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SOME REMARKS ON THE HIGH ENERGY BEHAVIOUR OF
INCLUSIVE DIFFERENTIAL CROSS-SECTIONS IN QCD

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ABSTRACT

Differential cross-sections in QCD of quasi-exclusive processes, in the sense as introduced by Block and Nordsieck, are shown by an explicit perturbative calculation in lowest order to go as $d\sigma/dt \sim s^2$ at fixed t . This phenomenon is entirely traced back to the emission of massless quanta.

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Perturbative calculations in QCD of leading infra-red behaviour of inclusive differential cross-sections are supporting the growing belief that in the infra-red QCD behaves in much the same manner as QED in the sense that although in a less trivial way and much more complicated manner the infra-red singularities exponentiate and drop out in suitably defined differential cross-sections ¹⁾. Much less is known about the non-leading infra-red singularities ²⁾.

We will adopt the attitude that if confinement of quarks and gluons has its origin in an infra-red mechanism, it should come from perturbative untractable infra-red singularities.

As a matter of fact, people are already looking at a much more stringent cancellation scheme in trying to relate QCD to the phenomena of jet structure in hadron production ³⁾. However, and unlike in QED, the addition of bremsstrahlung processes in QCD might hold some peculiar implications which reflect the intrinsic non-Abelian character of the theory.

In this respect, let us focus our attention on the finite part of differential cross-sections in QCD, and in this note we will deal with the asymptotic high energy behaviour of some inclusive processes. Let us recall, for the good order, in what respect QCD is found to deviate from QED.

First, the emission of gluons does not, unlike in QED, exhibit a Poisson distribution ⁴⁾ reflecting independent emission of soft quarks, on the contrary the emission of gluons occurs to be correlated even in leading infra-red calculations. As a consequence it is likely to be the case that the differential cross-section exhibits a cut structure starting not at elastic threshold but at a displaced point in the s plane. Secondly, the analytic structure is rather more complicated than the one in QED ⁵⁾. To be more definite, end-point infra-red singularities like the Coulomb peak $1/t^2$ in a differential cross-section which involves exchanges and/or emission of massless quanta are enhanced by factors of the type $\ln(-t)$. These terms might bend the static potential into a confining potential. Processes involving only the scattering of colour singlet states are expected not to have these properties ⁶⁾.

In this note we shall exhibit a much more troublesome and unexpected feature of QCD, namely its high s behaviour. In this respect we calculated the finite part of the quasi-elastic quark-quark scattering, up to $\alpha_s^3(u)$, where u is the normalization point, and $\alpha_s = g^2/4\pi$. Avoiding technicalities, let us however state the important premises of the derivation.

It is important to stress the fact that in order to get rid of infra-red singularities, emission processes have to be taken into account. We have confined ourselves to a small region of t near the forward direction ($0 < -t \ll m^2$) neglecting terms which modulo $1/t^2$ vanish as $t \rightarrow 0$. We integrate covariantly, this means over full phase space. However, for exceptional momenta $t \approx 0$ this still implies the emitted gluons to be soft. Indeed for a configuration like in Fig. 2, the minimum value of $-t$ is given by

$$\left(\frac{-t}{m^2} \right)_{\min} = \frac{\epsilon^2}{s^2} \quad (1)$$

therefore, confining ourselves to small values of t implies the emission of soft gluons.

We mention also some less essential premises. We used an $SU(N)$ invariant Lagrangian where we have put the quarks in the fundamental representation. A summation over all colours in the final states and an average of colours of the initial state has been performed.

The differential cross-section then reads (disregarding $\ln -t$ terms) :

$$\frac{d\sigma}{dt} \Big|_{0 < -t \ll m^2} = \left(\frac{d\sigma}{dt} \right)_0 \left\{ 1 + \frac{\alpha_s C_{YM}}{\pi} \frac{6 p_2^2}{m^2} \right\} \quad (2)$$

where

$$\left(\frac{d\sigma}{dt} \right)_0 = \frac{2\pi\alpha_s^2 C_F}{N} \cdot \frac{1}{t^2} \cdot \frac{E_2^2}{p_2^2} \quad (3)$$

$$E_2^2 = \left(\frac{\Delta - 2m^2}{2m} \right)^2$$

$$p_2^2 = \frac{\Delta(\Delta - 4m^2)}{4m^2}$$

C_F, C_{YM} = casimir of fundamental (adjoint) representation.

Therefore

$$\left(\frac{d\sigma}{dt}\right)_{t=0} \xrightarrow{s \rightarrow \infty} \frac{\alpha_s^3}{m^4} \frac{C_F C_{YM}^2}{N^2} \frac{3s^2}{t^2} \quad (4)$$

For a massive theory, such an s behaviour would spoil the Froissart bound in a catastrophic way. However, for massless theories, such a bound may or may not exist, although QED does not spoil the Froissart bound. Let us state some reasons why this result might not be so dramatic as it looks. First, the result is only valid for a small region of t around the forward direction where everything is dominated by the Coulomb peak and the $\log(-t)$ enhancement factors in such a way that the s behaviour might be irrelevant. However, we will state that also for finite t the result holds.

Furthermore, this typical s behaviour comes entirely from the bremsstrahlung processes. Therefore these mass singularities will not be cancelled by considering degenerate incoming states in the limit where $m \rightarrow 0$.

Finally, it is a perturbative result, and we have no idea about a summed over all orders expression (one of the modifications would be the replacement of α_s by the running coupling constant).

Keeping these remarks in mind, we forced ourselves to calculate the high s behaviour of the emission processes for a finite value of t , to be definite we took the region where

$$s \gg -t > m^2$$

Introducing some dimensionless variables

$$s_R = \frac{s}{m^2} \quad , \quad t_R = \frac{-t}{m^2}$$

the result of a long and tedious calculation can be summarized as follows. We calculated for $s_R(d\sigma/dt)_{BR}$ (for $s_R \rightarrow \infty$ and $1 < t_R \ll s_R$) all terms of the form

$$\left\{ \frac{C_{YM}^2}{C_F^2} \right\} \left(\frac{s_R}{t_R} \right)^n \log s_R \quad , \quad \left\{ \frac{C_{YM}^2}{C_F^2} \right\} \left(\frac{s_R}{t_R} \right)^n \log t_R \quad , \quad \left\{ \frac{C_{YM}^2}{C_F^2} \right\} \left(\frac{s_R}{t_R} \right)^n$$

where n took on the values two and three. The brackets are meant to distinguish between terms which have a non-Abelian and an Abelian origin. (We did not calculate terms with $n=0$ or 1 for all diagrams). We noticed that the first two sets dropped out completely for $n=3$. Also terms like $C_F^2(s_R/t_R)^n$ cancelled, leaving us with an expression of non-Abelian origin which is given by (disregarding the infra-red divergent terms)

$$\left(\frac{d\sigma}{dt}\right)_{BR} \xrightarrow[1 < t_R \ll s_R]{s_R \rightarrow \infty} \frac{\alpha_s^3}{m^4} \frac{C_F C_{YM}^2}{N^2} \frac{13}{24} \frac{s_R^2}{t_R^3} \quad (5)$$

Therefore an s^2 behaviour seems to be a general feature for emission processes in QCD. Of course since t_R is finite, the emitted gluons are allowed to be less soft than in the foregoing case and therefore $(d\sigma/dt)_{BR}$ may not be added to the elastic one for all experimental set-ups.

What might be the significance of such a power behaviour (which is against all intuitive ideas of confinement) ? As indicated by Dolgov et al. ⁷⁾ such a power behaviour might mean the appearance of a new type of t singularities besides the threshold ($t=0$) singularities, which they called shrinkage singularities. These conclusions were derived for exclusive processes using dispersion relation techniques. On the other hand, as we stressed earlier, one needs to consider quasi-exclusive processes to handle the infra-red behaviour, implying a modification of the threshold singularity at $t=0$ ⁵⁾, this might as well imply the appearance of other singularities for which there is no direct signal because of the lack of a non-perturbative treatment.

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$$s = (p_1 + p_2)^2$$

$$t = (p_1 - p_3)^2$$

FIG.1

$$v = p_4' + k$$

$$v^2 = m^2 + \epsilon$$

FIG.2

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