Electromagnetic production of trimuons in deep-inelastic muon scattering. II. Hadronic-final-state interactions

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We convert a hadronic neutrino-interaction trimuon model into a model for trimuon production by muon beams. This process is assumed to proceed in two independent steps: the initial electromagnetic production of a hadronic shower and its subsequent decay into $\mu^+\mu^-X$ by hadronic-final-state interactions. Event rates are calculated and we also give results for relevant distributions. The experimental determination of a rate for trimuon production in muon beams would be very useful because it would permit a cross check between neutrino and muon physics.

I. INTRODUCTION

Several muon experiments are in different stages of completion at CERN and Fermilab.¹ One of the channels which will be studied in these experiments is to measure rates for the production of J/ψ and associated charm production, for instance, a pair of D mesons. The analysis of such signals will be very easy in the case of a sharp resonance, but other channels have several backgrounds and it will be more difficult to extract a signal. For instance, the Bethe-Heitler and Compton pair production has a large cross section and some suitable subtraction procedure is required to remove these events.

The situation is much clearer in neutrino physics. MultimMuon production in neutrino collisions has been studied in some detail.² It is known that dimuon production principally arises from the production and subsequent decay of a single charmed particle. On the other hand, trimuon production³ has at least two components, namely radiative electromagnetic⁴ and pair production by hadronic-final-state interactions⁵ in normal inclusive collisions. There is no evidence for any new heavy-lepton or heavy-quark production. In fact, there is hardly any need at all for even associated charm production and decay, except for possibly explaining a like-sign dimuon signal.⁶

If we concentrate on the two mechanisms largely responsible for trimuon production in neutrino collisions, we observe these are induced by a charged weak current by the exchange of a W boson. We thus expect the analogs of these reactions to contribute in muon collisions, where the W boson is replaced by a photon (Fig. 1). For instance, the electromagnetic radiative production of muon pairs from the outgoing muon and the

struck quark in the target nucleon corresponds to trident production and virtual Compton pair production. [Fig. 1(a)]. In a previous paper,⁷ which will be referred to as I, this was investigated. These calculations were carried out with the so-called radiative quark-parton model. The corresponding hadronic reaction also has an analog in muon physics, and that is what we investigate in this paper.

In neutrino physics the hadronic model under consideration⁵ says that the muon pairs are produced in the hadron shower. Thus the reaction is assumed to have two parts, namely the production of a hadronic shower which depends only on the structure functions of the weak charged-current reactions, and the decay of this hadronic state into the channel $\mu^+\mu^-X$. This second process is taken to be a universal process, independent of the way it is produced. This enables us to utilize the distributions obtained in the reaction $\pi p \rightarrow \mu^+ \rightarrow \mu^+\mu^-X$ at the same center-of-mass energy, to obtain distributions for the neutrino interaction.

This particular model explains most of the neutrino trimuon events, especially those where the secondary dimuon pair is correlated with the hadron-shower direction.³ We expect similar considerations to hold for muon induced trimuon events. The only question is one of input. Whereas the πp reaction has the appropriate quantum numbers for the neutrino reaction, in order to study muon reactions we need information on vector-meson production of dimuon pairs. This is precisely what will come out of the muon reactions. However, the present data on hadronic dimuon production by p , π^+ , and π^- beams indicate that the magnitude of the cross section and the dimuon mass distributions are very similar.⁸ Thus as a first approximation it seems reasonable to assume a similar

FIG. 1. Analogous reactions in neutrino and muon tri-muon production. (a) electromagnetic radiative production, (b) hadronic-final-state interaction.

cross section for vector-meson production. Even if the cross section does not turn out to be the same, the fact that several mechanisms will be present simultaneously means that distributions extracted from such a model will be helpful in understanding precisely how to isolate a particular signal. Thus it seems worthwhile to do such a calculation to see what kind of results are expected. Here we concentrate on the region of low secondary dimuon masses because that is where the cross section is largest. Modifications to examine the higher mass region can easily be made at a later date.

In Sec. II we discuss the details of the model we employ. In Sec. III we present in detail the various distributions we obtained and compare them to those calculated in I. We finally give our conclusions in Sec. IV.

II. DETAILS OF THE MODEL

As mentioned in the Introduction, the reaction is assumed to proceed in two independent stages. The initial stage in which the hadronic shower is produced, governed by the electromagnetic struc-

ture functions, and the final stage in which the hadronic shower produces two muons, which only depends on a universal decay function determined by the quantum numbers of the state.

In order to calculate the cross section for the process which is schematically indicated in Fig. 2(a), we proceed as follows. Since we are looking at states $\mu^+\mu^-X$ the hadron shower X' has the appropriate quantum numbers of this state. The matrix element for the reaction can be written as

$$\mathcal{M} = \bar{u}(p')\gamma_\mu u(p)(1/Q^2)\langle\mu^+\mu^-X|S|X'\rangle\langle X'|J^\mu|N\rangle, \quad (1)$$

where the factor $\langle\mu^+\mu^-X|S|X'\rangle$ represents the amplitude for the process $X' \rightarrow \mu^+\mu^-X$. Now the final-state decay is assumed to be independent of its mode of production. So we may use information obtained from reactions having the same quantum numbers as X' . The only restriction being that we use the results at the same kinematic configurations, in particular, the invariant mass W of the hadronic state must be identical with the c.m. energy $\sqrt{s}$ of the reaction [Fig. 2(b)]. This enables us to write

$$\langle\mu^+\mu^-X|S|X'\rangle\langle X'|S|\rho N\rangle = \langle\mu^+\mu^-X|S|\rho N\rangle. \quad (2)$$

Thus we obtain for the total cross section for the process $\mu^-N \rightarrow \mu^-\mu^+\mu^-X$ as

$$\sigma(\mu^-N \rightarrow \mu^-\mu^+\mu^-X) = \int \frac{d^2\sigma}{dx dy} \frac{1}{\sigma_{\text{tot}}} \frac{E d^4\sigma}{d^3p dm} dx dy dp^3 dm, \quad (3)$$

where the only link between the two cross sections is through the W dependence of the hadronic cross sections.

The muon deep-inelastic cross section for spin- $\frac{1}{2}$ partons, [i.e., $2F_1(x) = F_2(x)/x$; $\sigma_L/\sigma_T = 0$] has the form

$$\frac{d^2\sigma}{dx dy} = \frac{8\pi\alpha^2}{Q^4} ME[1 + (1-y)^2]F_2(x, Q^2). \quad (4)$$

We assume that the structure functions are determined by the quark-parton model with scaling violations expected from quantum chromodynamics

FIG. 2. (a) Schematic representation of the process $\mu^-N \rightarrow \mu^-\mu^+\mu^-X$. (b) Factorization of the processes into $\mu^-N \rightarrow \mu^-X$ and $\rho N \rightarrow \mu^+\mu^-X$. The kinematical variables used are shown too.

(QCD). On heavy targets such as iron the scale-dependent structure functions have been determined from neutrino scattering. We have in mind the recent CERN-Dortmund-Heidelberg-Saclay results.⁹ For our purpose it is sufficient to assume a simple polynomial fit in the variable

$$\bar{s} = \ln[(\ln Q^2/0.47)/\ln(10.64)].$$

Hence we assume

$$F_2(x, Q^2) = 3x^{\eta_1(\bar{s})}(1-x)^{\eta_2(\bar{s})}/D(\bar{s}) + A(\bar{s})(1-x)^{B(\bar{s})}, \quad (5)$$

where the parameters are

$$\begin{aligned} \eta_1(\bar{s}) &= 0.56 - 0.147\bar{s}, \\ \eta_2(\bar{s}) &= 2.71 + 0.813\bar{s}, \\ D(\bar{s}) &= 0.77 + 0.29\bar{s}, \\ A(\bar{s}) &= 0.99 + 1.71\bar{s}, \\ B(\bar{s}) &= 8.1 + 4.76\bar{s}, \end{aligned} \quad (6)$$

for the region above Q^2 equal to 5 GeV² and assume exact scaling below this value.¹⁰

Now we need information for ρN collisions, but data is not available. We note, however, that we expect the cross section to be similar to that in πp collisions. We assume a parametrization for isotropic decay into dimuons

$$E \frac{d^4\sigma}{dp^3 dm} = a(m) \{ \exp[-b(m)m_t] \} (1 - |x_F|)^{c(m)}, \quad (7)$$

with E , p , and m the energy momentum and mass of the dimuon pair, in the c.m. frame, where $m_t^2 = p_\perp^2 + m^2$ and $x_F = p_\parallel/p_{\max}$ is the usual Feynman scaling variable.⁵ The function $a(m)$ (units $\mu\text{b}/\text{GeV}^3$) includes some smoothly falling background, with Breit-Wigner terms superimposed for the (ρ , ω), ϕ , and J/ψ resonances. As most of the cross section comes from the low-mass part, we ignore the J/ψ at present. To be specific we assume these functions $a(m)$, $b(m)$, and $c(m)$ are the same as those in $\pi p \rightarrow \mu^+ \mu^- X$. When data are available they will probably have to be modified. Therefore we assume the fit based on the parametrization of Barger *et al.* in Ref. 5:

$$\begin{aligned} a(m) &= a_c + a_\rho \omega + a_\phi, \\ a_c &= 2.3 \times 10^4 \left[\left(\frac{m}{2m_\mu} \right) - 1 \right]^2 \exp \left[-3.2 \frac{m}{2m_\mu} \right], \\ a_\rho \omega &= 2m / [(m^2 - m_\rho^2)^2 + m_\rho^2 \Gamma_\rho^2], \\ a_\phi &= 0.15m [(m^2 - m_\phi^2)^2 + m_\phi^2 \Delta \Gamma^2], \\ b(m) &= 5.7, \\ c(m) &= 1 + (0.5/m)^2, \end{aligned}$$

m ranges from $2m_\mu$ to about 1.2 GeV. To normal-

TABLE I. Total cross section (in nb) for the process $\mu^- N \rightarrow \mu^- \mu^+ \mu^- X$ for a few energies and energy cuts.

Energy (GeV)	Energy cut (GeV)	4.5	10	15
200		2.4		
275		4.0	0.8	0.3
400		6.8		

ize the dimuon production to the total pion nucleon cross section we will divide by $\sigma^{\text{tot}}(\pi N) = 25$ mb.

One problem which immediately comes to mind is the question of the Pauli principle, namely, in an actual experiment how do we know which muon is which? Fortunately there is practically no overlap in phase space between the two like charge muons for the situation we consider here. The falloff in Q^2 in deep-inelastic muon scattering means that, at small Q^2 , the initial muon is very energetic and the hadron shower has very low mass. Therefore the secondary muons have very little energy in the laboratory. We will see these effects in the distributions in Sec. III. Of course the situation will be different if we impose drastic cuts to look at the larger- Q^2 region, so that a "leading" muon has to be defined experimentally.

III. RESULTS

We now give the results of our calculation. We consider in detail the distributions for a muon-beam laboratory energy of 275 GeV, which is approximately the energy used in the experiments. We assume a 4.5-GeV cut on each of the muon energies. When substantially different we present alongside the results with an additional cut $Q^2 > 0.1$

FIG. 3. Invariant-mass distribution for the slow muon pair M_{23} .

FIG. 4. Invariant-mass distribution of the fast and slow muon M_{12} and 3 muons M_{123} . (a) No Q^2 cut. (b) $Q^2 > 0.1$ GeV².

GeV². When appropriate we also compare them with the distributions for radiative Compton production calculated in I. Table I gives values for the total cross section at different energies and energy cuts. The cross section for a beam energy of 275 GeV with energy cut of 4.5 GeV and a Q^2 cut of 0.1 GeV² is 0.2 nb. That is a drop by a factor of 20. However, the signal is still large enough to be observable, and it is larger than the Compton cross section by a factor of 100. These, however, are to be considered only as very crude estimates since we lack an overall normalization. So we turn to the differential cross sections to see what features arise.

As in I we refer to the primary muon as 1 (fast μ^-), the secondary muons as 2 (slow μ^-) and 3 (μ^+). The invariant-mass distribution (Fig. 3) of the secondary dimuons reflects the input into the calculation. The Q^2 cut changes this distribution very

FIG. 5. Energy distribution of (a) slow muon (b) fast muon. Solid lines: no Q^2 cut. Dashed line: $Q^2 > 0.1$ GeV².

FIG. 6. Energy distribution of hadronic shower. Solid line: no Q^2 cut. Dashed line: $Q^2 > 0.1$ GeV².

little and thus is a rather distinct signal for this process. The other invariant-mass distributions are shown in Fig. 4. The Q^2 cut lowers the average value of M_{12} from 1.6 to 1.2 GeV and M_{123} from 2.5 to 1.9 GeV. That this is to be expected is best seen by observing that the Q^2 cut largely affects the fast muon, and all distributions involving this will be expected to change substantially.

Shown in Fig. 5 are the energy distributions. We note a resemblance to the Compton process where there is a very fast muon ($\langle E_1 \rangle = 195$ GeV) and slow secondary muons ($\langle E_2 \rangle = 13$ GeV). On imposing a Q^2 cut we note the rather dramatic change in the energy distribution of the fast muon while the distributions of the slow muon does not change appreciably. ($\langle E_1 \rangle = 140$ GeV; $\langle E_2 \rangle = 13$ GeV.) The hadron energy distributions are shown in Fig. 6. We consider next the distribution in Q^2 and W . We also show distributions in x and y where their definitions coincide with that in Eq. 3. We see that the average values for $\langle Q^2 \rangle = 0.07$ GeV² and $\langle x \rangle = 0.004$ (Figs. 7 and 8). These averages are very low which is why a small Q^2 cut (0.1 GeV²) causes such a severe drop in cross section. On the other

FIG. 7. Distribution in Q^2 .

FIG. 8. Distribution in x .

hand, $\langle W \rangle = 11.8$ GeV and $\langle y \rangle = 0.29$ (Figs. 9 and 10). These values are more reasonable. Indeed, a Q^2 cut gives higher averages $\langle W \rangle = 15.8$ GeV, $\langle y \rangle = 0.49$. Unlike Compton production the hadron invariant masses are rather high in this case. This should help distinguish the two processes.

The transverse momenta perpendicular to the direction of the incident beam are shown in Fig. 11(a) and the corresponding transverse momenta with respect to the virtual photon direction are given in Fig. 11(b). We see that the transverse momenta of the two secondary muons are smaller than the transverse momenta of the first muon when we look at the component along the direction of the virtual photon. The opposite is true for the muon beam direction. These results are consequences of small Q^2 and therefore small x typical in these reactions. Indeed, as a consequence of this we see that a Q^2 cut affects the fast muon distributions, whereas the slow muon distributions remain substantially the same (Fig. 11).

Now let us describe the distributions in the azimuthal angles between the $p_\perp$ vectors projected on the plane perpendicular to the beam direction.

FIG. 9. Distribution in W . Solid line: no Q^2 cut. Dashed line: $Q^2 > 0.1$ GeV 2 . FIG. 10. Distribution in y . Solid line: no Q^2 cut. Dashed line: $Q^2 > 0.1$ GeV 2 .

These particular distributions are very helpful in distinguishing the hadronic and electromagnetic components in neutrino trimuon production. Unfortunately, we now have such a small momentum transfer that there is not enough $p_\perp$ to distinguish clearly the incident muon and the hadron shower. Hence when the muons are emitted from the shower they do not show any peaking along the shower direction, so the ϕ distributions are essentially flat. The only one which shows a mild peaking is ϕ_{23} [Fig. 12(a)], because the muons are emitted back to back in the dimuon rest frame and there is still some memory of this fact when they are transformed to the laboratory frame. However, on imposing a Q^2 cut [Fig. 12(b)] we see a peaking in ϕ_{123} and ϕ_{12} . This is because the Q^2 cut leads us to observe events in which the primary muon has a lower energy. Thus there are greater correlations between the primary and secondary muons. We hope this information will be of use to experimenters.

FIG. 11. Transverse-momentum distributions perpendicular to (a) incident muon, (b) the virtual-photon direction. Solid line: no Q^2 cut. Dashed line: $Q^2 > 0.1$ GeV 2 .

FIG. 12. Distribution in the ϕ angles (a) no Q^2 cut. (b) $Q^2 > 0.1 \text{ GeV}^2$.

IV. CONCLUSIONS

The real motivation behind multimMuon studies is to try to detect signals due to charmed (or heavier) quark production and decay. Production mechanisms for such particles lead to final muons which are hadronlike in nature, i.e., they follow the hadron-shower direction. Hence it will be very difficult to see small effects when there is a naturally large source of low-energy muons. Our analysis (here and in I) will hopefully help elucidate the characteristics of trimuons produced via electromagnetic radiative production and production by hadronic-final-state interactions. The

presence of additional effects above the ones described will indicate the production of other heavier-quark states as $c\bar{c}$.

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¹At CERN there are two experiments, namely a CERN-Dubna-Munich-Saclay Collaboration, Expt. No. NA4, and a CERN-Hamburg-Freiburg-Kiel-Lancaster-Annecy-Liverpool-Munich-Orsay-Oxford-Rutherford-Sheffield-Shrivenham-Turin-Wuppertal Collaboration, Expt. No. NA9. At Fermilab there is experiment No. E203/391, by a Berkeley and Princeton Collaboration. Previous results on multimMuon physics have been published by C. Chang, K. W. Chen, and A. van Ginneken, Phys. Rev. Lett. **39**, 519 (1977); see also K. W. Chen, in *Proceedings of the International Symposium on Lepton and Photon Interactions at High Energies, Hamburg 1977*, edited by F. Gutbrod (DESY, Hamburg, 1977), p. 467.

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¹⁰This is essentially the same fit advocated by A. J. Buras and K. J. F. Gaemers in Nucl. Phys. B **132**, 249 (1978). However, to avoid evaluating beta functions at

every point, we normalize using a polynomial $D(\bar{s})$.

¹¹A model for multimMuon production which involves the production and decay of charmed particles, and is similar in spirit to the model here, has been investigated by F. Bletzacker, H. T. Nieh, and A. Soni, Phys. Rev. Lett. 38, 1241 (1977); see also F. Bletzacker and

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