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A FIT OF UPSILON AND CHARMONIUM SPECTRA

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A B S T R A C T

We show that upilon and charmonium spectra can be fitted by a common potential of the form $A + Br^\alpha$, with α close to 0.1. In particular the mass difference of T''' and T is reproduced within 10 MeV. In this way it is confirmed that for the $b\bar{b}$ and $c\bar{c}$ systems the relevant part of the potential is flavour-independent and approximately though not exactly logarithmic. At the same time the claims, according to which the short-distance part of the potential could be shown to agree with the predictions of QCD by a comparison of charmonium and upilon spectra, are put in serious doubt.

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1. - INTRODUCTION

Recently the spectacular results of the CLEO ¹⁾ and CUSB ²⁾ experimental groups at the CESR machine, shortly after the not less spectacular start of the machine itself, have confirmed the idea that the T system is a bound state of a pair of heavy non-relativistic quarks.

There have been, in the past, numerous fits of the charmonium and/or upilon spectra with various potentials, some of which being inspired by theoretical prejudices, for instance those containing in the short-range part the effect of one gluon exchange ³⁾, some much more crude, like $V = A + Br^\alpha$. In the latter category we find in particular the fits by Quigg and Rosner ⁴⁾ which point towards a value of α which is close to zero [which corresponds to a logarithmic potential because $\lim(r^\epsilon - 1)/\epsilon = \log r$], but positive. The advantage of that kind of fit is that it is simple and that one has simple properties of scaling with respect to the quark mass. The other advantage is that, if it is successful, it allows to appreciate the statements according to which the comparison of the $b\bar{b}$ and $c\bar{c}$ spectra gives evidence for a short-range part of the potential behaving as predicted by QCD, i.e., $V \sim (-8\pi/27) r^{-1} |\log r/r_0|^{-1}$. There are in fact two predictions of QCD. One is the short-range potential, which, as we shall see, is not yet clearly exhibited. The second is the approximate flavour independence of the potential, which, on the contrary, will be seen to be very well tested.

The reason for repeating a fit with a power-like potential is that new and very accurate data have been accumulated on the T system on the presumably $L=0$ $J=1$ states $1S, 2S, 3S$ ^{1),2)} and $4S$ ²⁾, and that these states considered as $b\bar{b}$ bound states, are highly non-relativistic, as opposed to charmonium (the charmed quark mass may be as low as 1.08 GeV).

The non-relativistic character of the upilon can be seen by using the following information :

- 1) from the Quigg-Rosner fit $\alpha > 0$ we know that the kinetic energy of the T system is less than the kinetic energy of the charmonium ;
- 2) the kinetic energy of the $J\psi$ can be estimated, in a model-independent way to be ~ 0.36 GeV by an argument of Bertlmann and I ⁵⁾ ;
- 3) for

$$m_c > 1.08 \text{ GeV} , \quad (1)$$

we also get a model-independent lower limit for m_b which is

$$m_b > 4.49 \text{ GeV} \quad (2)$$

Hence in the T ground state we have

$$\frac{v^2}{c^2} < 0.08. \quad (3)$$

Another argument in favour of the T is that there are more "stable" states and the coupling to open channels is less important.

The strategy will be therefore to try to get the T potential first, and see then if it is compatible with the $c\bar{c}$ spectrum. However, for the best fit, we have to eliminate the hyperfine effects for which we know only one number, the $J/\psi - \eta_c$ mass difference which is ⁶⁾:

$$3.095 - 2.983 = 0.112 \text{ GeV}. \quad (4)$$

In a first step we estimate α by using the WKB approximation. This value of α is then used to scale the hyperfine splittings according to the law

$$M_{\text{Triplet}} - M_{\text{Singlet}} = C \frac{|\psi(0)|^2}{M_V^2} \quad (5)$$

where M_V is the vector meson mass.

Then the mean masses ($1/4$ singlet + $3/4$ triplet) of the states are obtained and from them a final potential determined and confronted with both spectra. Here, we use the exact solution of the Schrödinger equation. Finally the quark mass values are discussed.

2. - WKB FIT OF THE SPECTRA

For a power potential the WKB approximation has the advantage of giving a particularly transparent expression for the energy levels and showing explicitly what can and what cannot be obtained from the knowledge of the

energy levels. It has also the advantage of being relatively accurate (a few per cent for the ground state already), but as we shall see, not enough for our best fit.

We have, for the $\ell = 0$ n states

$$M(n, m_q) = 2m_q + A + C_\alpha B^{1 - \frac{\alpha}{\alpha+2}} m_q^{-\frac{\alpha}{\alpha+2}} \left(n - \frac{1}{4}\right)^{\frac{2\alpha}{\alpha+2}}, \quad (6)$$

with

$$V = A + B r^\alpha, \quad \alpha > 0 \quad (7)$$

and where C_α depends only on α .

On (6) we see that there is no hope to get the quark mass from the spectrum of a single $Q\bar{Q}$ system. A change of the quark mass can be accommodated by a change of A and B . On the other hand, a single spectrum will overdetermine α . The scaling properties with respect to m_q , valid for the energy differences, hold in fact rigorously, without making the WKB approximation.

In principle the masses of the quarks can be obtained from the comparison of two systems : once α is determined and found to be the same for both systems, one gets the ratio of the quark masses from

$$\frac{(E_{2s} - E_{1s})_{b\bar{b}}}{(E_{2s} - E_{1s})_{c\bar{c}}} = \left(\frac{m_c}{m_b}\right)^{\frac{\alpha}{\alpha+2}}. \quad (8)$$

However, for α small, this determination is highly unstable and sensitive to small errors in the energy differences. From the knowledge of $2m_b + A$ and $2m_c + A$, obtained from separate spectra, $m_b - m_c$ is fixed and hence m_b and m_c . Naive application of WKB approximation gives for the T system 4)

$$\frac{E_{3s} - E_{1s}}{E_{2s} - E_{1s}} = \frac{0.8911}{0.5607} = \frac{(11)^{\frac{2\alpha}{\alpha+2}} - (3)^{\frac{2\alpha}{\alpha+2}}}{(7)^{\frac{2\alpha}{\alpha+2}} - (3)^{\frac{2\alpha}{\alpha+2}}} \quad (9)$$

In this way we get

$$\alpha_{WKB, r} = 0.157 \quad (10)$$

and, hence "predict,"

$$E_{3S} - E_{1S} = (E_{3S} - E_{1S}) \frac{(15)^{\frac{2\alpha}{\alpha+2}} - 3^{\frac{2\alpha}{\alpha+2}}}{7^{\frac{2\alpha}{\alpha+2}} - 3^{\frac{2\alpha}{\alpha+2}}} = 1.131 \text{ GeV} \quad (11)$$

Experiment gives us, using the mass scale of CESR (not necessarily accurate in absolute value) ²⁾

$$E_{4S} - E_{1S} = 10.546 - 9.434 = 1.112 \text{ GeV} \quad (12)$$

This is already not too bad ! We can predict then the energy of the 5S state (hoping that its total width is sufficiently small to allow its observation !)

$$E_{5S}(\text{CESR SCALE}) = 10.736 \text{ GeV} \quad (13)$$

(Here we have used the experimental value of E_{4S}).

If we try to determine α from the $c\bar{c}$ system we have to use the 1S state (3.095 GeV) 2S state (3.684) and 3S state (4.030 GeV), the latter being pretty broad and high above the $D\bar{D}$ threshold (3.74 GeV). The analogue of Eq. (9) gives

$$\alpha_{WKB, c\bar{c}} = 0.151 \quad (14)$$

This represents what we may consider a too good agreement with (10). We must remember that the 3S state of $c\bar{c}$ is very broad and that the hyperfine splitting $J/\psi - \eta_c$ is of the order of 100 MeV. Attempts to predict the 4S $c\bar{c}$ state give 4.28 GeV instead of 4.4 GeV but we believe that this was to be expected, owing to the width of the 4.4 state, its coupling to open channels, its large kinetic energy. Also it may be that the distant confining part is felt.

We do not push further this simple approach and estimate, in the next section, hyperfine effects.

3. - HYPERFINE EFFECTS AND MASSES OF "SPINLESS" $Q\bar{Q}$ STATES

Here what we have to do is rather dirty but cannot be avoided if we want to go further. We argue that the spin-dependent effect can be removed by considering the combination

$$\frac{3}{4} M(\text{Triplet}) + \frac{1}{4} M(\text{Singlet})$$

i.e.,

$$M = M_{\text{Triplet}} - \frac{1}{4} (M_{\text{Triplet}} - M_{\text{Singlet}}) \quad (15)$$

Furthermore we decide that we have

$$M_{\text{Triplet}} - M_{\text{Singlet}} = C \times \frac{|\psi_n(0)|^2}{m_q^2} \quad (16)$$

What is secretly behind this is that the hyperfine splitting comes from a potential $C(\Delta V_g/m_q^2)$, V_g being a one-gluon exchange potential. However (16) might be more general, We adjust C to the $J/\psi - \eta_c$ mass difference and use WKB for $\psi_n(0)$. We then get

$$\Delta M(\text{Triplet} - \text{Singlet}, n, m_q) = 0.112 \text{ GeV} \times \left(\frac{m_q}{m_c} \right)^{\frac{-1-2\alpha}{2+\alpha}} \left(\frac{4n-1}{3} \right)^{2\frac{\alpha-1}{\alpha+2}} \quad (17)$$

in which we take $\alpha = 0.16$, obtained from the considerations of Section 2.

There is, of course, an uncertainty on m_b/m_c which will be obtained only at the end of the fit. However, since α is close to zero this problem is not as serious as it looks, and changing m_b/m_c from 3 to 4 changes the mean mass of the $b\bar{b}$ 1S state by only 2 MeV. We decide to take $m_b/m_c = m_T/m_{J\psi}$ in Eq. (17). Then we get the following values from the energy differences :

$c\bar{c}$	$b\bar{b}$
$E_{2S} - E_{1S} = 0.6025 \text{ GeV}$	$E_{2S} - E_{1S} = 0.5675 \text{ GeV}$
$E_{3S} - E_{1S} = 0.952 \text{ GeV}$	$E_{3S} - E_{1S} = 0.900 \text{ GeV}$
$E_{4S} - E_{1S} = 1.339 \text{ GeV}$	$E_{4S} - E_{1S} = 1.122 \text{ GeV}$

(18)

Furthermore we have

$M_{1S}(c\bar{c}) = 3.067$	$M_{1S}(b\bar{b}) = 9.420 \text{ (Cornell scale)}$
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3. - BEST DETERMINATION OF α AND CONCLUSIONS

The simplest thing to do is to use the universal graphs of Quigg and Rosner for $(E_{3S}-E_{1S})/(E_{2S}-E_{1S})$. We have

$$\rho = \frac{E_{3S} - E_{1S}}{E_{2S} - E_{1S}} = 1.551 \quad \text{for } V = \log r \quad (19)$$

and

$$\frac{d\rho}{d\alpha} = 0.28 \quad (20)$$

We get therefore from Table (18)

$\alpha_{c\bar{c}} = 0.104$	$\alpha_{b\bar{b}} = 0.125$
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(21)

If we use these values of α , we can get a value for the quark mass ratio from

$$\frac{(E_{2S} - E_{1S})_{b\bar{b}}}{(E_{2S} - E_{1S})_{c\bar{c}}} = \left(\frac{m_b}{m_c} \right)^{-\frac{\alpha}{\alpha+2}}$$

we get

$$\frac{m_b}{m_c} = \begin{cases} 2.92 & \text{for } \alpha = 0.125 \\ 3.35 & \text{for } \alpha = 0.104 \end{cases} \quad (22)$$

This gives an idea of the lack of stability of the process, but it shows that the assumption $(m_b/m_c) \simeq m_T/m_{J\psi}$ used to scale the hyperfine structure effects was not grossly wrong. We could iterate the process and recalculate the hyperfine splittings with $\alpha = 0.125$, however, if we do that we get a change of less than 1 MeV on all energy differences.

The question is now what is the best value of α . In principle we prefer the one coming from the T. However, the present experiments quote systematic errors of ± 3 MeV on $E_{3S} - E_{1S}$ and $E_{2S} - E_{1S}$. Even if these errors are correlated this gives sufficient flexibility to the ratio $(E_{3S} - E_{1S})/(E_{2S} - E_{1S})$. It happens that if we take $\alpha = 0.125$ and use the values of Table (18) for $E_{2S} - E_{1S}$, we find

$$E_{4S} - E_{1S} = 1.141 \text{ GeV} \quad (23)$$

19 MeV above the "experimental" number.

If we take

$$\alpha = 0.104 \quad (24)$$

and use $E_{2S} - E_{1S} = 0.569$ GeV, compatible with errors we get $E_{3S} - E_{1S} = 0.899$ GeV, also perfectly compatible with errors, and

$$E_{4S} - E_{1S} = 1.131 \text{ GeV} \quad (25)$$

only 9 MeV above the "experimental" number. We believe that a shift appreciably smaller than the total width of the state (~ 24 MeV) is acceptable.

Naturally $\alpha = 0.104$ is also quite compatible with the $3S$ state of $c\bar{c}$ which is wide, and with the P state mean $c\bar{c}$ mass, 3.52 GeV. It leads to

$$\frac{m_b}{m_c} = 3.35 \quad (26)$$

It is also possible to get $m_b - m_c$, by using the equation

$$E_{1S}(m_b) - E_{1S}(m_c) = - \int_{m_c}^{m_b} \frac{T(m)}{m} dm, \quad (27)$$

where $T(m)$ is the kinetic energy of the 1S state.

For $T(m_c)$, with $\alpha = 0.104$ we have

$$T = 0.357 \text{ GeV} \quad (28)$$

and

$$T(m) = \left(\frac{m_c}{m} \right)^{\frac{\alpha}{2+\alpha}} T(m_c) \quad (29)$$

from which one gets, using (26)

$$E(m_c) - E(m_b) = 0.4195 \text{ GeV}$$

and hence

$$m_b - m_c = \frac{1}{2} \left(M_{b\bar{b}}(\text{deq}) - M_{c\bar{c}} + E(m_c) - E(m_b) \right) = 3.41 \text{ GeV} \quad (30)$$

which is within the allowed limits ⁵⁾ and finally, using (26)

$$\alpha = 0.104, \quad \left. \begin{array}{l} m_c = 1.45 \text{ GeV} \\ m_b = 4.86 \text{ GeV} \end{array} \right\} \quad (31)$$

However, we must stress that (26) and (31) are rather unstable :
 $\alpha = 0.125$ gives

$$\begin{aligned}
 m_b - m_c &= 3.38 \text{ GeV} \\
 m_c &= 1.76 \text{ GeV} \\
 m_b &= 5.14 \text{ GeV}
 \end{aligned}
 \tag{24}$$

As expected ⁵⁾ the quark mass difference is rather stable, but the masses themselves, especially the charm mass are not. The only way to fix the charm mass accurately is to decide to believe the absolute magnitude of the leptonic width given by the Van Royen-Weisskopf formula. This leads to a small charmed quark mass. However, the formula is subject to large but not quite certain corrections. ⁷⁾

We notice that the largest allowed value for $m_b - m_c$ leads to a prediction of the B meson mass

$$1.87 + 3.41 = 5.28 \text{ GeV},$$

which would place the T''' 20 MeV above the threshold for $B\bar{B}$ production.

We want now to make a few simple predictions rather insensitive to the particular α chosen.

- 1) The $J=1$, $5S$ T state is predicted to have a mass

$$10.716 \text{ GeV (CESR scale)}$$

with a guessed uncertainty of ± 25 MeV. It may or may not be seen depending on its total width.

- 2) The $1P$, T state mean mass is around

$$9.78 \text{ GeV (CESR scale)}$$

and the $2P$, T state mass around

$$10.21 \text{ GeV (CESR scale)}.$$

Now what can we conclude from this fit ? It is reasonable and tests flavour independence in the sense that three independent determinations of α lead to the same result : the fit of the charmonium spectrum,

the fit of the upsilon spectrum, the comparison of the level spacings of the two spectra with a reasonable quark mass ratio. The absolute magnitude of the quark masses is still rather vague, while the quark mass difference is pretty well fixed, as expected. One could, of course, fix these masses by using the Van Royen-Weisskopf formula for the leptonic widths. However, this formula should not be too much trusted in absolute value, as it is affected by various corrections ⁷⁾.

Another conclusion is that it is difficult to deduce from experimental data on charmonium and upsilon the short-range behaviour of the potential since the present potential is completely different from what one expects from QCD, which is $-\text{const} \times r^{-1}$, or in the more refined versions $-\text{const} r^{-1} |\log(r/r_0)|^{-1}$. All one can say is that the data point towards a potential less singular than Coulomb. The question of the short-range can only be settled by looking at the $t\bar{t}$ spectrum. Concerning the long-range confining potential we are still more pessimistic. It seems that it is essentially impossible to get it from experiment. The only possible indication for a confining potential rising faster than $r^{0.1}$ might be the discrepancy observed for the probable 4S state of $c\bar{c}$ at 4.414 GeV. This discrepancy, however, is not terribly meaningful since the state is high above the thresholds for $D\bar{D}$, D^*D^{-*} , $F\bar{F}$, F^*F^{-*} .

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