Formula (13b) leads to the prediction of photoproduction cross-section $a_H = 190 \mu\text{b}$, not far from the experimental value of $201 \mu\text{b}$ ¹⁷⁾.

Then

$$a_{^{14}\text{N}}/a_H = \left(\frac{1+x}{1+x'} \right)^2 c_\pi \times \frac{1}{3} \times 2 F_A^2(q_1^2). \quad (14)$$

Taking $F_A(q_1^2 \approx \mu^2) = 0.051$ from Ref. 8, and with $[(1+x)/(1+x')]^2 \approx 1.4$ ¹⁶⁾, $c_\pi \approx 0.5$ estimated for ^{14}N , we have:

$$a_{^{14}\text{N}} / a_H \sim 12 \times 10^{-4} , \quad (15)$$

compared with the experimental value ⁶⁾:

$$a_{^{14}\text{N}} / a_H = (7.0 \pm 2.3) \times 10^{-4} . \quad (16)$$

Thus, the "elementary particle" approach overestimates the photoproduction cross-section by a factor ~ 2 . As we will see later, this is far better than the result in the KR limit with theoretical nuclear wave functions, but comparable to that obtained using empirical ones: we consider this as a first indication that effects not included in the impulse approximation treatment with the shell-model nuclear wave functions are important in process (1), the axial-vector form factor recipe ^{*}) including at least a part of such contributions. On the other hand, the remaining disagreement with experiment points probably to the occurrence here of supplementary terms in the amplitude: in particular, the factorization of distortion effects in a single coefficient c_π cannot be correct when the pseudoscalar contributions (which are related to the distortion because the pion acquires a momentum in the nuclear potential) become important, as will be shown to be the case here (see Section 3.4).

2.3 The pseudoscalar terms in the photoproduction Hamiltonian and their non-local treatment

Terms involving the operators $(\vec{\sigma} \cdot \vec{k})(\vec{q} \cdot \vec{\epsilon})$, $(\vec{\sigma} \cdot \vec{q})(\vec{q} \cdot \vec{\epsilon})$ and $(\vec{\sigma} \cdot \vec{k})(\vec{q} \cdot \vec{\epsilon})(\vec{q} \cdot \vec{k})$ in the effective Hamiltonian (8) have a transparent physical origin. They come from the pseudoscalar contribution to the photoproduction amplitude, described by the Feynman graph in Fig. 1. The corresponding amplitude is proportional to $\vec{\sigma} \cdot (\vec{k} - \vec{q}) \vec{q} \cdot \vec{\epsilon} [(\vec{k} - \vec{q})^2 + m_\pi^2]^{-1}$, and it gives rise to the three terms $C_8 O_8$, $C_{11} O_{11}$, and $C_{14} O_{14}$ ^{**)} of the effective Hamiltonian (8), when the denominator is developed in powers of $\vec{k} \cdot \vec{q}$. However, the rigorous treatment of this amplitude leads to a non-local form for the operator in H_{eff} ; this comes from the Yukawa fall-off of the pion propagator. Since this contribution is very important for a precise calculation of the cross-section (2), we have taken care to do an exact treatment

^{*}) The dependence on the momentum transfer of the axial vector form factor is taken from that of the magnetic form factor F_M : the basis of this approximation is doubtful for the transition studied here, since the matrix elements, which are common to both F_A and F_M , are suppressed in this case. See Ref. 18 for a more detailed discussion on this point.

^{**)} It takes a simple calculation to check that the coefficients C_8 , C_{11} , and C_{14} as phenomenologically determined ¹⁵⁾ are completely dominated by the contributions calculated from this pion-pole term.

of the pseudoscalar term; this amounts essentially to making the following change in Eq. (8), and then in the radial integrals appropriate to the pseudoscalar terms:

$$\begin{array}{ccc} \delta(\vec{r}-\vec{r}_3) & \longrightarrow & \frac{e^{-\mu|\vec{r}-\vec{r}_3|}}{|\vec{r}-\vec{r}_3|} \\ \text{[local form]} & & \text{[non-local form]} \end{array} \quad (17)$$

In the present case, this change, from the local to the non-local treatment of the radial integrals, results in a significant difference in the cross-section, affecting the matrix elements of the operators O_8 , O_{11} , and O_{14} , as indicated in the Appendix.

The sensitivity of the treatment of the amplitude in Fig. 1 and its importance in the process (1) is theoretically very interesting. It is precisely the same graph, with (γ, π) external lines replaced by (μ, ν) ones, that appears in the pseudoscalar contribution to the nuclear muon capture³⁾, where it makes a large contribution²⁾ in the reaction analogous to (1), viz., $^{14}\text{N}(\mu^-, \nu_\mu)^{14}\text{C}_{\text{gs}}$. Therefore it will not be a surprise that the graph in Fig. 1 *does* make important contributions to the reaction (1) as well, the advantage of photoproduction being its easier observability.

3. RESULTS

There are three important ingredients in our calculation that need detailed discussion in this section: nuclear configuration mixing, nuclear radial functions, and the distortion of the outgoing pion wave due to the strong and Coulomb interactions of the residual nucleus. There is also the question of treating the pseudoscalar non-locality, mentioned in Section 2.3. Our calculation aims at finding out the accuracy that can be reached in dealing with these problems.

3.1 Nuclear configuration mixing scheme

Here we shall confine ourselves to the models that ignore any excitation of the 1p nucleons to the 2s-1d shell. There are many wave functions available in this category, obtained by diagonalizing the appropriate nuclear Hamiltonian (see Ref. 1 for a list of these, together with Ref. 10 for a more recent work). We have used them all to see the sensitivity of our results to the nuclear configuration mixing. We have also included the theoretically interesting limiting cases¹⁾ of the LS *) and jj coupling schemes -- none of these, however, provides

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*) In the naïve LS coupling scheme, the $^{14}\text{N}_{\text{gs}} \rightarrow ^{14}\text{C}_{\text{gs}}$ transition is $^{13}\text{S}_1 \rightarrow ^{31}\text{S}_0$ (we are using here the standard representation $^{2T+1}2S+1L_J$, in which T, S, L, J are isospin, spin, orbital angular momentum, and total spin quantum numbers, respectively, of the $(1p)^{-2}$ configuration).

a correct description of the nuclear problem at hand. Finally, we use the phenomenological "wave functions" of the MIT group¹¹⁾ obtained by *fitting* other weak and electromagnetic processes relating the nuclear states (or their analogues) involved in reaction (1). The latter effectively absorb meson exchanges and other corrections that we cannot include in our conventional impulse approximation treatment with *real* nuclear wave functions. Note, however, that only if these corrections are not very different, for a given momentum transfer range, for these diverse processes, the use of MIT "wave functions" does make some sense^{*)}.

The most important difference between the MIT wave functions¹¹⁾ and the shell model ones^{1,10)} is in the rôle of the p wave ($L = 1$) components (of orbital symmetry [11]), both in the initial ($^{14}\text{N}_{\text{gs}}$) and the final ($^{14}\text{C}_{\text{gs}}$ or its analogue in ^{14}N) states. Whereas the MIT description makes the reaction (1) dominantly a $^{13}\text{D}_1 \rightarrow ^{33}\text{P}_0$ transition, the shell-model schemes describe it essentially as $^{13}\text{D}_1 \rightarrow \alpha ^{31}\text{S}_0 + \beta ^{33}\text{P}_0$, with $\alpha^2 \approx \beta^2 \approx \frac{1}{2}$. The consequence of this is reflected in the computed values of the quantities $\omega(0)$ and $\omega(2)$ defined in Eqs. (11) (see Table 2). The small Gamow-Teller matrix element in the β decay $^{14}\text{C}_{\text{gs}} \rightarrow ^{14}\text{N}_{\text{gs}}$ is reflected in the rather small values of $\omega(0)$ yielded by realistic models [in contrast with the much larger result (0.136) obtained in the LS coupling scheme, which is relevant, for instance, in the superallowed β decay of ^6He]. On the other hand, shell models give large absolute values (≈ 0.09) to $\omega(2)$, which result in much too large predictions for the cross-section of process (1). The MIT wave functions instead predict a lower absolute value $|\omega(2)| \approx 0.05$ and come closer to the experimental result.

The near equality of $\omega(2)$ in the predictions of all realistic shell-model wave functions and their consequent larger predictions for the cross-section of (1), in contrast to the MIT scheme's success in lowering the value of $\omega(2)$, suggests that the corrections to the impulse approximation itself, effectively absorbed in the MIT scheme but not in the shell-model approach, should go in the direction of *lowering* the cross-section from a value predicted in the impulse approximation, as in related weak and electromagnetic processes. A similar conclusion has been reached by Goulard et al.⁸⁾, from some results in their investigation of the meson exchange contributions to the β decay of $^{14}\text{C}_{\text{gs}}$.

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*) That is to say, the MIT "wave functions" are *not* the true wave functions, in the sense that they do not satisfy the relevant nuclear eigenvalue problem. Thus, their applicability is not *a priori* guaranteed. See, for a further discussion on this point, Ref. 19.

3.2 Nuclear radial effects

The biggest problem of using a one-parameter harmonic oscillator model is its inadequacy to treat the spin-orbit splitting and the consequent differences in the $1p_{3/2}$ and $1p_{1/2}$ radial wave functions. For example, the density-dependent Hartree-Fock approach or Woods-Saxon model yields equivalent harmonic oscillator parameters which are nearly 20% different²⁰⁾ for the $1p_{3/2}$ and $1p_{1/2}$ orbits in ^{12}C -- in ^{16}O , these values are 5% higher for the corresponding orbits. In this work we use the value 1.68 fm for the oscillator parameter b , extracted from the elastic scattering, as a canonical value. The MIT group¹¹⁾ also use this value in their extraction of the phenomenological wave functions.

Variation of the cross-section for the process (1) around this value of b is indicated in Figs. 2a and 2b, for two sets of nuclear wave functions -- those of Lee and Vergados¹⁰⁾ and those of the MIT group. The treatment of the pseudoscalar terms in local or non-local manner (Section 2.3) does not change the slope of these curves, which points to a relatively mild importance of the parameter b in fixing the cross-section.

3.3 Treatment of distortion of the outgoing pion wave

The general effect of taking the pion distortion into account is well known²¹⁾: the nuclear Coulomb effect changes the pion photoproduction cross-section, roughly linear with the pion momentum in the free pion case, into an approximately energy independent quantity near threshold. The same is true in the present case. Also, as in the "strong" Gamow-Teller transitions like $^6\text{Li}(\gamma, \pi^+) ^6\text{He}_{\text{gs}}$, one gets an increase in cross-section by turning off the nuclear strong potential, for example, by a factor ~ 2.5 in the MIT configuration of nuclear wave functions.

We take the pion-nuclear optical potential in the Krell-Ericson²²⁾ form, taking also into account the Coulomb potential for the extended charge of the nucleus. We will consider the following parametrizations of the strong potential, to illustrate the sensitivity of the theoretical cross-section for the process (1) to this input:

- (A) $b_0 = -0.0162$; $b_1 = -0.0873$; $c_0 = 0.21$; $c_1 = 0.16$;
 $B_0 = -0.056 + 0.041i$; $C_0 = 0.07i$ [our canonical choice, based on the analysis²³⁾ of the 1s levels in light pionic atoms]:
- (B) $b_0 = -0.03$; $b_1 = -0.08$; $c_0 = 0.22$; $c_1 = 0.18$;
 $B_0 = 0.040i$; $C_0 = 0.08i$ [this set results from an older fit to pionic atom data for light nuclei²⁴⁾]:
- (C) strong optical potential = 0.

(All units are given in a system for which $m_\pi = 1$.)

The following values of a_s [$\approx a$, defined in Eq. (12)*)] are obtained for the above choices of the nuclear strong potentials and the MIT wave functions:

$$a_s = 0.272 \text{ (A)}, \quad 0.303 \text{ (B)}, \quad 0.660 \text{ (C)}, \quad (18)$$

all in units of μb .

Thus, the effect of nuclear strong potential is at most a factor of two in the cross-section and the sensitivity to the variation of its parameters is not large, as can be seen in Fig. 3, where we plot a_s as a function of the parameter c_0 , for the nuclear wave functions of Lee and Vergados, and those of the MIT group.

3.4 Treatment of the pseudoscalar terms

The need for a precise treatment of the terms in the effective Hamiltonian directly arising from the pseudoscalar graph has already been mentioned (Section 2.3). The relevant matrix elements are affected through their radial integrals as indicated in the Appendix. Below, we compare the results obtained in the local and non-local treatments of this contribution:

- a) For a given set of input parameters, the cross-section is larger in the non-local treatment than in the local one (Table 1): owing to subtle changes in the appropriate radial integrals, the compensation between the KR and the pseudoscalar contributions to the cross-section is much larger in the latter case.
- b) The cross-section is relatively much more sensitive to the variation of input parameters when the local approximation for the pseudoscalar term is used (see Figs. 2 and 3, which show the sensitivity to the harmonic oscillator parameter b and to the nuclear strong interaction potential parameter c_0 , respectively). This obviously results from the larger compensations in this case.
- c) We also note that the Kisslinger singularity appears for a much larger value of c_0 in the non-local case than in the local one.

3.5 Dominant terms and effects in the photoproduction cross-section

In Table 1, we show the effect of including $\ell = 1$ and 2 terms, besides the $\ell = 0$ contribution, in the expansion of the pion wave function. The former clearly have a negligible effect and they can be safely ignored. Thus the cross-section near threshold $\sigma = a(q/k)$ can be written to a high accuracy as $a_s(q/k)$.

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*) This is the photopion production cross-section obtained by retaining only the $\ell = 0$ (s wave) pion terms in the amplitude, which are the most dominant ones near threshold.

From Table 1 it is also clear that the contribution of the KR (or $\vec{\sigma} \cdot \vec{E}$) term is not enough to explain the measured cross-section for the process (1): it produces a cross-section which, in realistic models, is too large by a factor ~ 20 compared with the experimental result. The dominant effect of the remainder of terms in the effective Hamiltonian comes from the pseudoscalar terms, whose separate contribution to the cross-section is comparable to that of the KR term alone, but interferes destructively with the latter.

To better than 20%, the cross-section for the process (1) near threshold is a combination of the contributions from the KR and pseudoscalar terms alone and can be written as:

$$\begin{aligned} \sigma \approx 8.4 \times 10^4 \mu b \times & \left| \omega(0) \left[C_1 I_1^{0,0} - \frac{1}{3} C_{14} (J_5^{2,0} + J_7^{0,0}) \right] \right. \\ & + \omega(2) \left[C_4 I_4^{2,0} - \frac{3k_0}{5} C_8 (J_2^{1,0} + J_2^{3,0}) \right. \\ & \left. \left. - \frac{1}{3} C_{14} (2J_4^{0,0} + J_4^{2,0} + J_6^{2,0}) \right] \right|^2, \end{aligned} \quad (19)$$

where $\omega(0)$ and $\omega(2)$ are given by Eqs. (11), and k_0 is the photon momentum at threshold. We recall that C_1 , C_8 , and C_{14} are the coefficients of the terms $\vec{\sigma} \cdot \vec{E}$, $(\vec{\sigma} \cdot \vec{k})(\vec{q} \cdot \vec{E})$, and $(\vec{\sigma} \cdot \vec{q})(\vec{q} \cdot \vec{E})$, respectively, in the effective Hamiltonian; I 's and J 's are the radial integrals defined in the Appendix.

4. CONCLUSIONS

From our study of the pion photoproduction near threshold in the ^{14}N target leading to the ^{14}C ground state, we can conclude the following:

i) As expected, this anomalous transition shows strong sensitivity to all the inputs (nuclear configuration mixing and radial wave functions, pion distortion), and hence it is hard to predict its cross-section theoretically with much reliability. This sensitivity is somewhat exaggerated in the usual treatment of the pseudoscalar term (Sections 2.3 and 3.4), which approximates this contribution by a local operator in the effective Hamiltonian.

ii) The Kroll-Ruderman or $\vec{\sigma} \cdot \vec{E}$ term is not adequate in describing the process. It makes a prediction which is too large by an order of magnitude, compared to the experiment. The pseudoscalar (or pion pole) terms make contributions comparable in magnitude to this KR term, and reduce the cross-section. This effect, expected from a similar strong rôle of the pseudoscalar term in the muon capture reaction $^{14}\text{N}(\mu^-, \nu_\mu)^{14}\text{C}_{\text{gs}}$, is the first *experimentally verifiable* evidence of such a contribution from the pion pole graph. This stands out *above all uncertainties* of the inputs.

iii) While it is easy to exclude unrealistic nuclear configuration mixing schemes, realistic ones predict cross-sections that are one order of magnitude too large. Only the phenomenological wave functions of the MIT group reduce the cross-section to an acceptable order of magnitude. Considering the uncertainties of inputs, these wave functions predict a cross-section ($a_{14N}/a_H = 12.3 \times 10^{-4}$), which is in fair agreement with experiment [$a_{14N}/a_H = (7 \pm 2.3) \times 10^{-4}$]. This probably points towards a strong rôle of meson exchange and other effects not included in the impulse approximation treatment with the shell-model nuclear wave functions, but which might have been effectively taken into account in the MIT scheme.

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APPENDIX

Starting from the pion-nucleon amplitude (4), we obtain the effective Hamiltonian (8) for photoproduction in nuclei. Retaining terms to order q^2 and $1/m$, this Hamiltonian is the sum of 19 operators O_i ($i = 1, \dots, 19$). Below we give a list of these together with their coefficients C_i :

$$\begin{array}{ll}
 O_1 = i \vec{\sigma} \cdot \vec{\epsilon} & O_2 = i \vec{\sigma} \cdot \vec{\epsilon} \vec{p} \cdot \vec{k} \\
 O_3 = i \vec{\sigma} \cdot \vec{\epsilon} \vec{p} \cdot \vec{q} & O_4 = i \vec{\sigma} \cdot \vec{\epsilon} \vec{q} \cdot \vec{k} \\
 O_5 = i \vec{\sigma} \cdot \vec{\epsilon} \vec{q} \cdot \vec{k} \vec{p} \cdot \vec{k} & O_6 = i \vec{\sigma} \cdot \vec{\epsilon} \vec{q} \cdot \vec{k} \vec{q} \cdot \vec{k} \\
 O_7 = i \vec{\sigma} \cdot \vec{k} \vec{p} \cdot \vec{\epsilon} & O_8 = i \vec{\sigma} \cdot \vec{k} \vec{q} \cdot \vec{\epsilon} \\
 O_9 = i \vec{\sigma} \cdot \vec{k} \vec{p} \cdot \vec{\epsilon} \vec{q} \cdot \vec{k} & O_{10} = i \vec{\sigma} \cdot \vec{k} \vec{q} \cdot \vec{\epsilon} \vec{p} \cdot \vec{k} \\
 O_{11} = i \vec{\sigma} \cdot \vec{k} \vec{q} \cdot \vec{\epsilon} \vec{q} \cdot \vec{k} & O_{12} = i \vec{\sigma} \cdot \vec{p} \vec{q} \cdot \vec{\epsilon} \\
 O_{13} = i \vec{\sigma} \cdot \vec{q} \vec{p} \cdot \vec{\epsilon} & O_{14} = i \vec{\sigma} \cdot \vec{q} \vec{q} \cdot \vec{\epsilon} \\
 O_{15} = (\vec{q}, \vec{k}, \vec{\epsilon}) & O_{16} = (\vec{q}, \vec{k}, \vec{\epsilon}) \vec{p} \cdot \vec{k} \\
 O_{17} = (\vec{q}, \vec{k}, \vec{\epsilon}) \vec{q} \cdot \vec{k} & O_{18} = (\vec{p}, \vec{k}, \vec{\epsilon}) \\
 O_{19} = (\vec{q}, \vec{p}, \vec{\epsilon}) &
 \end{array} \tag{A.1}$$

$$C_1 = \left(1 + \frac{\mu}{m}\right) E_{0+} + \left(R - \mu - \frac{\mu^2}{2m} + \frac{\mu^2}{Am}\right) \frac{\partial}{\partial R} E_{0+} - \left(\frac{\mu^3}{m} - \frac{\mu^3}{Am}\right) \frac{M_{1+} + 3E_{1+} - M_{1-}}{qR}$$

$$C_2 = -\frac{1}{m} \frac{\partial}{\partial R} E_{0+} - \frac{\mu}{m} \frac{M_{1+} + 3E_{1+} - M_{1-}}{qR}$$

$$C_3 = -\frac{\mu}{m} \frac{M_{1+} + 3E_{1+} - M_{1-}}{qR} - \frac{\mu^3}{m} \frac{6M_{2-} + 2E_{2-} - 6M_{2+} - 3E_{2+}}{q^2 R^2}$$

$$C_4 = \left(1 + \frac{\mu}{Am}\right) \frac{M_{1+} + 3E_{1+} - M_{1-}}{qR} + \left(R - \mu - \frac{\mu^2}{2m} + \frac{\mu^2}{Am}\right) \frac{\partial}{\partial R} \left(\frac{M_{1+} + 3E_{1+} - M_{1-}}{qR} \right) - \left(\frac{2\mu^3}{m} - \frac{2\mu^3}{Am}\right) \frac{6E_{2+} + 3M_{2+} + E_{2-} - 3M_{2-}}{q^2 R^2}$$

$$C_5 = -\frac{1}{m} \frac{\partial}{\partial R} \left(\frac{M_{1+} + 3E_{1+} - M_{1-}}{qR} \right) - \frac{2\mu}{m} \frac{6M_{2+} + \frac{15}{2} E_{2+} - 6M_{2-}}{q^2 R^2}$$

$$C_6 = \frac{6M_{2+} + \frac{15}{2} E_{2+} - 6M_{2-}}{q^2 R^2}$$

$$C_7 = \frac{1}{\mu m} E_{0+} - \frac{\mu}{m} \frac{3E_{1+} - M_{1+} + M_{1-}}{qR} \quad (A.2)$$

$$C_8 = \left(1 + \frac{\mu}{Am}\right) \frac{3E_{1+} - M_{1+} + M_{1-}}{qR} + \left(R - \mu - \frac{\mu^2}{2m} + \frac{\mu^2}{Am}\right) \frac{\partial}{\partial R} \left(\frac{3E_{1+} - M_{1+} + M_{1-}}{qR} \right) - \left(\frac{\mu^3}{m} - \frac{\mu^3}{Am}\right) \frac{12E_{2+} - 3M_{2+} + 3M_{2-} - 3E_{2-}}{q^2 R^2}$$

$$C_9 = \frac{1}{\mu m} \frac{6E_{1+}}{qR} - \frac{\mu}{m} \frac{15E_{2+} - 6M_{2+} + 6M_{2-}}{q^2 R^2}$$

$$C_{10} = -\frac{1}{m} \frac{\partial}{\partial R} \left(\frac{3E_{1+} - M_{1+} + M_{1-}}{qR} \right) - \frac{\mu}{m} \frac{15E_{2+} - 6M_{2+} + 6M_{2-}}{q^2 R^2}$$

$$C_{11} = \frac{15E_{2+} - 6M_{2+} + 6M_{2-}}{q^2 R^2}$$

$$C_{12} = -\frac{\mu}{m} \frac{3E_{1+} - M_{1+} + M_{1-}}{qR} - \frac{\mu^3}{m} \frac{3M_{2+} - 3E_{2+} - 3E_{2-} - 3M_{2-}}{q^2 R^2}$$

$$C_{13} = -\frac{\mu^3}{m} \frac{3M_{2+} - 3E_{2+} - 3E_{2-} - 3M_{2-}}{q^2 R^2}$$

$$C_{14} = \hbar^2 \frac{3M_{2+} - 3E_{2+} - 3E_{2-} - 3M_{2-}}{q^2 \hbar^2}$$

$$C_{15} = \left(1 + \frac{\mu}{Am}\right) \frac{2M_{1+} + M_{1-}}{q\hbar} + \left(\hbar - \mu - \frac{\mu^2}{2m} + \frac{\mu^2}{Am}\right) \frac{\partial}{\partial \hbar} \left(\frac{2M_{1+} + M_{1-}}{q\hbar} \right) - \left(\frac{\mu^3}{m} - \frac{\mu^3}{Am} \right) \frac{9M_{2+} + 6M_{2-}}{q^2 \hbar^2}$$

(A.2 cont'd)

$$C_{16} = -\frac{1}{m} \frac{\partial}{\partial \hbar} \left(\frac{2M_{1+} + M_{1-}}{q\hbar} \right) - \frac{2\mu}{m} \frac{9M_{2+} + 6M_{2-}}{q^2 \hbar^2}$$

$$C_{17} = \frac{9M_{2+} + 6M_{2-}}{q^2 \hbar^2}$$

$$C_{18} = -\frac{\mu}{m} \frac{2M_{1+} + M_{1-}}{q\hbar}$$

$$C_{19} = -\frac{\mu}{m} \frac{2M_{1+} + M_{1-}}{q\hbar} + \frac{\mu^3}{m} \frac{9M_{2+} + 6M_{2-}}{q^2 \hbar^2}$$

A correction must still be made to account for the nuclear centre-of-mass motion, since we use independent particle (harmonic oscillator) nuclear wave functions; it results in three modifications: first one has to introduce one extra term:

$$O_{20} = \vec{\sigma} \cdot \vec{\varepsilon} \frac{\vec{q}^2}{q}, \quad C_{20} = \frac{b^2}{4A} C_1, \quad (A.3)$$

b being the harmonic oscillator parameter. Then other coefficients have to be modified by terms in $1/A$ (A atomic number of the nucleus) :

$$\begin{aligned} C_1 &\longrightarrow C'_1 = C_1 + \frac{1}{2A} C_2 \\ C_4 &\longrightarrow C'_4 = C_4 + \frac{1}{2A} (-b^2 C_1 - C_2 + C_3 + C_5) \\ C_5 &\longrightarrow C'_5 = C_5 - \frac{b^2}{2A} C_2 \\ C_6 &\longrightarrow C'_6 = C_6 - \frac{b^2}{2A} C_4 \\ C_8 &\longrightarrow C'_8 = C_8 + \frac{1}{2A} (-C_7 + C_{10} + C_{11}) \\ C_9 &\longrightarrow C'_9 = C_9 - \frac{b^2}{2A} C_7 \\ C_{11} &\longrightarrow C'_{11} = C_{11} - \frac{b^2}{2A} C_8 \end{aligned} \quad (A.4)$$

./.

$$\begin{aligned}
 C_{15} &\longrightarrow C'_{15} = C_{15} + \frac{1}{2A} (C_{16} - C_{18} + C_{19}) \\
 C_{16} &\longrightarrow C'_{16} = C_{16} - \frac{b^2}{2A} C_{18} \\
 C_{17} &\longrightarrow C'_{17} = C_{17} - \frac{b^2}{2A} C_{15} \\
 C_{19} &\longrightarrow C'_{19} = C_{19} + \frac{b^2}{2A} C_{18}
 \end{aligned}
 \tag{A.4 cont'd}$$

(We have made the approximation $|\vec{k}| \approx \mu \equiv 1$, since the difference $|k| - 1$ would give here terms of order $1/m^2$). Finally, the complete amplitude has to be divided by the well-known factor $\exp[-k^2 b^2/4A]$.

The radial integrals corresponding to these 20 operators are of twelve different types, the simplest of which is written in the text, formula (10). The others are derivable from this form by applying derivatives to $\psi(r)$ or $\phi_q(r)$ according to the nature of the operator (each $\vec{p}$ or $\vec{q}$ is considered as a $\vec{\nabla}$ on the appropriate quantity). For example, the integrals

$$\begin{aligned}
 I_2^{l'l}(k) &= \int_0^\infty j_{l'}(kr) \psi^*(r) \left[\frac{d\psi(r)}{dr} - \frac{1}{r} \psi(r) \right] \Phi_l^*(r) r^2 dr \\
 I_3^{l'l}(k) &= \int_0^\infty j_{l'}(kr) \psi^*(r) \left[\frac{d\psi(r)}{dr} + \frac{2}{r} \psi(r) \right] \Phi_l^*(r) r^2 dr
 \end{aligned}
 \tag{A.5}$$

will appear in the calculation of matrix elements for the operators O_2 , O_7 , and O_{18} .

In the case of an exact treatment of the pseudoscalar contribution (operators O_8 , O_{11} , and O_{14}), the nucleon and pion wave functions are to be evaluated at different points ($\vec{x}_1$ and $\vec{x}$, respectively) before integration, with the δ function of the local approximation being replaced by the Yukawa form. This results in adding 10 types of radial integrals, derived from the local ones which involve only derivatives of the pion wave function (since the operators include $\vec{q}$ and no $\vec{p}$). Their prototype is

$$J_1^{l'l}(k) = \int_0^\infty j_{l'}(kr) B_0(r) \left[\frac{d\Phi_l^*(r)}{dr} - \frac{r}{r} \Phi_l^*(r) \right] r^2 dr, \tag{A.6}$$

where

$$\begin{aligned}
 B_\lambda^{l'l}(r) &= p_\lambda(\mu r) \mu^3 \int_0^r i_\lambda(\mu x) x^2 |\Psi(x)|^2 dx \\
 &\quad + i_\lambda(\mu r) \mu^3 \int_r^\infty p_\lambda(\mu x) x^2 |\Psi(x)|^2 dx
 \end{aligned}$$

In Eq. (A.6), k_λ and i_λ are related to modified spherical Bessel functions and

$$k_0(z) = e^{-z}, \quad k_1(z) = e^{-z} \left(1 + \frac{1}{z}\right), \quad i_0(z) = \frac{\sinh z}{z}, \dots$$

$J_2(k)$ is defined as J_1 , but $B_2(r)$ replaces $B_0(r)$. The general form of the others is the same, except for the expression in brackets, which is, respectively:

$$\begin{aligned} J_{3,4} &: \left[\frac{d\Phi_\ell^*(r)}{dr} + \frac{\ell+1}{r} \Phi_\ell^*(r) \right] \\ J_{5,6} &: \left[\frac{d^2\Phi_\ell^*(r)}{dr^2} - \frac{2\ell+1}{r} \frac{d\Phi_\ell^*(r)}{dr} + \frac{\ell(\ell+2)}{r^2} \Phi_\ell^*(r) \right] \\ J_{7,8} &: \left[\frac{d^2\Phi_\ell^*(r)}{dr^2} + \frac{2}{r} \frac{d\Phi_\ell^*(r)}{dr} - \frac{\ell(\ell+1)}{r^2} \Phi_\ell^*(r) \right] \\ J_{9,10} &: \left[\frac{d^2\Phi_\ell^*(r)}{dr^2} + \frac{2\ell+1}{r} \frac{d\Phi_\ell^*(r)}{dr} + \frac{\ell^2-1}{r^2} \Phi_\ell^*(r) \right]. \end{aligned} \quad (A.7)$$

In all cases, the non-locality appears in the same way through the term B_0 or B_2 , which replaces $|\psi(r)|^2$ of the equivalent local integrals.

Finally, the coefficients C_8 , C_{11} , and C_{14} can be given here their exact theoretical value, obtained from the graph in Fig. 1:

$$C_8 = -C_{14} = 2\sqrt{2} e f, \quad C_{11} = 0, \quad (A.8)$$

where $e^2 = 1/137$ and $f^2 = 0.08$ are the electromagnetic and strong interaction (pion-nucleon) coupling constants, respectively.

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Table 1

Contributions of the dominant terms of the effective Hamiltonian to the process (1) as obtained in the nuclear models of Lee and Vergados (Ref. 10) and the MIT fit (Ref. 11). The experimental number to be compared with is $R = 10^4 a_{1N}/a_H = (7 \pm 2.3)$; R_S and R_{spd} refer respectively to the inclusion of $\ell = 0$ and $\ell = 0, 1, 2$ terms in the partial wave expansion of the pion wave function.

	Lee-Vergados	MIT
$\vec{\sigma} \cdot \vec{\epsilon}$ term alone	$R_S = 161.8$ $R_{spd} = 162.7$	$R_S = 35.9$ $R_{spd} = 36.3$
All terms in H_{eff} included, with the non-local treatment of the pseudoscalar term	$R_S = 71.4$ $R_{spd} = 71.8$	$R_S = 12.3$ $R_{spd} = 12.3$
All terms in H_{eff} included, with the (approximate) local treatment of the pseudoscalar term	$R_S = 23.6$ $R_{spd} = 23.6$	$R_S = 2.2$ $R_{spd} = 2.2$

Table 2

The nuclear structure constants $\omega(0)$ and $\omega(2)$ [defined in Eqs. (11a) and (11b)], involved in the process (1), as calculated in various nuclear models. The jj and LS coupling limits are unrealistic. See Ref. 1 for original references to the older works.

Nuclear scheme	$\omega(0)$	$\omega(2)$
1) jj coupling	+0.0453	-0.0908
2) LS coupling	+0.136	0
3) Soper	+0.0394	-0.0915
4) Sherr et al.	+0.000417	-0.0841
5) Elliot	-0.000095	-0.0898
6) Visscher-Ferrel	-0.000984	-0.0896
7) Cohen Kurath I	-0.000065	-0.0870
8) Cohen Kurath II	-0.00735	-0.0854
9) Bojarkina	+0.01178	-0.07712
10) Lee-Vergados (Ref. 10)	-0.00631	-0.0894
11) MIT fit (Ref. 11)	-0.000216	-0.0508

Figure captions

- Fig. 1 : The pion pole graph which gives a pseudoscalar contribution to the photopion production amplitude.
- Fig. 2 : The sensitivity of the pion photoproduction cross-section a_s (defined in Section 3.3) to the variation of the harmonic oscillator parameter b . In Fig. 2a, we use the nuclear wave functions of Lee and Vergados (Ref. 10) and, in Fig. 2b, those of the MIT group (Ref. 11). The solid line represents the non-local treatment of the pseudoscalar term and the dashed line the local one. The canonical value $b = 1.68$ fm is indicated by the dotted line.
- Fig. 3 : The sensitivity of the pion photoproduction cross-section a_s to the variation of the pion-nucleus strong optical potential parameter c_0 (defined after Ref. 22). See the caption of Fig. 2 for other explanations. Notice the sharp increase of a_s near the Kisslinger singularity.

FIG.1

FIG. 2

FIG. 3